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Detecting Failures in Granular Materials via Nonaffine Displacements

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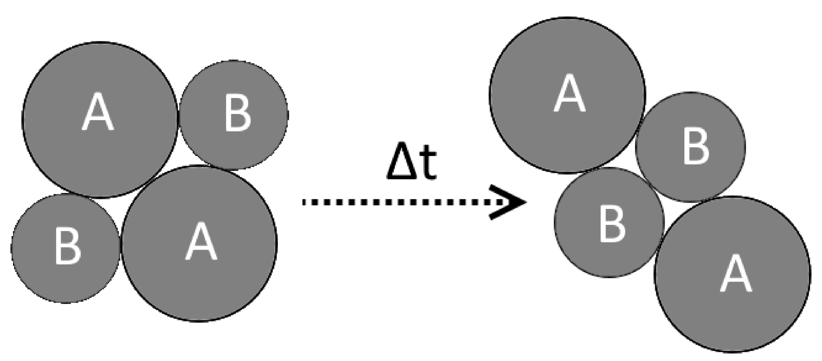
Context

- Detecting and ultimately being able to forecast failures within granular materials presents the opportunity to safeguard many structures as well as provide more efficient methods to processes that use granular materials.
- Experimenting on 2-dimensional photoelastic disks gives an idealized system to study failures



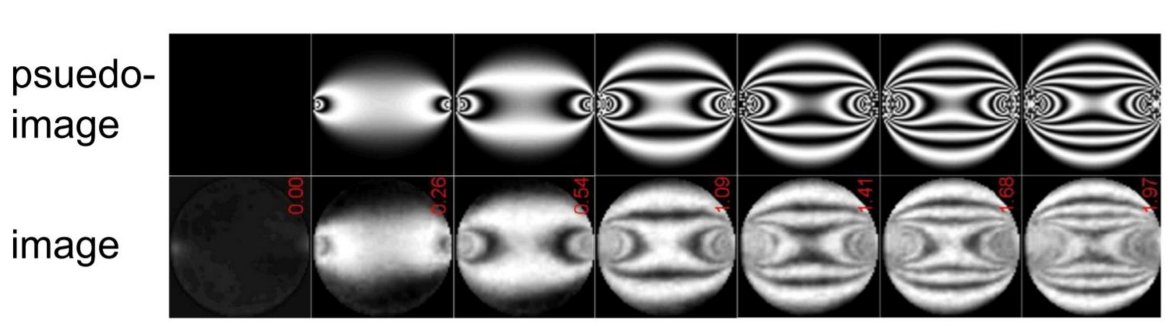
An example of a failure event for a four circular grains with exaggerated radii. Note that without a complete inversion of the forces, the particles will not return to their original positions.

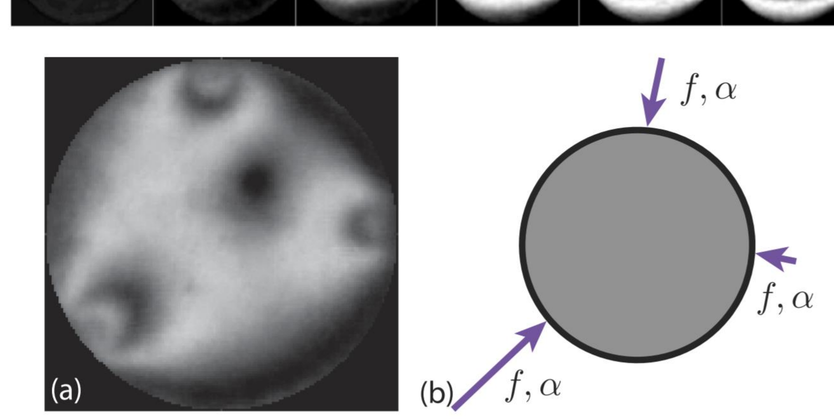
Goals

1. Track particles as a function of time
2. Calculate the D² values for tracked particles
3. Correlate D² values to force networks.
4. Study D² values relation to networks and communities. [5]

Method

- Data is taken using polarized green light .
  - Green light → Force networks
  - Red light → Particle locations
- 2-dimensional particles are put on an air table.
  - Compressed using stepper motors
  - Controlled via LabView
  - Uniaxial compression, biaxial compression, pure shear, etc.
- PeGS preprocessing (J E. Kollmer) [3]
  - Disk finding, neighbor detection, and disk solving
  - Takes images and extracts particle and force information.
  - First step to processing our data.
- Take PeGS data and apply D² calculations and force network analyzations (Estelle Berthier)





J E. Kollmer et al. - Review of Scientific Instruments - 2017 [1]

The D² Calculation

- Proposed by Falk and Langer [5]
- Tracks the movement of one particle relative to its neighbours.
- Minimizes this difference between neighbouring particle motion and the movement in a uniform strain field.

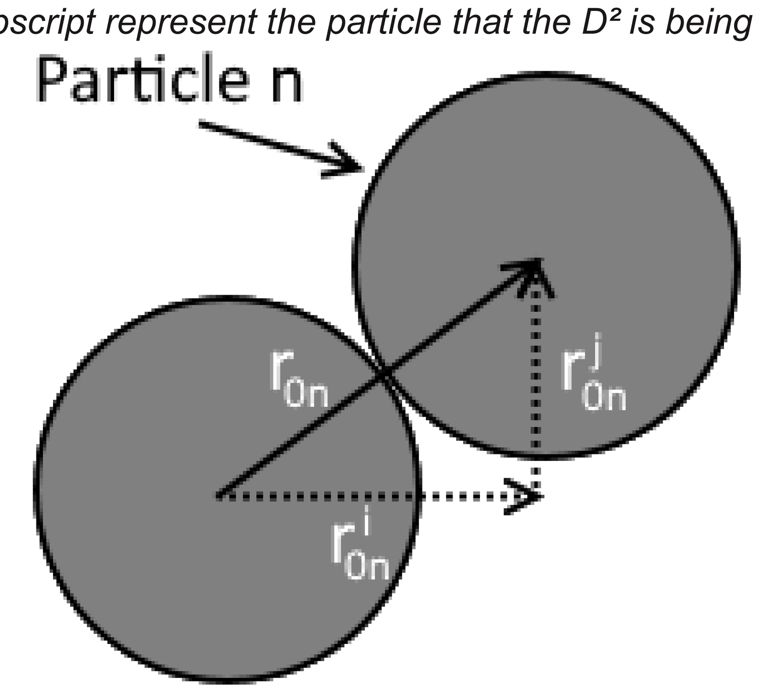
$$D^2(t,\Delta t) = \sum_n \sum_i \left(r_n^i(t) - r_0^i - \sum_j (\delta_{ij} + \varepsilon_{ij}) \times \left[r_n^j(t-\Delta t) - r_0^j(t-\Delta t) \right] \right)^2$$

$$X_{ij} = \sum_n \left[r_n^i(t) - r_0^i(t) \right] \times \left[r_n^j(t-\Delta t) - r_0^j(t-\Delta t) \right]$$

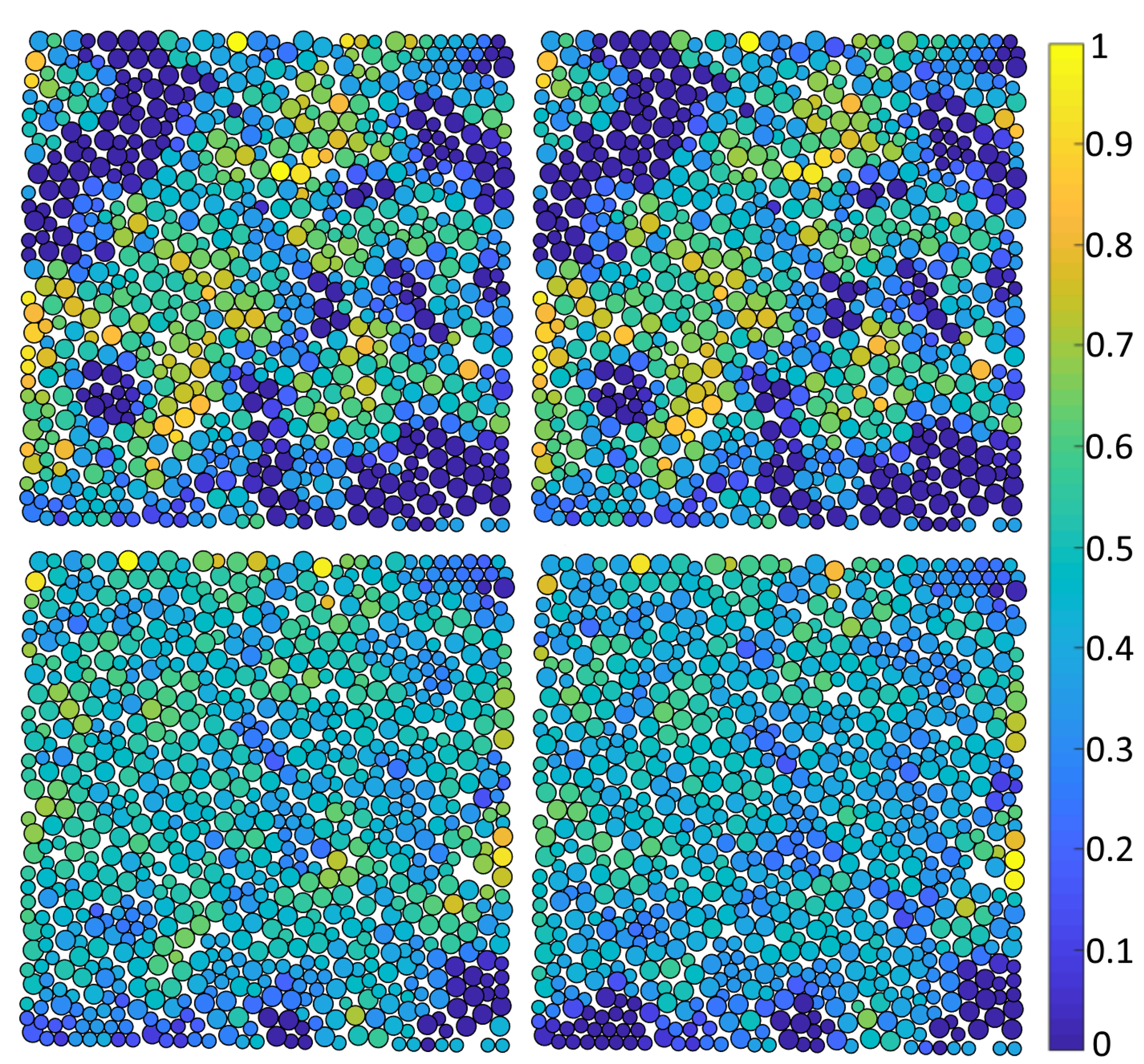
$$Y_{ij} = \sum_n \left[r_n^i(t-\Delta t) - r_0^i(t-\Delta t) \right] \times \left[r_n^j(t-\Delta t) - r_0^j(t-\Delta t) \right]$$

$$\varepsilon_{ij} = \sum_k X_{ik} Y_{jk}^{-1} - \delta_{ij}$$

The D² calculation. Here, i and j are spatial variables akin to the x and y coordinates, n is a variable used to define the closest neighbours to the particle and k is a dummy variable for summation. Position vectors denoted with a "0" subscript represent the particle that the D² is being calculated for.



An example of the vectors used in the D² calculation between two particles (the one being studied being the bottom left).

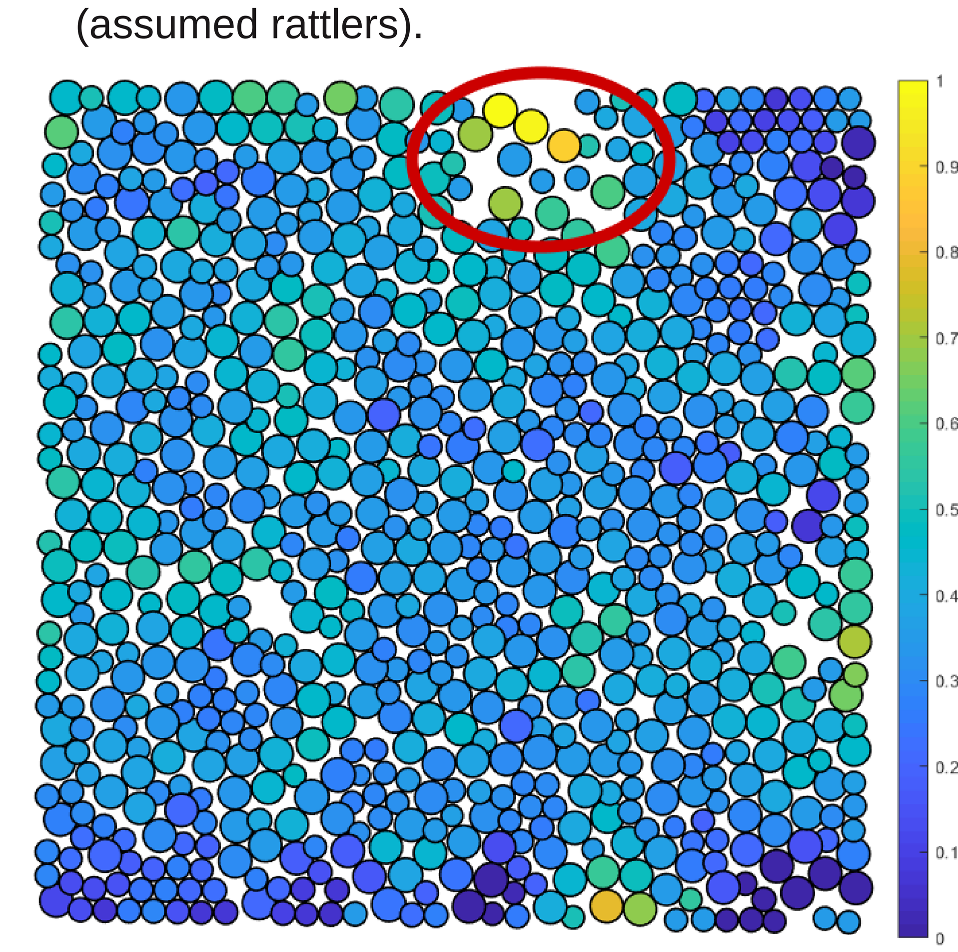


Normalized D² values for the same image with different time steps (1, 2, 3, and 5). Higher values (brighter colors) represent areas where the particles have moved contrary to a uniform strain field.

Rattler Detection

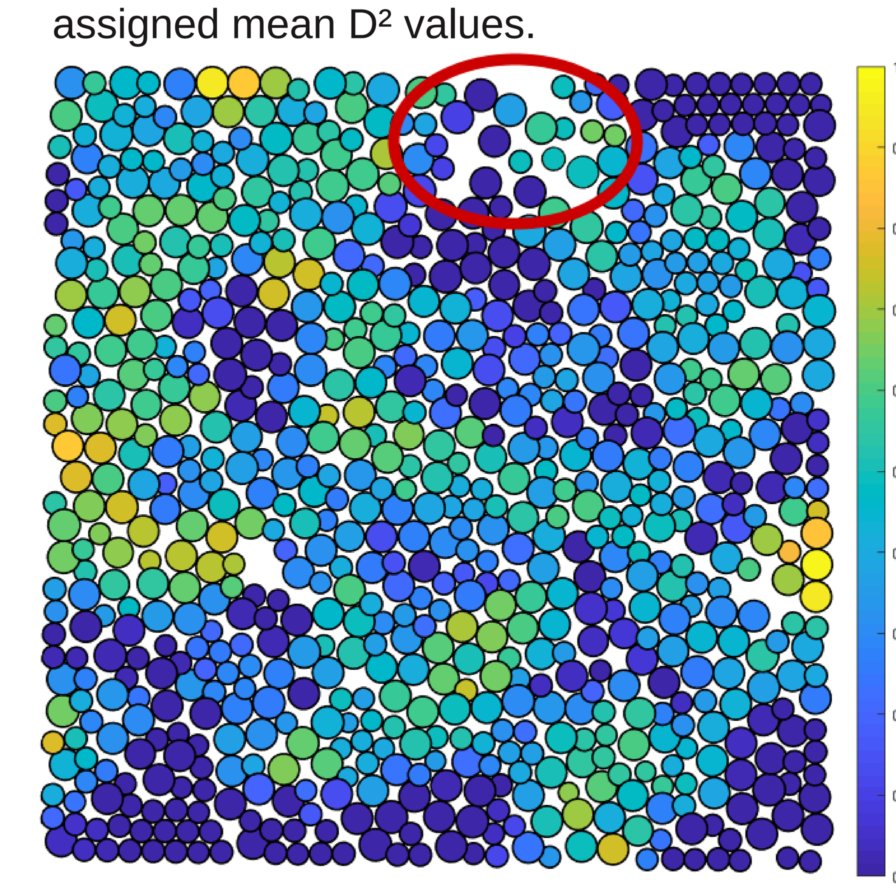
- Some particles that are not bound by geometric constraints.
  - These particles are called "rattlers"
- Rattlers create problems for D² calculations.
  - Move large distances → large D² values.
- There are multiple approaches to remedying this.

1. Exclude outlying D² values.
   - Calculate D² values without constraints.
   - Order D² values for each particle and its neighbours.
   - Detect outlier values.
   - Rerun D² calculations without counting outlier values (assumed rattlers).



D² calculations via the above method with rattlers circled. The above method fails for this image as the rattlers had a D² value that was not considered to be an outlier

2. Ignore particles without 3 neighbours.
   - Cheapest method.
   - Particles with too few neighbors are ignored and assigned mean D² values.

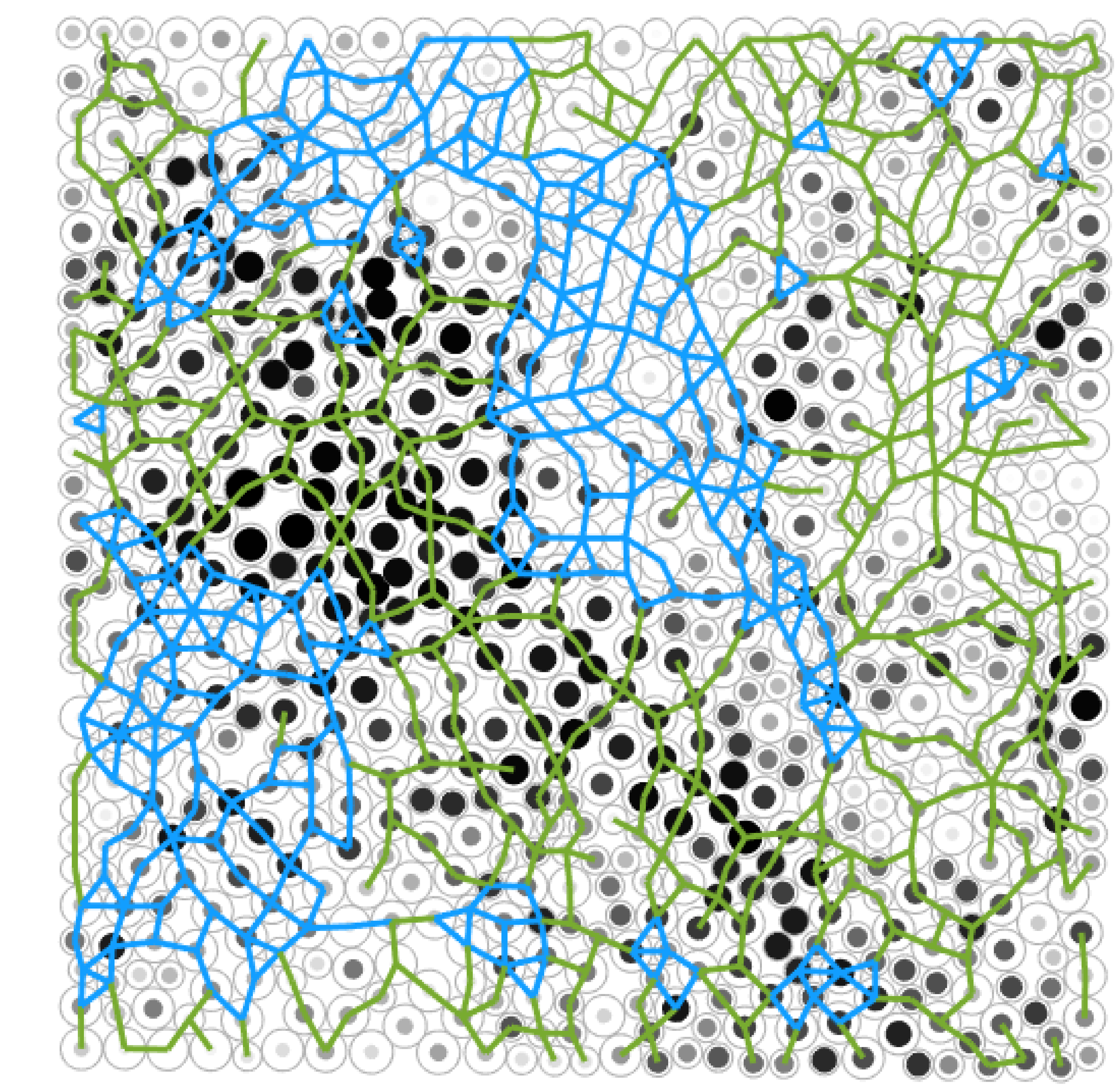


An example of calculating D² via the second method. Here, the rattlers were properly identified allowing for the other D² values to have a larger range when normalized.

3. Detection via radical Voronoi tessellation.
   - Requires voro++ [4]
   - Steps outside of main program and main language

Networks

- Combining the D² calculations with analysis of networks of force chains allows for failure events to be studied.
- Done by looking at force networks the step before failure .
  - Shows behavior that leads up to a failure event.



The combination of D² calculations and the analyzed force network. D² value are represented as the internal circles and become larger and darker as D² values approach 1. Normal force chains are represented as green lines while blue lines represent force chains within a rigid cluster.

Conclusions

- Some rigid clusters of force chains are found to have low D² values.
  - Could mean they are less prone to failure events.
- Studying systems in multiple scenarios (uniaxial compression, different shears, etc.) will give more insight to failure behaviors.
- Future work hinges on studying additional systems and applying community detection algorithms.
- Since multiple frameworks exist, it is now a question of seeing what each one tells us and how they are connected.
- Ultimate goal is to learn more about granular materials and systems while producing methods to study them with.

References

1. Daniels, Puckett, Kollmer, Review of Scientific Instruments **88**, 051808 (2017)
2. M. L. Falk and J. S. Langer, Phys. Rev. E **57**, 7192 (1998).
3. J. E. Kollmer, Photo-elastic Grain Solver (PEGS), https://github.com/ jekollmer/PEGS
4. Chris H. Rycroft, Voro++, http://math.lbl.gov/voro++/
5. Bassett, Owens, Porter, Manning, Daneils., "Extraction of Force-Chain Network Architecture in Granular Materials Using Community Detection." *Soft Matter* 11: 2731-2744 (2015).