a)

$T(n) = T\left(\frac{n}{2}\right) + \sqrt{n}$

$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$

$a = 1 \quad b = 2 \quad f(n) = n^{\frac{1}{2}}$

$n^{\log_b a} = n^0$

$1 \cdot f\left(\frac{n}{2}\right) \leq c \cdot f(n)$

$\left(\frac{n}{2}\right)^{\frac{1}{2}} \leq c \cdot n^{\frac{1}{2}}$

$\frac{\sqrt{n}}{\sqrt{2}} \leq c \cdot \sqrt{n}$

$c \geq \frac{1}{\sqrt{2}}$

$T(n) = \Theta(f(n)) = \Theta\left(n^{\frac{1}{2}}\right)$

$n^0 \quad \not\in \quad n^{\log_b a - \varepsilon}$

$n^{\log_b}$

$n^{\log_b + \varepsilon}$

$\varepsilon = \frac{1}{4}$

$f(n) = \Omega\left(n^{0+\varepsilon}\right) = \Omega\left(n^{\frac{1}{4}}\right)$

① $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$a = 3 \qquad b = 2 \qquad f(n) = n^2$

② $n^{\log_2 3} > n^2$

③ Ω

$a \cdot f\left(\frac{n}{b}\right) \leq c \cdot f(n)$

$3 \cdot f\left(\frac{n}{2}\right) \leq c \cdot n^2$

$3 \cdot \frac{n^2}{4} \leq c \cdot n^2$

$\frac{3}{4} \leq c$

④ $T(n) = \Theta(f(n))$

$T(n) = \Theta(n^2)$

d)

② $T(n) = 8T\left(\frac{n}{4}\right) + 3n\sqrt{n}$

$a = 8 \quad b = 4 \quad f(n) = n\sqrt{n}$

②

$n^{\log_4 8} = 3n^{\frac{3}{2}} \cdot n^{\frac{3}{2}} = n^{\frac{3}{2}}$

$n^{\frac{3}{2}} = 3n^{\frac{3}{2}}$

③

$T(n) = \Theta\left(n^{\frac{3}{2}} \cdot \log n\right)$

$4^x = 8$

$2^{2x} = 2^3$

$2x = 3$

$x = \frac{3}{2}$

c) $T(n) = 5T\left(\frac{n}{2}\right) + n^2$

① $a = 5 \quad b = 2 \quad f(n) = n^2$

② $n^{\log_2 5} \not\equiv n^2$

$n^{\log_2 5} > n^2 = f(n)$

③ $T(n) = \Theta\left(n^{\log_b a}\right)$

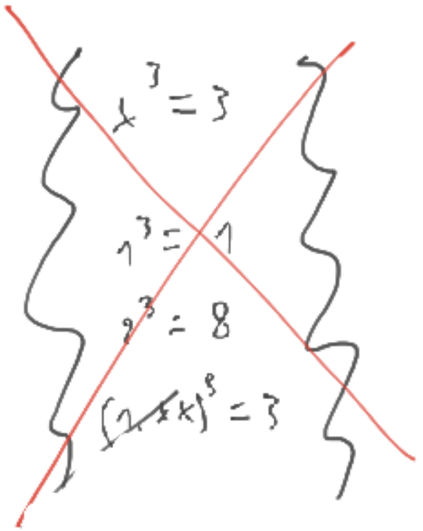
$= \Theta\left(n^{\log_2 5}\right)$

e) $T(n) = 3T\left(\frac{n}{3}\right) + n$

① $a = 3 \quad b = 3 \quad f = n^1$

② $n^{\log_3 3} = n^1$

$n^{\log_3 3} = n^1 = f(n)$



③

$T(n) = \Theta\left(n^{\log_b a} \cdot \log n\right)$

$= \Theta(n \cdot \log n)$

$\log_a b = x$

$a^x = b$

$3^x = 3$

$x = 1$

g) $T(n) = 10\,T\left(\frac{n}{3}\right) + n^2$

① $a = 10 \quad b = 3 \quad f = n^2$

② $n^{\log_3 10} = n^2$

$n^{\log_3 10} > n^2 = f(n)$

③

$T(n) = \Theta\left(n^{\log_3 10}\right)$

2 a d 3

2 a d $\overline{3}$ a

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + \Theta(n)$$

$a = 2 \quad b = 2 \quad f(n) = \Theta(n)$

$n^{\log_2 2} \stackrel{?}{=} \Theta(n)$

$n^1 = \Theta(n)$

n

$L = n^{\log_2 2} = n^1 = n$

$P = \Theta(n)$

$L = P$

$L = \Theta(P)$

$T(n) = \Theta(n^{\log_b a} \cdot \log n)$

$= \Theta(n \cdot \log n)$

$n^1 = n = \Theta(n) \rightarrow$

$n \in \Theta(n)$

$f(n) \in \Theta(n)$

$n \in \Theta(n)$

$\Theta(n) \stackrel{?}{=} \Theta(n)$

$\Theta(n) \in \Theta(n)$

Zad 6

$M(n) = \Theta(m)$ // $M(n, m) = \Theta(m)$

$T(n) = \Theta(n+m)$ // $T(n, m) = \Theta(n+m)$

2cd 6 ol

② $(x - m)$

zad 7

3073 306

$h_1(33) = 0$

$h_2(33) = 9 + 1 = 10$

!!

$(h_1(x) + i\,h_2(x)) \% m$

$i = 0$

$i = 1$

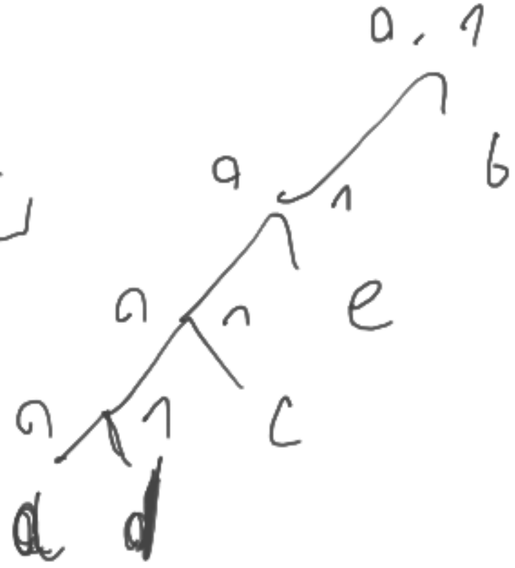
$i = 2$

Zad 8

3	a	7	3	a	6
x	a	b	c	d	e
	1	8	4	1	7

21

8 → b	000
7 → e	001
4 → c	010
1 → a	011
1 → d	100



a	0000	4 : 1	4
d	0001	4 : 1	4
c	001	3 : 4	12
e	01	2 : 7	14
b	1	1 : 8	8

20

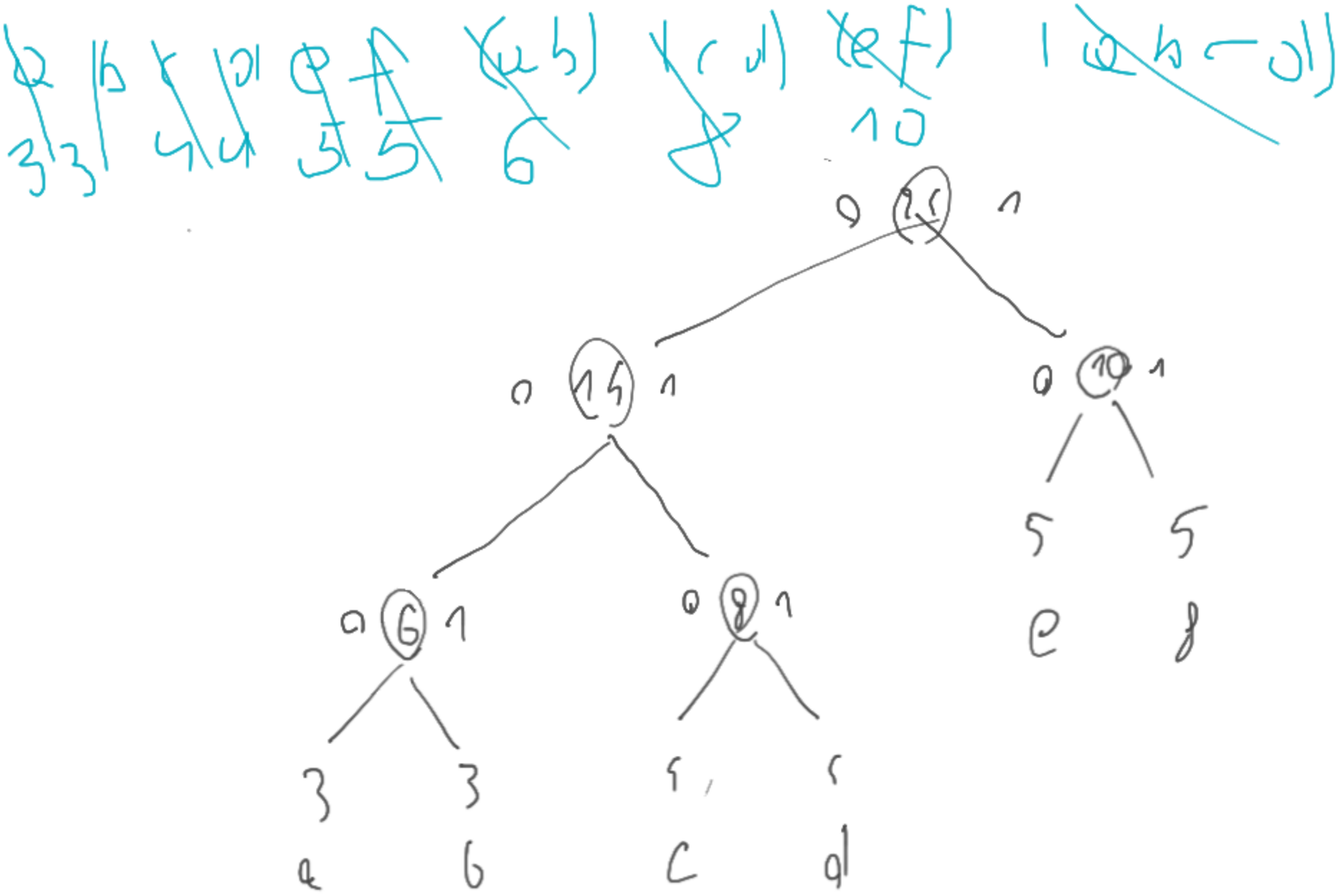
35

3 · 21 = 63 vs 42

>

63 - 42 = 21

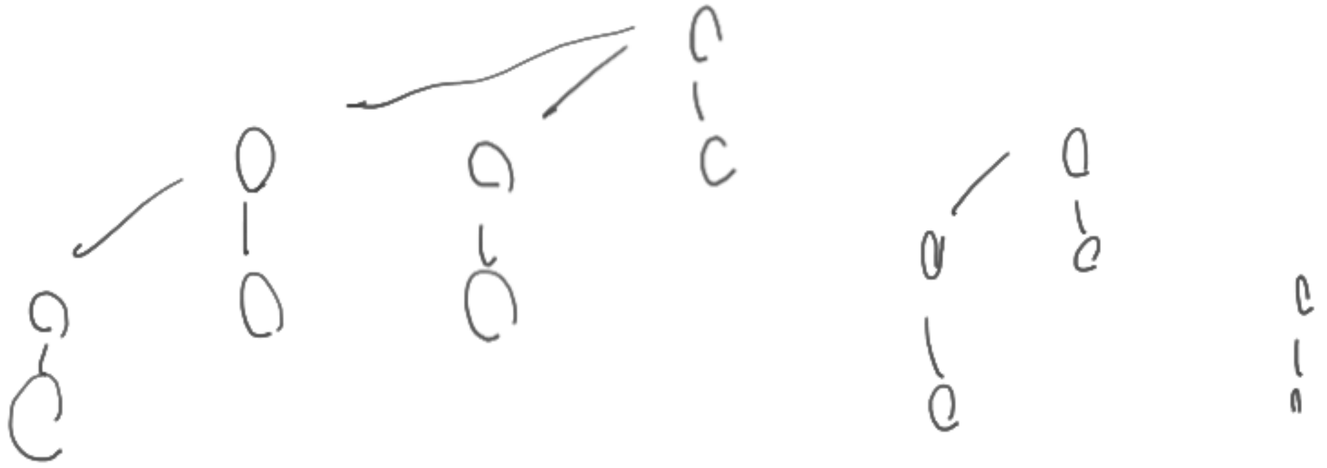
vs 42


a b c d e f g h
3 3 4 4 5 5 6 7 8 10
0 1
0 1
0 1
0 1
3 3 5 5
a b c d e g

Zad 9

306

2^0	1		
2^1	2	1	
2^2	4		
2^3	8		
2^4	16	1	18 − 16 = 2
2^5	32	1	50 − 32 = 18
2^6	64		
2^7	128		
2^8	256	1	306 − 256 = 50

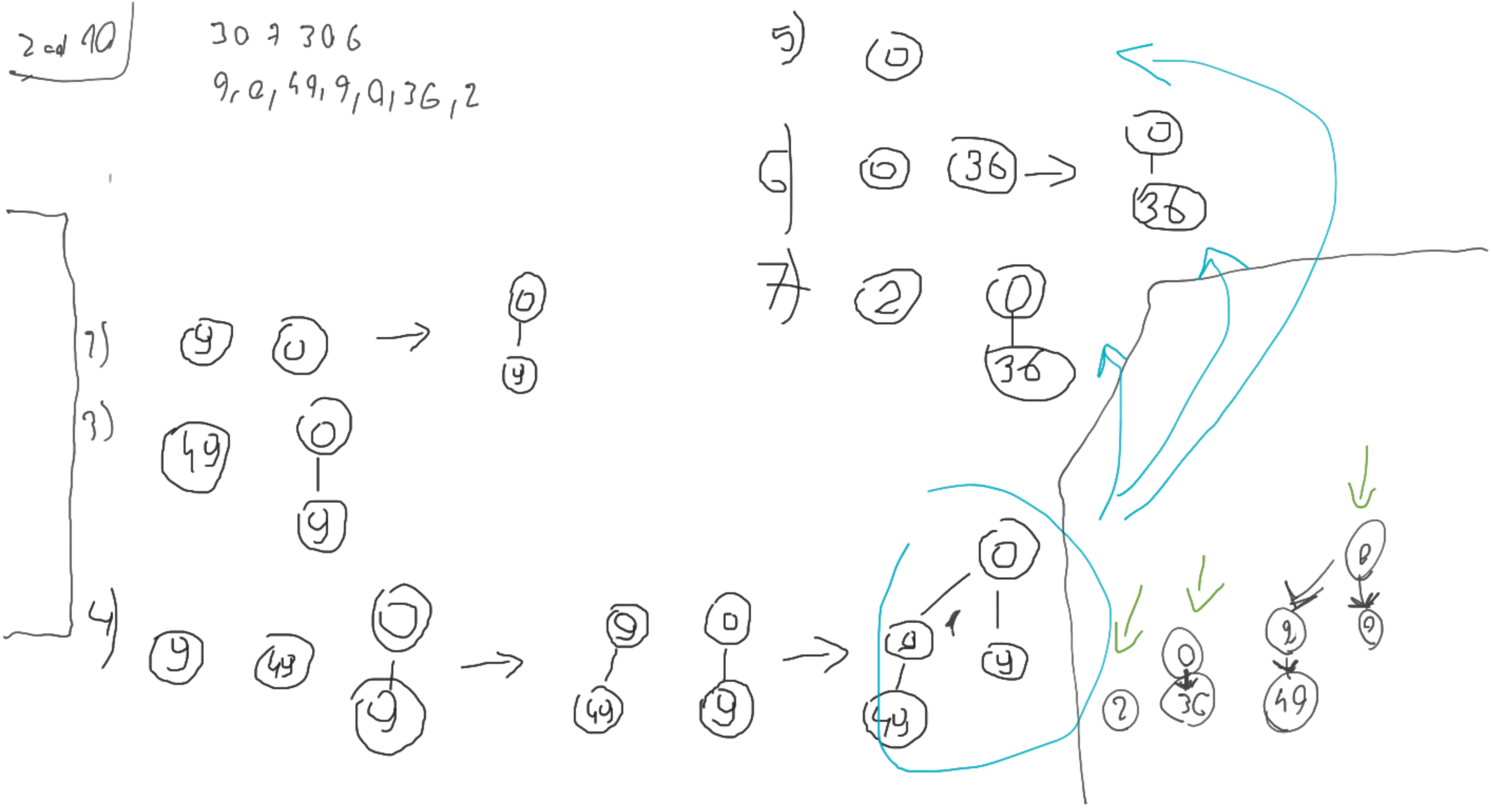


odp 1, 4, 5, 8

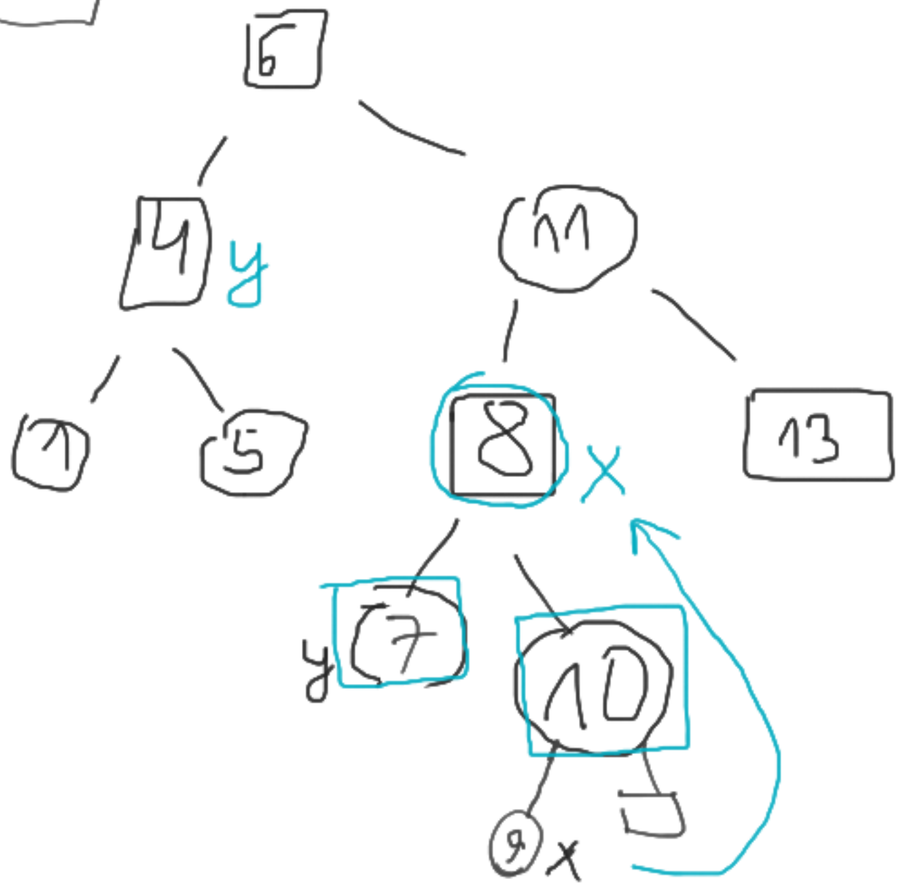
2 col 10

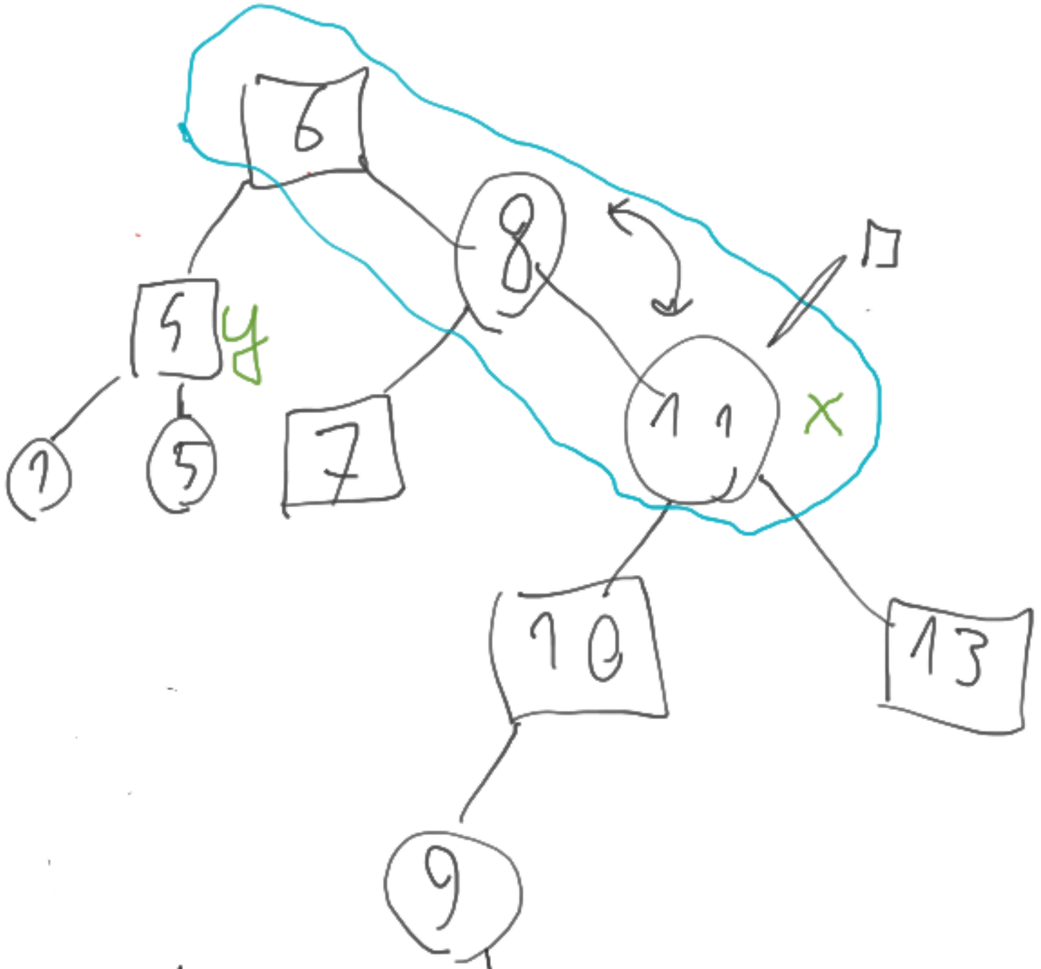
30 7 306

9, 0, 49, 9, 0, 36, 2



Zad 11





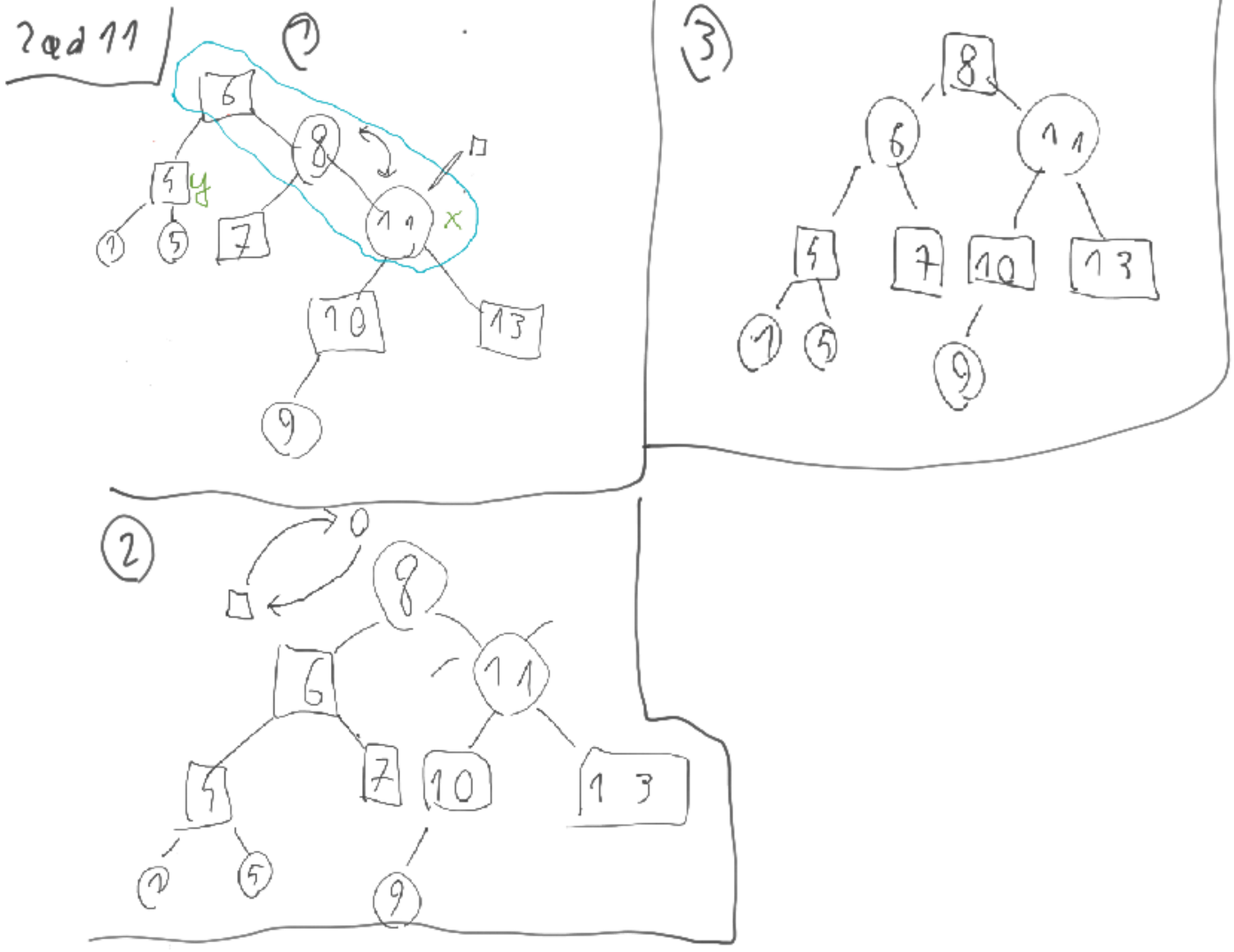
1) y - ○ p[x] - ○ → y - □ p[x] - □ d - ○ x → d

2) y - □ p[x] - ○ x - prawym synem → p[x] - prawym → lewy → right-rot(p[x])

3) y - □ p[x] - ○

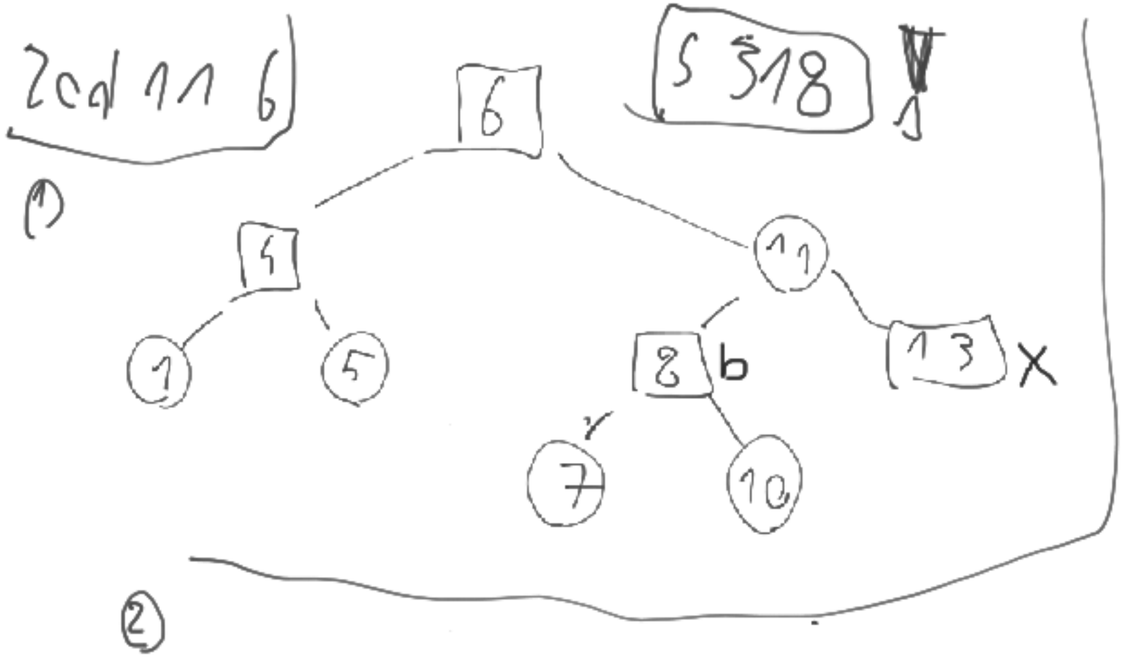
→ d - ○ p[x] - □ left-rot(d)

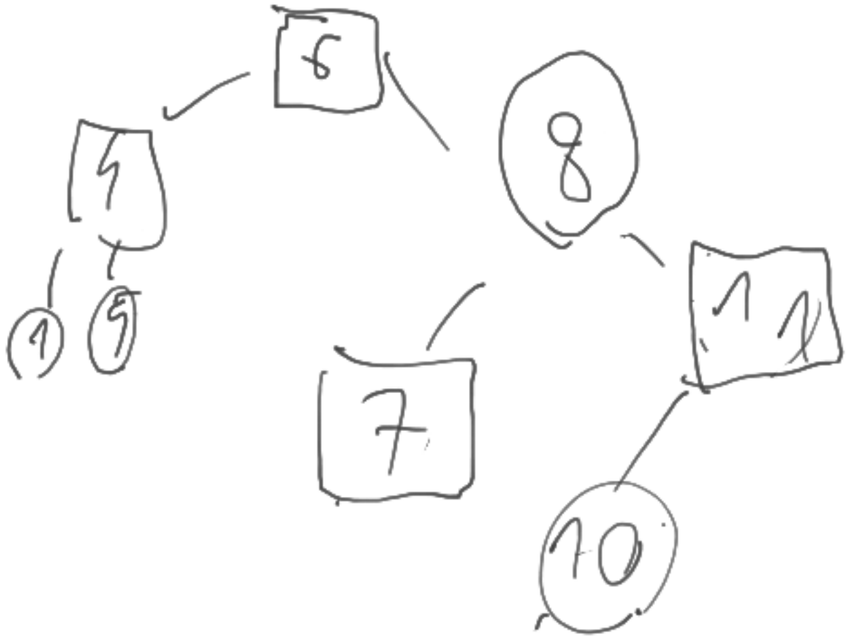
Zad 11

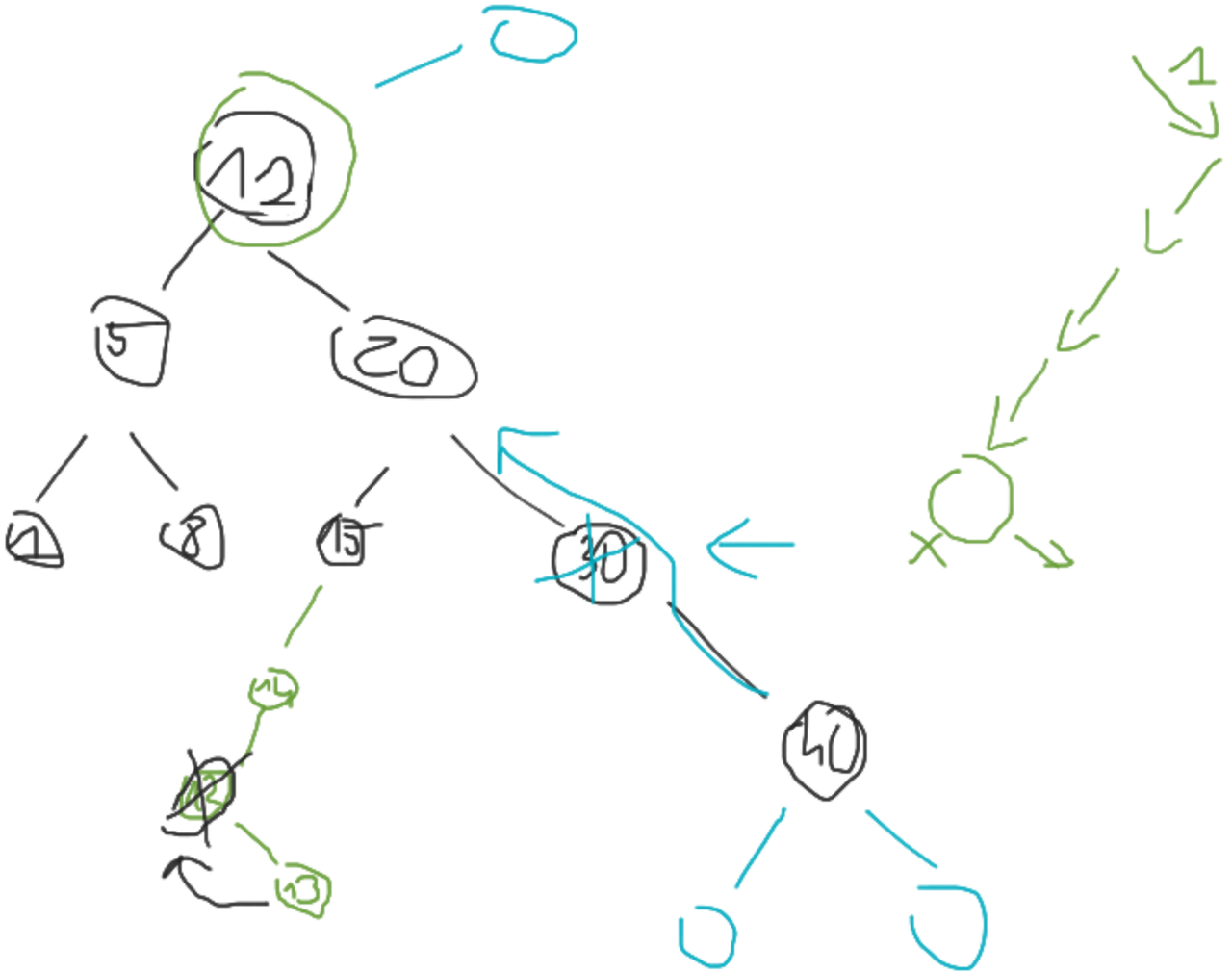


Zad 11 6

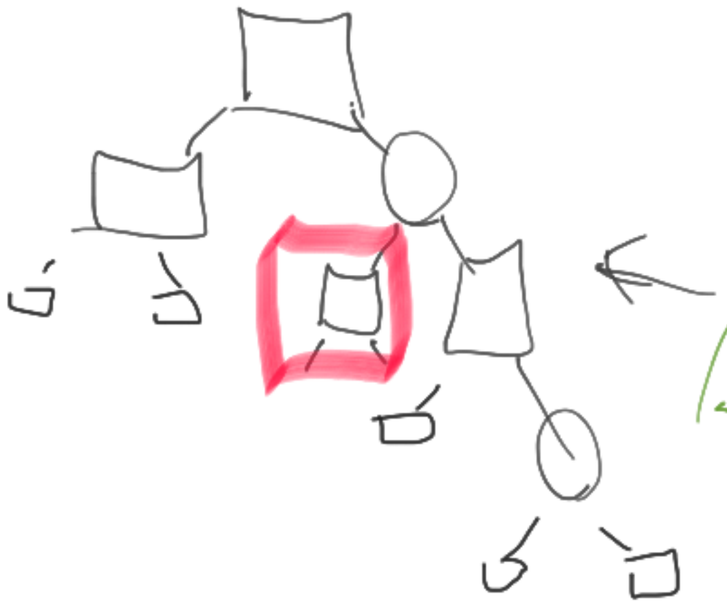
5 518






12
5
20
1
8
15
30
14
13
40

2 cd 12 | 1

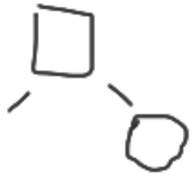


$$\left(2^{(2x-1)}-1\right)-\left(2^{(x-1)}-1\right)$$

$$2^{2x-1}-1-2^{x-1}+1=$$

$$=2^{2x-1}-2^{x-1}=2^{x-1}\left(2^{x}-1\right)$$

$$h(x)=2^{x-1}\left(2^{x}-1\right) \quad \infty$$



~~X~~

H_6	L	P	
1	0	$1 \to 2-1$	2^1
2	1	$7 \to 8-1$	2^3
3	3	$31 \to 32-1$	2^5

$$\Rightarrow 2^{(2x-1)}-1$$

P L

$$2^{x-1}-1$$