

$\mathcal{B}_k$

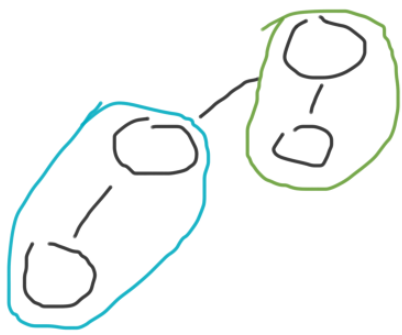
$k=0$



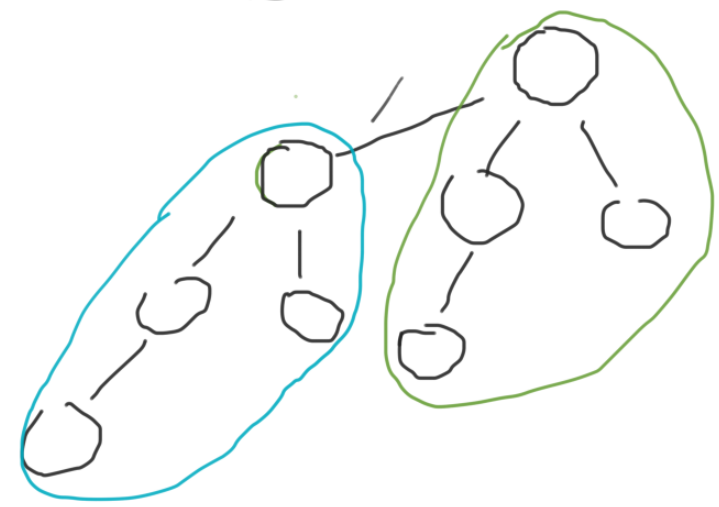
1



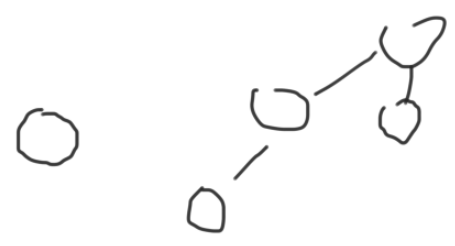
2



3



$[\mathcal{B}_0, \mathcal{B}_2, \mathcal{B}_5]$



ne

$B_n$

$n=0$

$\square$

$$2^0 = 1$$

$n=1$

$\square - \square$

$$2^1 = 2$$

$n=2$

$\square - \square - \square$

$$2^2 = 4$$

Base:  $n=0$

$$\square \quad 2^0 = 1 \quad \checkmark$$

kwk ind.

$n = k-1$

$$B_{k-1} \neq 2^{k-1}$$

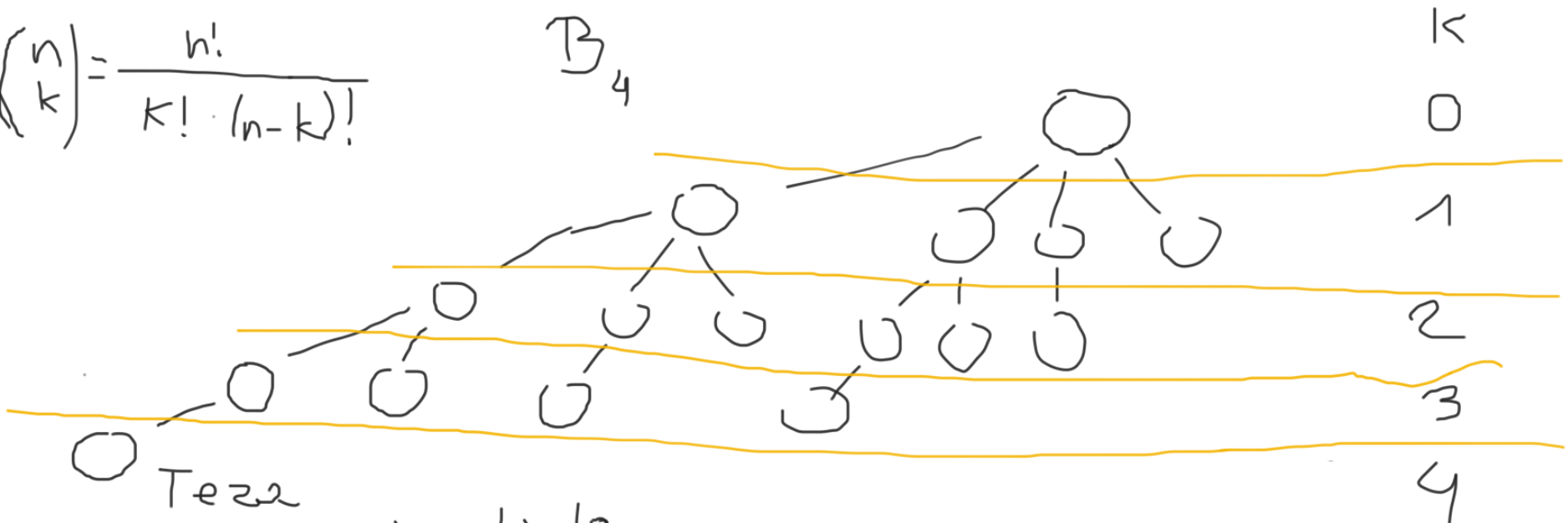
zu induzieren

$$B_k := B_{k-1} \oplus B_{k-1}$$

$$2^{k-1} + 2^{k-1} = 2^k \quad \square$$

$$1b) \binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

$B_4$



Teza

$D(n, k)$  - liczba wierz. na prz.  $k$  w  $B_n$

$$D(n, k) = \binom{n}{k}$$

Dowód ind.

1) Baza

$$D(n, 0) = 1 = \binom{n}{0} \checkmark$$

2) krok ind

$$D(4, 1) = D(3, 0) + D(3, 1)$$

$$D(4, 2) = D(3, 1) + D(3, 2)$$

$\boxed{\text{Induktion}}$ 

$$D(n, k) = D(n-1, k-1) + D(n-1, k) \stackrel{\text{z.zat ind}}{=} \\
= \binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(k-1)! (n-k)!} + \frac{(n-1)!}{k! (n-k-1)!} = \\
= \frac{(n-1)!}{(k-1)! (n-k-1)!} \cdot \left( \frac{1}{n-k} + \frac{1}{k} \right) = \frac{n!}{k! (n-k)!} = \underline{\underline{\binom{n}{k}}}$$

$$\binom{n}{k} = \frac{n!}{k! (n-k)!}$$

$$\frac{k + (n-k)}{nk \cdot k^2} = \frac{n}{k(n-k)}$$

□

7  $\eta \tau \bar{\tau} \omega$

$$2^2 \quad 2^1 \quad 2^0 \quad 1^0 \quad 1^1$$

$$10110_2 \leftrightarrow 2^4 + 2^2 + 2^1 = 21$$

$$f(7) = 3$$

```
int d=0;
```

```

for(int i=0; i<32; i++){
    d += (int)(n & (1<<i)) != 0)
}

```

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$$111 = 2^2 + 2^1 + 2^0$$

int |  $1 \ll i$

$i = 0$	00001	&
1	00010	&
2	00100	
3	01000	
4	10000	

$n = 11010$

0  
2  
0  
8  
16

$(int)(n \& (1 \ll i) \neq 0)$

	11010
&	00010
<hr/>	
	00010

00 → 0	10 → 0
01 → 0	11 → 1

2a/3a [ ]

(1)  
 $B_0$

[ (1) ]

(1) (12)  
 $B_0$   $B_0$

↓

$B_1$

(1) — (12)

(12) (3)  
(1) —

[  $B_1, B_0$  ]

(12) 3 14  
1 —

↓

1<sup>12</sup> 3<sup>14</sup>  
1 — 3 —

↓

1<sup>12</sup> 1<sup>14</sup>  
1 — 3 — 3

2nd 3<sup>9</sup> 2

(1)

(2)

5

14  
12  
3

5

16

↓

16

$[B_7, B_0]$

↓  
11

(3)

14  
12  
3  
1

16

7

(5)

14  
12  
3  
1

16

7

20

↓

20

7

$B_3$

20

14  
12  
3  
1

16

7

20

16

7

(5)

25

(6)

25 13

↓

25

13

(7)

25

8

13

(8)

14

16

7

20

25

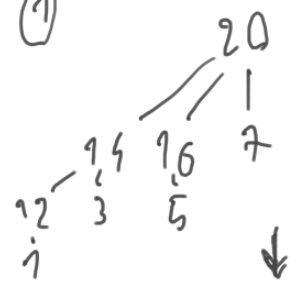
8

$[B_3, B_1, B_0]$



200136

①



11

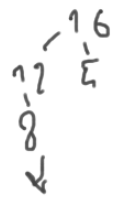
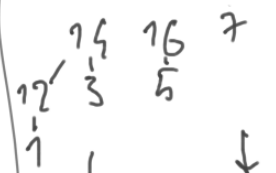
25

12

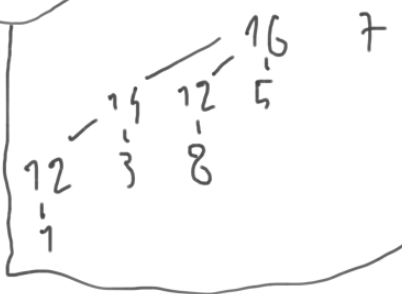
12  
8

②

20



12  
8



$$\{0\} \quad \{1, 2, 4, 7\} \quad \{5, 6\} \quad \{3\}$$

$$f(1) = f(2) = f(4) = f(7)$$

$$f(1) \neq f(5) \neq f(7) \neq f(1)$$

$$f(5) = f(6)$$

$f(x)$  - najmniejszy element w zbiorze w którym jest  $x$

$$f(1) = 1$$

$$f(5) = 5$$

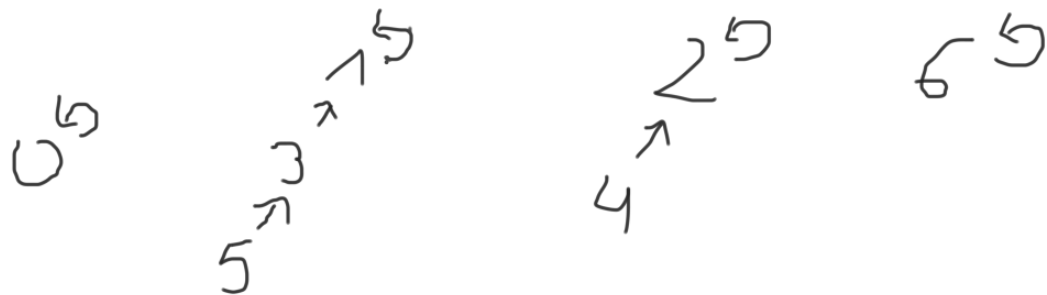
$f$

$$f(4) = 1$$

$$f(6) = 6$$

$$f(8) = 1$$

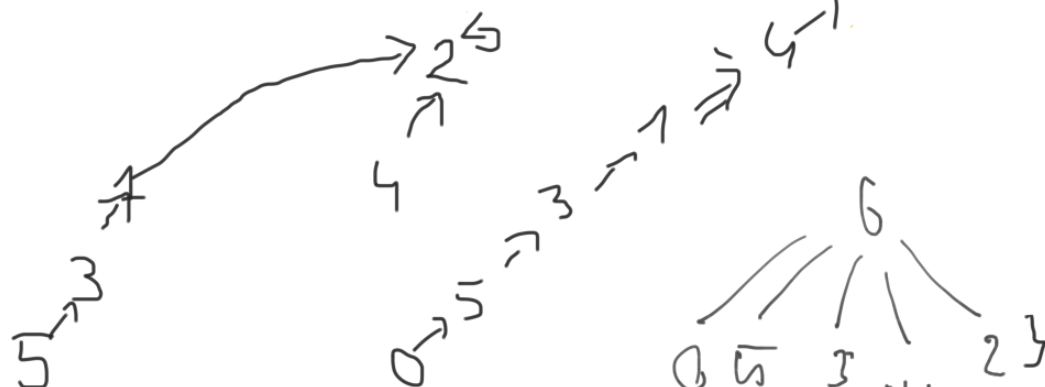
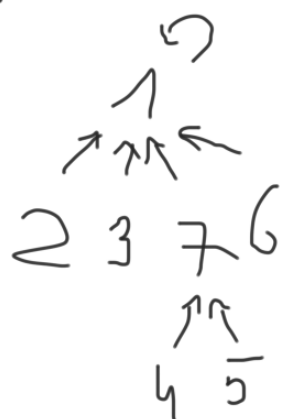
{0}    {1, 3, 5}    {2, 4}    {6}



```

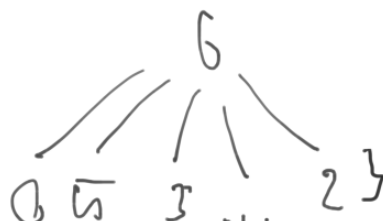
int find (n) {
  while (n != p[n])
    n = p[n]
  return n
}
  
```

n	0	1	2	3	4	5	6
p[n]	0	1	2	1	2	3	6



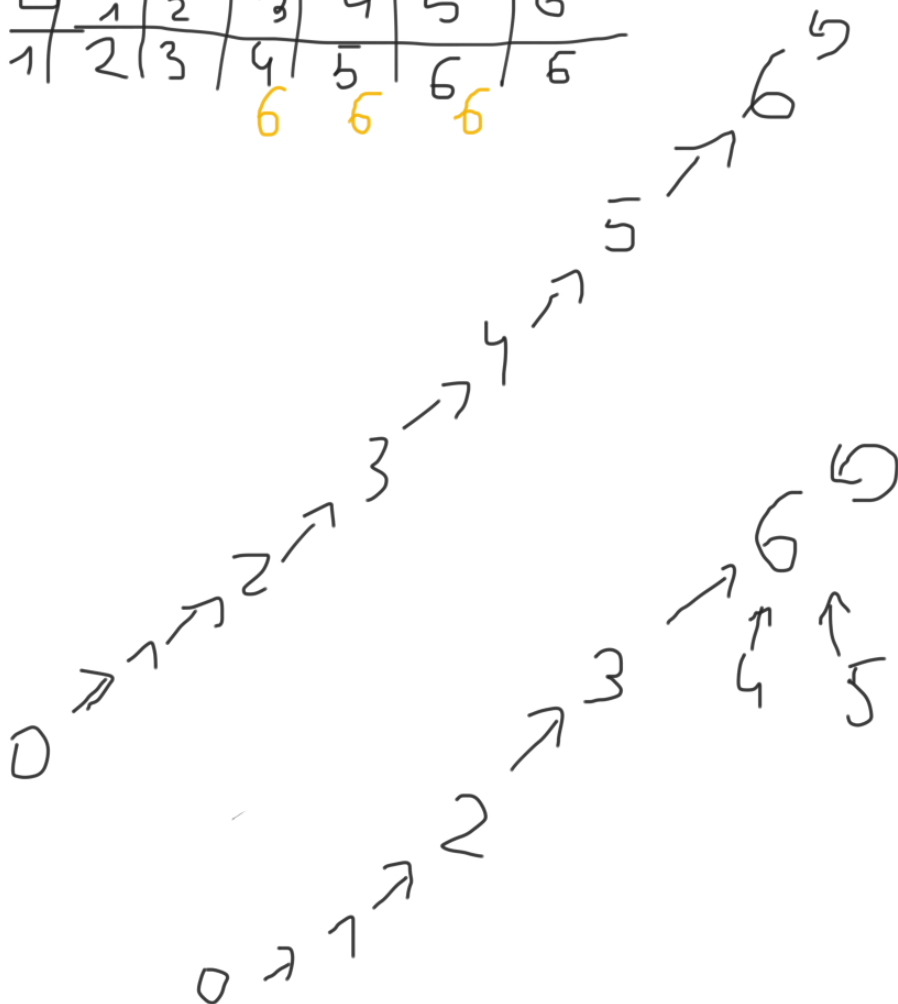
```

union (n, m) {
  if (find(n) == find(m)) {
    return
  }
  rn = find(n)
  p[rn] = m
}
  
```



1) Kompresja ścieżek

4	1	2	3	4	5	6
1	2	3	4	5	6	6



```

int find(n) {
    if (p[n] == n)
        return n
    else {
        p[n] = find(p[n])
    }
    return p[n]
}
  
```

```

int find(n) {
    if (p[n] != n)
        p[n] = find(p[n])
    return p[n]
}
  
```

2) RANK

	0	1	2	3	4	5	6
P	0	1	2	3	4	5	6
r	0	0	0	0	0	0	0

0 1 2 3 4 5 6

2  $\rightarrow$  3      3  $\rightarrow$  2

Union( $i, j$ ) {

$k_i = \text{find}(i)$

$k_j = \text{find}(j)$

if ( $k_i == k_j$ ) return

if ( $r[k_i] < r[k_j]$ ) {

$P[k_i] = k_j$

else if ( $r[k_i] > r[k_j]$ ) {

$P[k_j] = k_i$

} else {  $r[k_i]++$ ;  $P[k_j] = k_i$  }

5

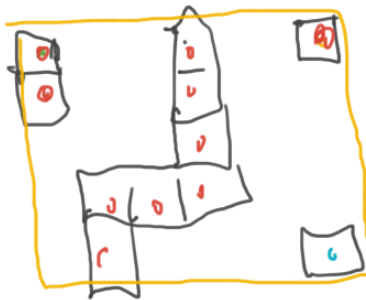
a)

	0	1	2	3	4
0	1	1		1	
1		1			1
2				1	
3			1	1	1

$UF(n^2)$

	a	
d	1	b
	c	

6 . . .



a

