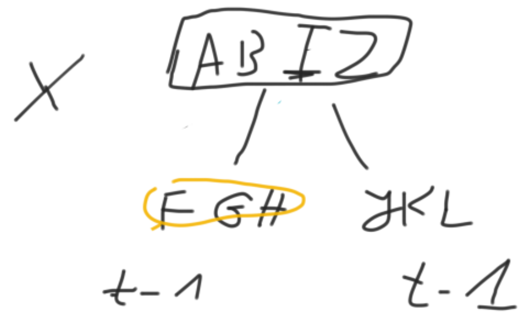
1) key[] - tab kluczy (m kluczy)
* m+1 wskaźników na synów jeśli wewnętrzny
leaf - T/F

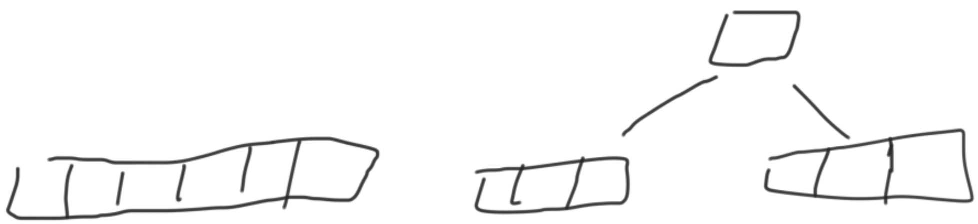
t - parametr

wierzchołek
1) jeśli w ≠ root to ma ~~...~~ $\geq t-1$ kluczy $t=2$
2) ~ $\leq 2t-1$ kluczy

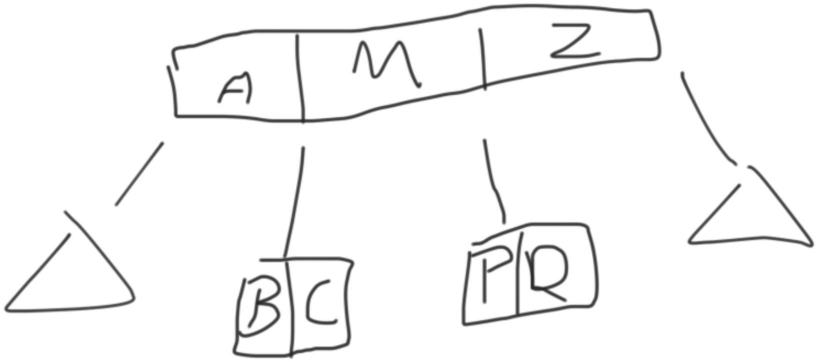
2a) split child


root



2b)

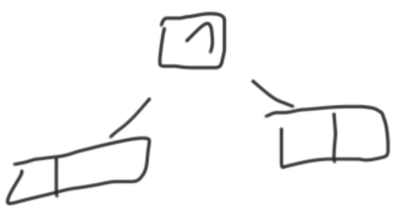



$t = 3$

$2, 5$

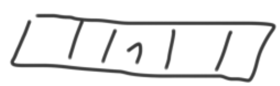

5

$$2 \cdot (t-1) + 1 = 2t - 1$$

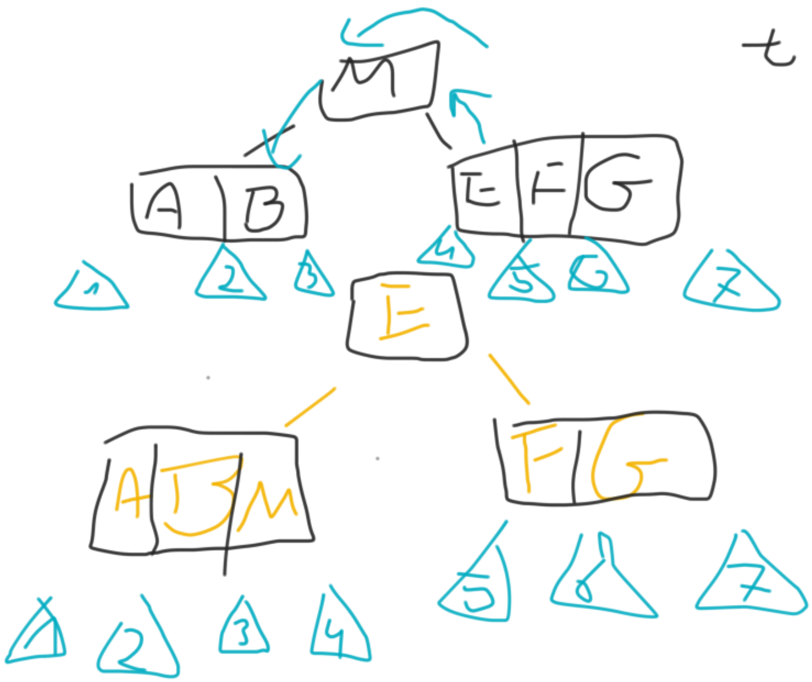



$n=1$

$\rightarrow$



2g)



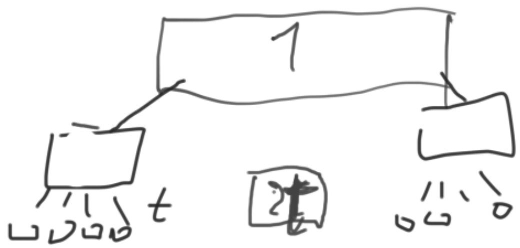

$t = 3$

3) a) $[0, 2t-1]$

b) $[0, 2t]$

c) $[t-1, 2t-1]$

d) $[t, 2t] \cup \{0\}$



$2t$

$2t$

$2t$

| | | |
|---|---|---|
| 1 | $\cancel{k}$ 0 | $\cancel{\#}$ 1 |
| 2 | 1 | $(2t)^1$ |
| $2t$ | 2 | $(2t)^2$ |
| $2t^2$ | 3 | $(2t)^3$ |
| $2t^3$ | | |

$t = 3$

wurzel

1

$2t$

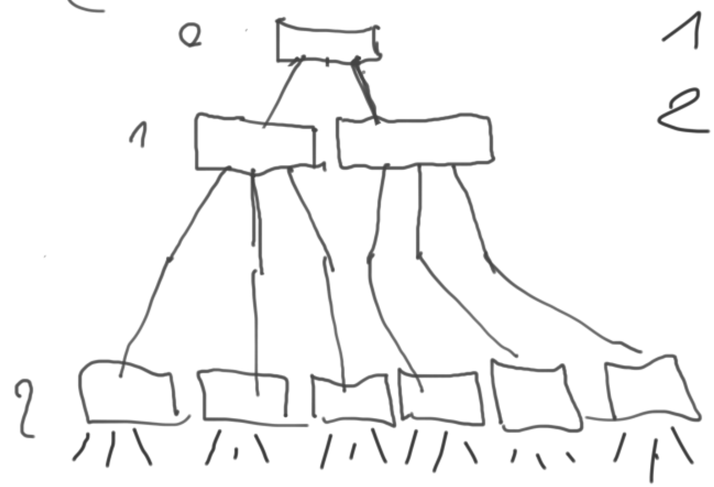
$(2t)^2$

$(2t)^3$

e) $(2t)^k$

f) $[2t^{k-1}, (2t)^k]$ $k>0$

$k=0$ 1



1

2

$2t$

$2t^2$

5a) $\min: S = 1 + 2(t-1) + 2t(t-1) + 2t^2(t-1) + \ldots =$

$$= 1 + (t-1) \cdot \sum_{k=1}^{n} 2t^{k-1} = 1 + 2(t-1) \sum_{k=1}^{n} t^{k-1} =$$

$$= 1 + 2\cancel{(t-1)} \cdot \frac{1-t^n}{\cancel{1-t}} =$$

$-1$

$$= 1 + 2\cancel{(t-1)} \cdot \frac{t^n - 1}{\cancel{t-1}} =$$

$$= 1 + 2t^n - 2 = 2t^n - 1$$

$$S = 1 + t + t^2 + \ldots + t^{n-1}$$

$$S = 1 + t(1 + t + \ldots + t^{n-2})$$

$$S = 1 + t(S - t^{n-1})$$

$$S = 1 + t \cdot S - t^n$$

$$S(1-t) = 1 - t^n$$

$$S = \frac{1-t^n}{1-t}$$

4b) max

$$S = 2t-1 + 2t(2t-1) + (2t)^2(2t-1) + \dots + (2t)^n(2t-1)$$
$$= (2t-1)\left(1 + 2t + (2t)^2 + \dots + (2t)^n\right)$$
$$= (2t-1)\sum_{k=0}^{n}(2t)^k = (2t)^{n+1} - 1$$

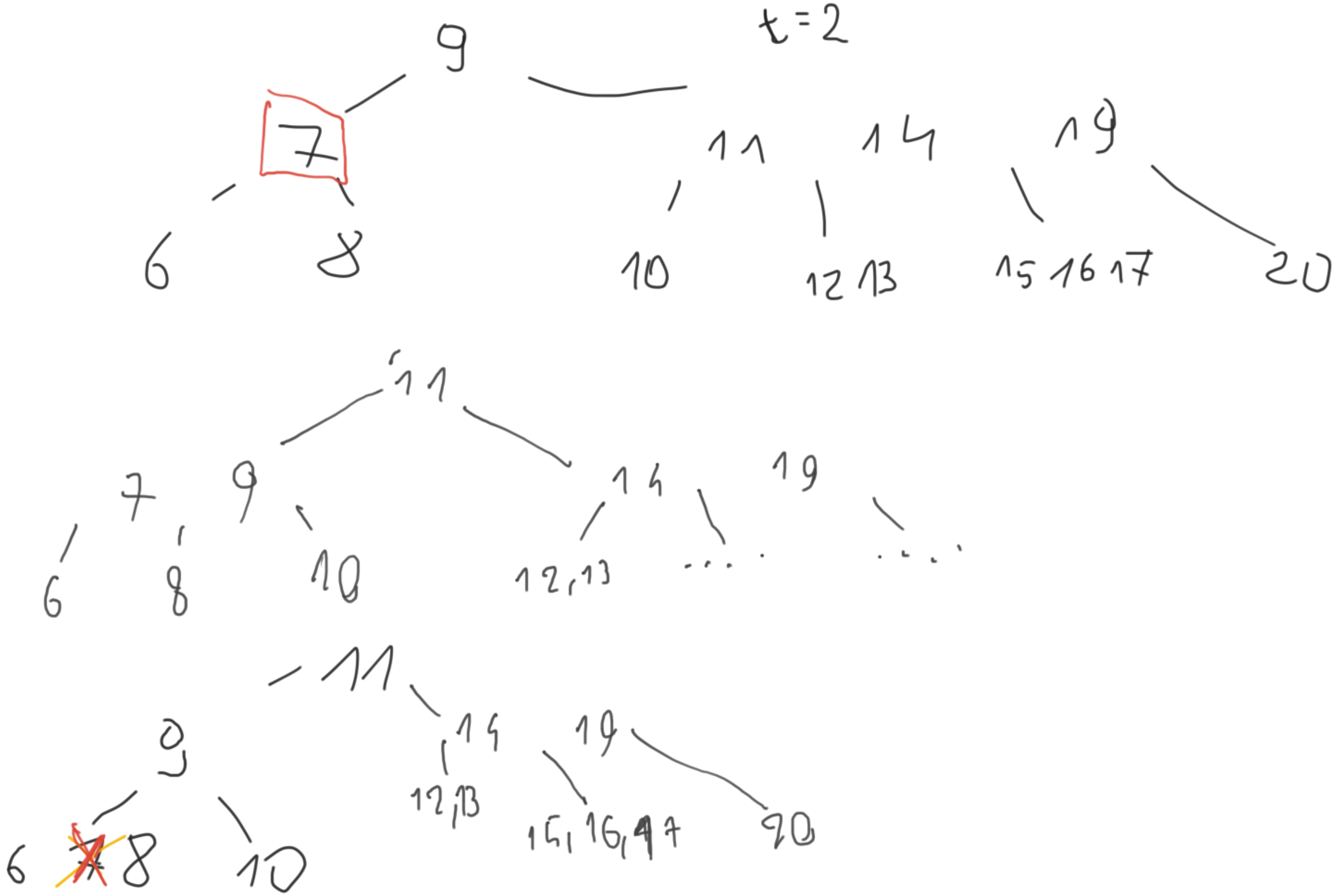
$$S = 1 + 2t + \dots + (2t)^n$$
$$S = 1 + 2t\left(1 + 2t + \dots + (2t)^{n-1}\right)$$
$$S = 1 + 2t\left(S - (2t)^n\right)$$
$$S = 1 + 2t \cdot S - (2t)^{n+1}$$
$$S - 2t \cdot S = 1 - (2t)^{n+1}$$

$$S(1-2t) = 1 - (2t)^{n+1}$$

$$S = \frac{1-(2t)^{n+1}}{1-2t} = \frac{(2t)^{n+1}-1}{2t-1}$$

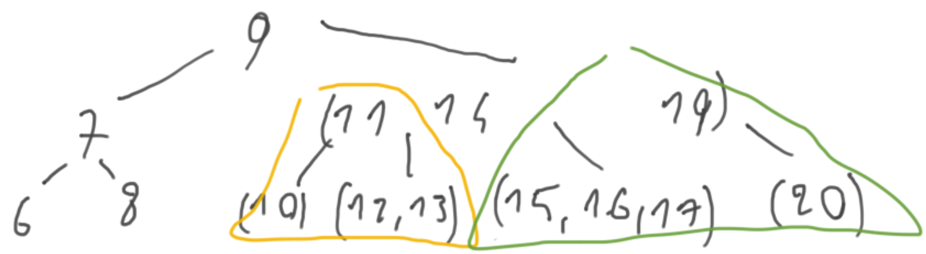
5a

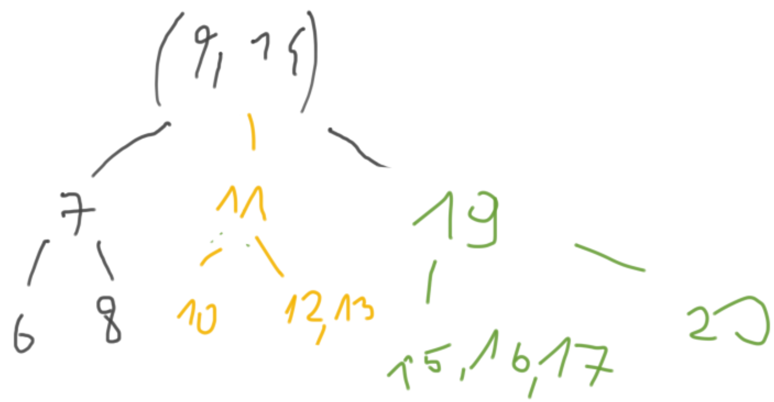
t = 2

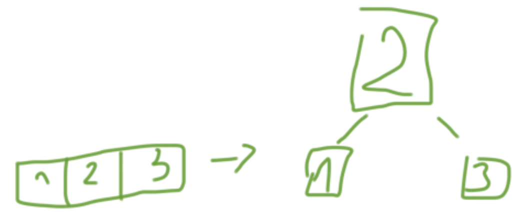

9
7
6
8
11 14 19
10
12 13
15 16 17
20
11
7 9
6
8
10
14 19
12,13
...
...
11
9
6 ~~7~~ 8
10
14 19
12,13
15,16,17
20

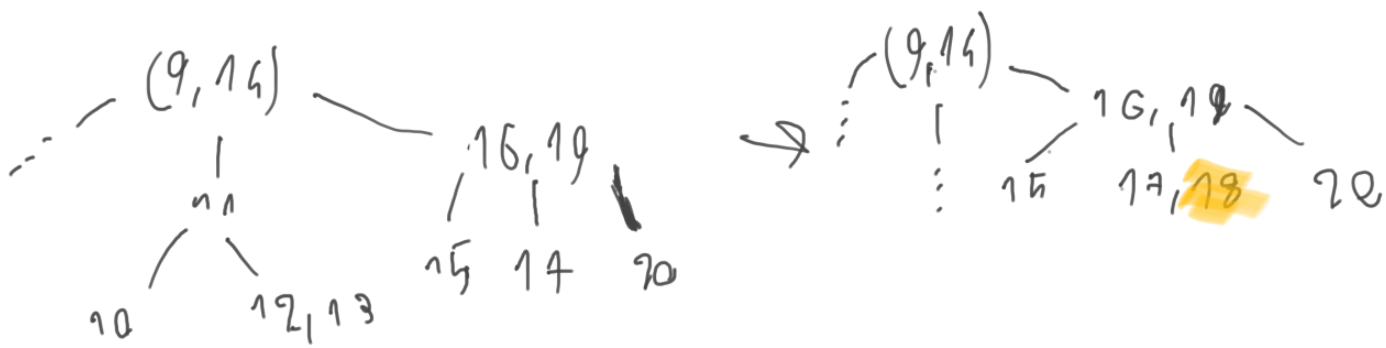
$\boxed{56}$

$t = 2$


9
7
11 14
19
6
8
10
12, 13
15, 16, 17
20


9, 14
7
11
19
6
8
10
12, 13
15, 16, 17
20


1
2
3
2
1
3


9, 14
11
16, 19
10
12, 13
15
17
20
9, 14
16, 19
15
17, 18
20

$t = 2$