Homework 4

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January 31, 2019

Problem 1

```
#Ryan Branagan
#Collaborators: N/A
#Branagan_hw4_p1.py
#1/29/19

import numpy as np

#Define the function we are trying to find the numerical derivative of
def costanh(x):
    return np.cos(x)*np.tanh(x)

#Parameters
f=costanh
x=2.
h=10**-4
ND3X = 1.231239865696003
#ND3X = 'No'

#Define the Central Difference
def CenDiffD(f,x,h,ND3X):
    if type(ND3X) == str:
        return (f(x+h)-f(x-h))/(2*h)
    if not type(ND3X) == str:
        x = (f(x+h)-f(x-h))/(2*h)
        y = (1/6)*ND3X*(h**2)
        return np.array([x,y])

print(CenDiffD(f,x,h,ND3X))
print(h**2)
```

This problem was pretty straight forward. I used the given formula to define the central difference and the error. Using Mathematica I calculated the third derivative evaluated at the given point in order to calculate error. Squaring h gives a value proportional to my calculated error. I wrote my code in a way that the error is calculated at the same as the numerical derivative but is optional. If ND3X is any string then the error will not be calculated. If needed this could be run through a loop to make a list of answers for different values of h and the error involved in each one all at the same time. For plotting you could use different methods for separating the values and error into two separate lists.

Problem 2

```
#Ryan Branagan
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#Branagan_hw4_p2.py
#1/31/19

import numpy as np

def costanh(x):
    return np.cos(x)*np.tanh(x)

f = costanh
x = 2
h = 10**-2
ND5X = 0.502625754665

#Define the Central Difference
def Cen5S(f,x,h,ND5X):
    if type(ND5X) == str:
        return (f(x-2*h)-8*f(x-h)+8*f(x+h)-f(x+2*h))/(12*h)
    if not type(ND5X) == str:
        x = (f(x-2*h)-8*f(x-h)+8*f(x+h)-f(x+2*h))/(12*h)
        y = -(1/30)*ND5X*(h**4)
        return np.array([x,y])

print(Cen5S(f,x,h,ND5X))
print(h**4)
```

To complete this problem I used a combination of Mathematica and hand calculations. Since I put in a lot of effort I'm going to include a scanned copy of my work. The results of all this though are expressions for the first derivative and the error. Modifying the code from Problem 1, this given the same answer. It does however give a more precise error with smaller h. The error is proportional to h to the fourth.

$$A\quad f(x-2h) = f(x) + \frac{f'(x)}{1!}(-2h)^1 + \frac{f''(x)}{2!}(-2h)^2 + \frac{f'''(x)}{3!}(-2h)^3 + \frac{f''''(x)}{4!}(-2h)^4$$
$$= \boxed{f(x)}\boxed{-2f'(x)h}\boxed{+2f''(x)h^2}\boxed{-\tfrac{4}{3}f'''(x)h^3}\boxed{+\tfrac{4}{6}f''''(x)h^4}$$

$$B\quad f(x-h) = f(x) + \frac{f'(x)}{1!}(-h)^1 + \frac{f''(x)}{2!}(-h)^2 + \frac{f'''(x)}{3!}(-h)^3 + \frac{f''''(x)}{4!}(-h)^4$$
$$= \boxed{f(x)}\boxed{-f'(x)h}\boxed{+\tfrac{1}{2}f''(x)h^2}\boxed{-\tfrac{1}{6}f'''(x)h^3}\boxed{+\tfrac{1}{24}f''''(x)h^4}$$

$$C\quad f(x) = \boxed{f(x)}$$

$$D\quad f(x+h) = f(x) + \frac{f'(x)}{1!}(h)^1 + \frac{f''(x)}{2!}(h)^2 + \frac{f'''(x)}{3!}(h)^3 + \frac{f''''(x)}{4!}(h)^4$$
$$= \boxed{f(x)}\boxed{+f'(x)h}\boxed{+\tfrac{1}{2}f''(x)h^2}\boxed{+\tfrac{1}{6}f'''(x)h^3}\boxed{+\tfrac{1}{24}f''''(x)h^4}$$

$$F\quad f(x+2h) = \boxed{f(x)}\boxed{+2f'(x)h}\boxed{+2f''(x)h^2}\boxed{+\tfrac{4}{3}f'''(x)h^3}\boxed{+\tfrac{4}{6}f''''(x)h^4}$$

$$f(x-2h) + f(x-h) + f(x) + f(x+h) + f(x+2h) =$$
$$(A+B+C+D+F)f(x) + (-2A-B+D+2F)f'(x)h +$$
$$(2A+\tfrac{1}{2}B+\tfrac{1}{2}D+2F)f''(x)h^2 + (-\tfrac{4}{3}A-\tfrac{1}{6}B+\tfrac{1}{6}D+\tfrac{4}{3}F)f'''(x)h^3 +$$
$$(\tfrac{4}{6}A+\tfrac{1}{24}B+\tfrac{1}{24}D+\tfrac{4}{6}F)f''''(x)h^4$$

$$A+B+C+D+F=0 \qquad A=\frac{1}{12h}$$
$$(-2A-B+D+2F)=\frac{1}{h} \qquad B=-\frac{2}{3h}$$
$$(2A+\tfrac{1}{2}B+\tfrac{1}{2}D+2F)=0 \qquad C=0$$
$$(-\tfrac{4}{3}A-\tfrac{1}{6}B+\tfrac{1}{6}D+\tfrac{4}{3}F)=0 \qquad D=\frac{2}{3h}$$
$$(\tfrac{4}{6}A+\tfrac{1}{24}B+\tfrac{1}{24}D+\tfrac{4}{6}F)=0 \qquad F=-\frac{1}{12h}$$

Error

$$\frac{f'''''(x)}{5!}(-2h)^5 = -\frac{4}{15}f^5(x)h^5$$
$$\frac{f'''''(x)}{5!}(-h)^5 = -\frac{1}{120}f^5(x)h^5$$
$$\frac{f'''''(x)}{5!}(h)^5 = \frac{1}{120}f^5(x)h^5$$
$$\frac{f'''''(x)}{5!}(2h)^5 = \frac{4}{15}f^5(x)h^5$$
$$Err = (-\tfrac{4}{15}A-\tfrac{1}{120}B+\tfrac{1}{120}D+\tfrac{4}{15}F)f^5(x)h^5$$

$$Af(x-2h) + Bf(x-h) + Cf(x) + Df(x+h) + Ff(x+2h) =$$
$$\frac{f(x-2h)}{12h} - \frac{2f(x-h)}{3h} + \frac{2f(x+h)}{3h} - \frac{f(x+2h)}{12h} = \frac{f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)}{12h}$$

$$Error = -\frac{1}{30h}f^5(x)h^5 = -\frac{1}{30}f^5(x)h^4$$

Figure 1: A lot of Taylor Expanding and Algebra.

Problem 3

```
#Ryan Branagan
#Collaborators: N/A
#Branagan_hw4_p3.py
#1/30/19

import numpy as np
import pylab as p

def lncosh(x):
    return np.log(x)/np.cosh(x)

f = lncosh
h = 10**-2
a = 2
b = 5
points = 1000

#Second Derivative
def d2(f,x,h):
    return (f(x+h)-2*f(x)+f(x-h))/(h**2)

#Array of xs
xs = np.linspace(a,b,points)

#Array of answers
ans = np.array([])
for x in xs:
    ans = np.append(ans,d2(lncosh,x,h))

#Plotting
p.plot(xs,ans,'o')
p.show()
```

I used the given formula to create a plot of the second derivative of the given function. I checked Desmos and my result matches Desmos.

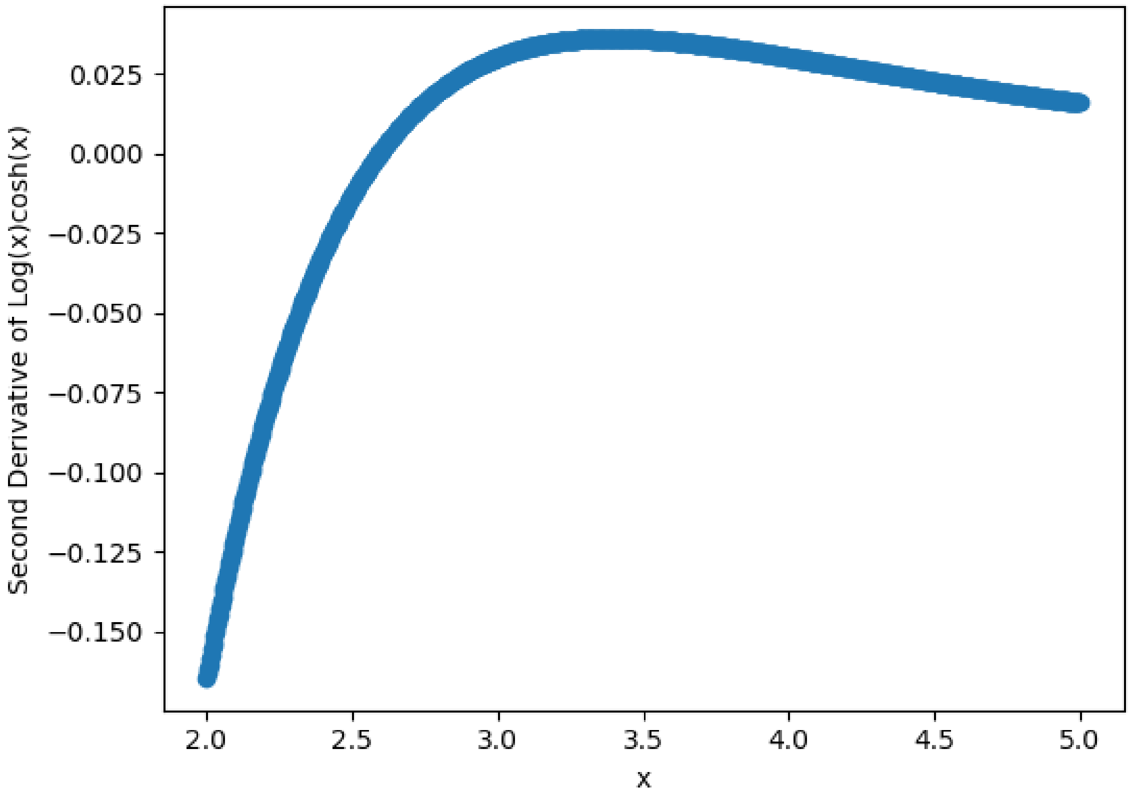

Figure 2: The graph of the second derivative of ln(x)sech(x).