L1 2 1

$w(x) = 1 + 2x + 3x^2$

$1 + x(2 + x \cdot (3 + x \cdot (4 +)$

```
t=1
s=0
for i in range(n+1):
    s+= t    ·a[i]
    t =x·t
```

```
s=a[n+1]
for i=n .. 0
    s = s·x
    s += a[i]
```

$x^{10} = x^2 \cdot ((x^2)^2)$

$10_{10} = 1010_2$

```
t=x
s=1
t=t·t   // x^2
s=s·t   // s=x^2
t=t·t   // x^4
t=t·t   // x^8
```

$s = s \cdot t$ // $x^2 \cdot x^8 = x^{10}$

x^b

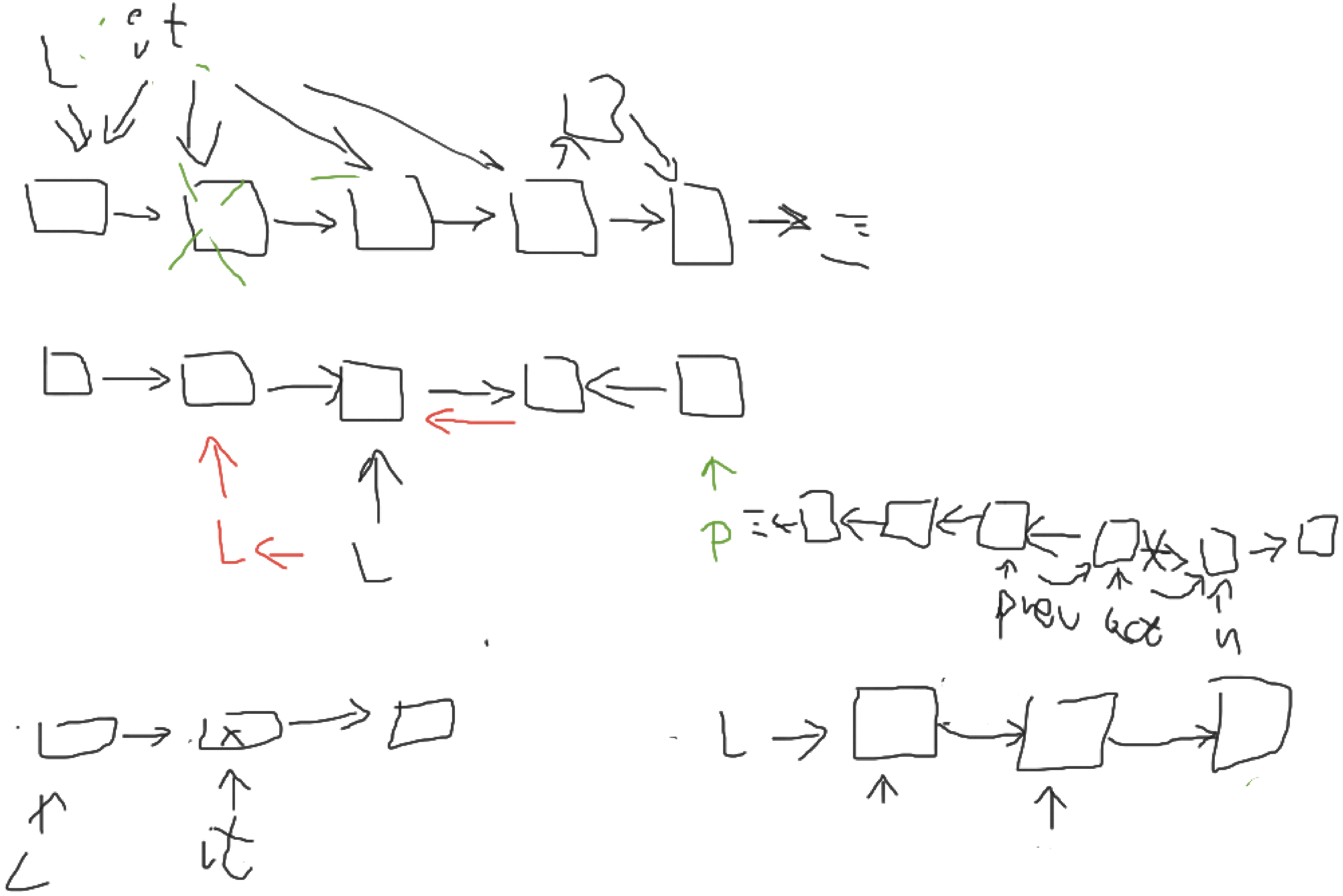
```
s=1
t=x
while b>0:
    if b % 2=1:
        s*=t
    t=t*t
    b /=2
```

L 1 2 4 | >?<

(3 ↔ 5) (2 1) (8 7) (6 4)

min? max?

3 por. na pary $\frac{3}{2}n$


L
it
L
L
it
P
prev
n
L

n
1
2
3
4
5
6
7
8
9
10

1
2
3
3
4
4
4
4
5

if $t[i]<x$

P

if $t[i]>x$

L

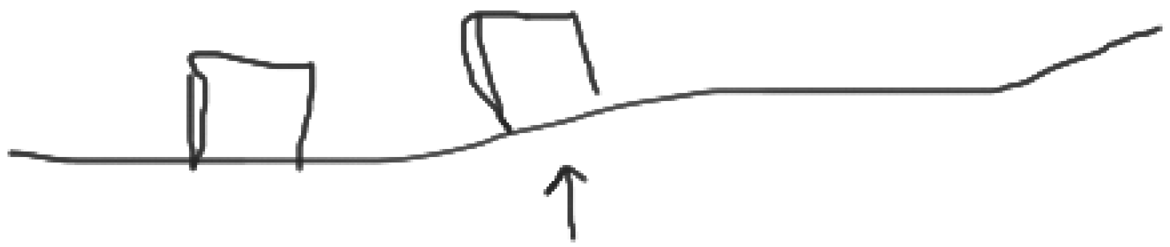
if $t[i] <= x$

P

else:

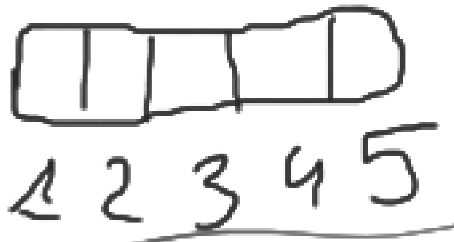
L

1 2 3 4



$O(\log_2 n)$

$\log_2 n + C$



$n=5$

i

1

AAS/5

$X \sim U$

$$\mu = \frac{\sum_{i=1}^{n} i}{n} = \frac{(1+n)}{2}$$

$$\frac{\sum_{i=1}^{n} (\mu - i)^2}{n}$$

2 coll 4

a) $(n-1)$; 4 bit

b) $T(n) = \begin{cases} 0 & n=1 \\ T(n-1)+1 \end{cases}$

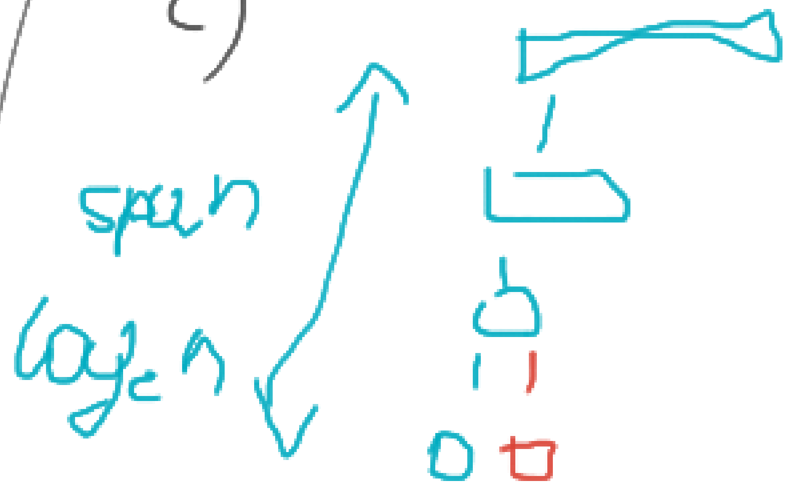
$T(n) = n-1$ $\boxed{4 \cdot (n-1) \text{ bit}}$

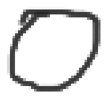
$T(2) = T(1) + 1 = 0 + 1 = 1$

$(n-1)$ 4 bit

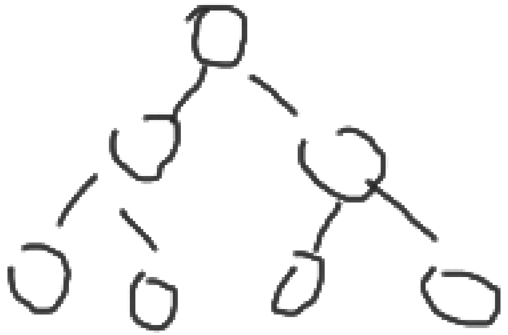
$T($

c)



$h = 1$  1 $2-1$

$h = 2$  3 $4-1$

$h = 3$  7 $8-1$

$h = 4$ 15 $16-1$

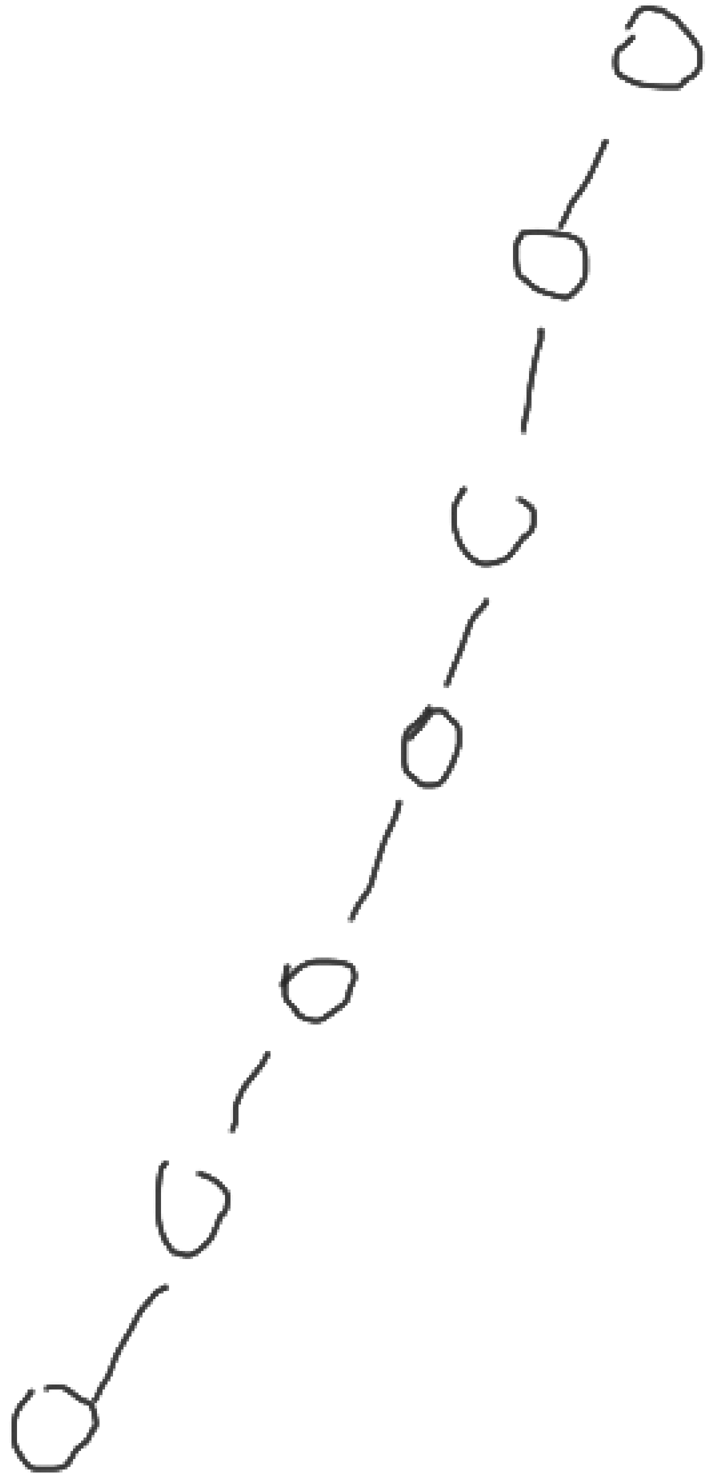
$h = 5$ 31

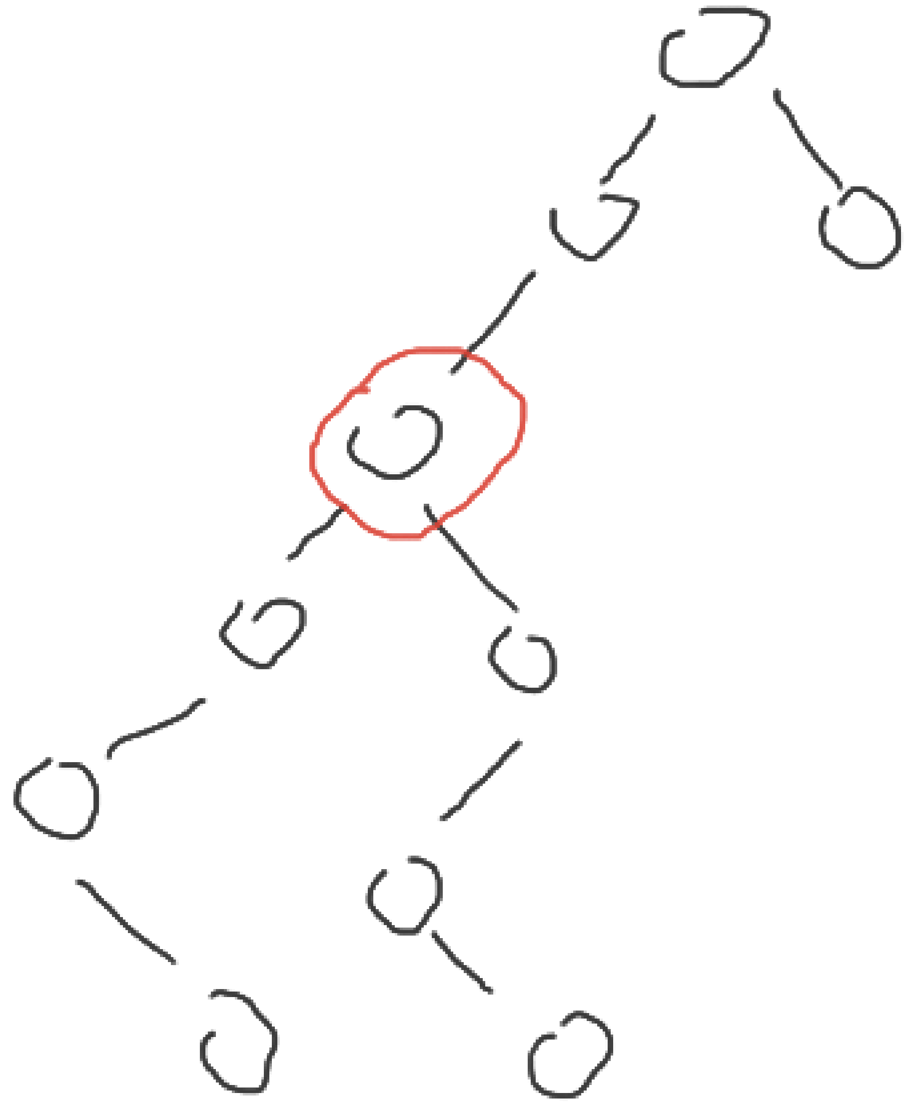
$2^h - 1$

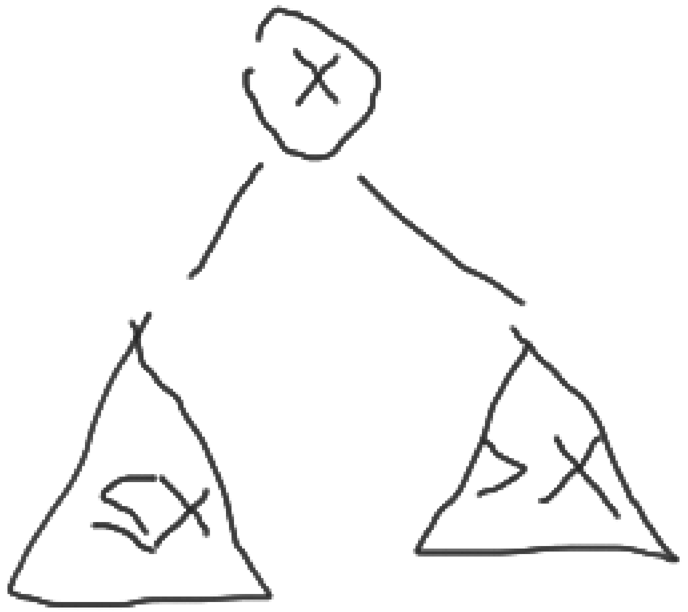
naikiellssa
$h = n$

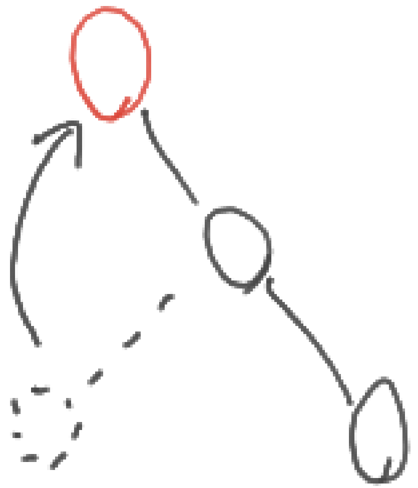
najmmissa

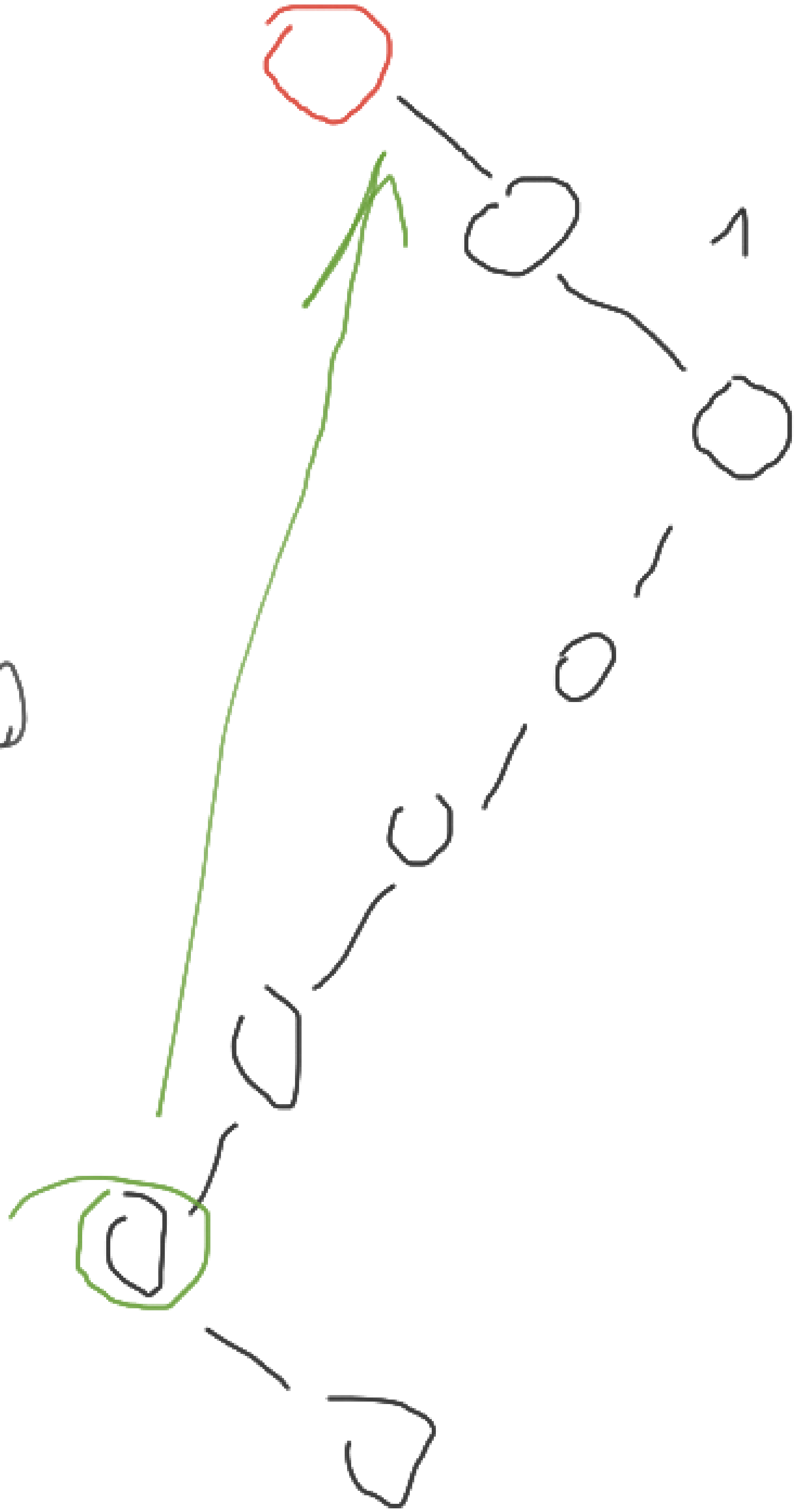
$\lceil \log_2(n+1) \rceil$






x
<x
>x




1

[insert Sort]

−∞ 5 | 4 3 1 6 2

i j

posort. nie-sort

−∞ 4 5 | 3 1 6 2

i j

3 4 5 | 1 6 2

1 3 4 5 | 6 2

1 3 4 5 6 | 2

1 2 3 4 5 6 |

L 4/23

$A \quad T(n) = 7T\left(\frac{n}{2}\right) + n^2 \qquad a = 7 \qquad b = 2 \qquad f = n^2$

$A' \quad T'(n) \; a'T'\left(\frac{n}{4}\right) + n^2 \qquad a' = ? \qquad b' = 4 \qquad f' = n^2$

$n^{\log_2 7} > n^2$

$T(n) = \Theta\left(n^{\log_b a}\right) = \Theta\left(n^{\log_2 7}\right)$

$\downarrow$

$n^{\log_4 49}$

$n^{\log_4 a} == n^2$

$\log_4 a == 2 \rightarrow \quad 4^x = a$

$\log_4 16 = 2 \qquad 4^2 = a = 16$

$a' = 16$

$T(n) = \Theta\left(n^{\log_b a} \cdot \log n\right) = \Theta\left(n^2 \cdot \log n\right)$

$a' > 16$

$T'(n) = \Theta\left(n^{\log_b a'}\right) = \Theta\left(n^{\log_4 a'}\right)$

④ $n^{\log_4 49} > n^{\log_4 a'} = n^{\log_4 48}$

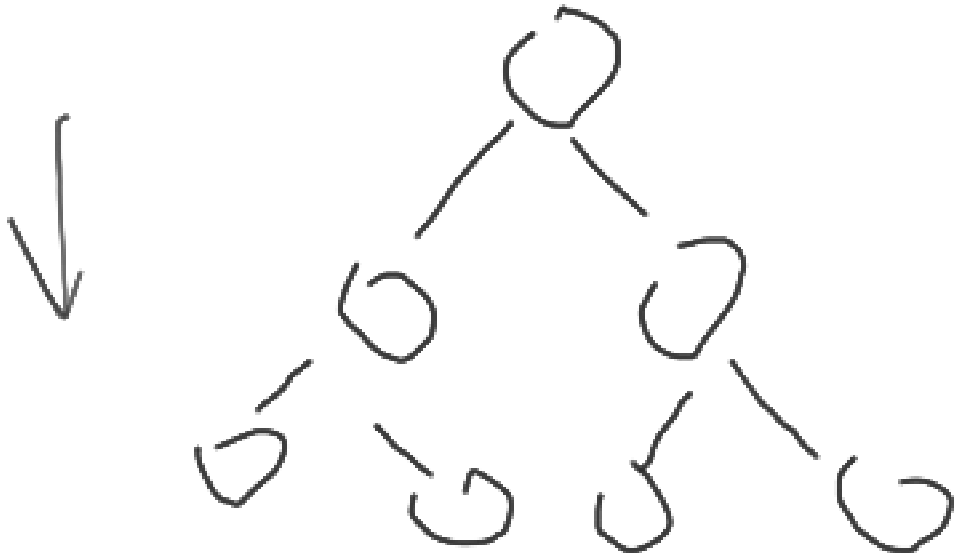
$49 > a' = 48$

T 5 2 1

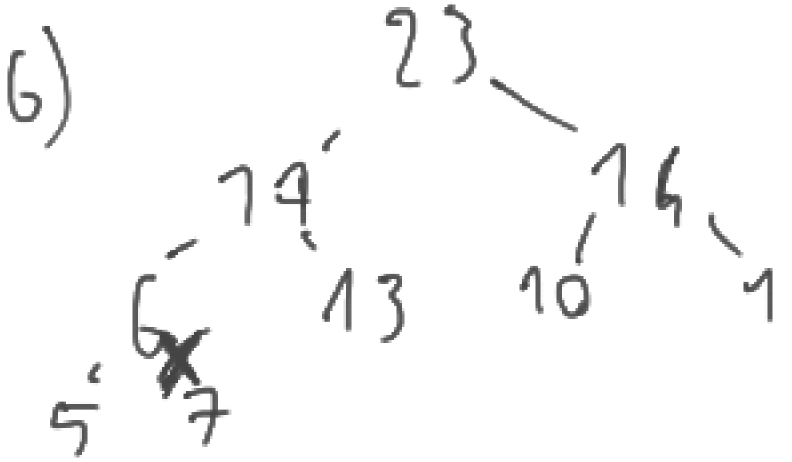
a) 5 4 3 2 1

↓

YES

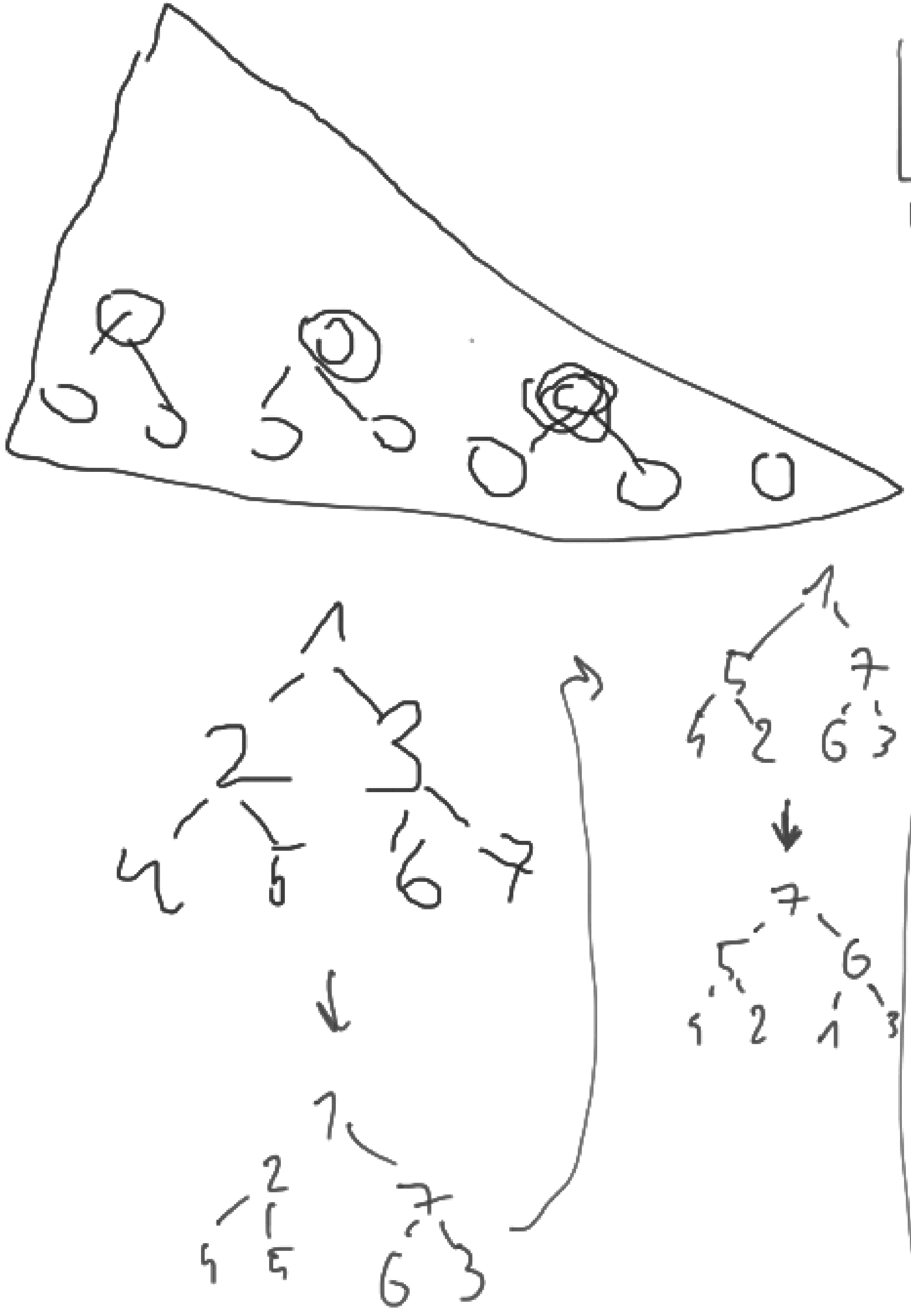


b)

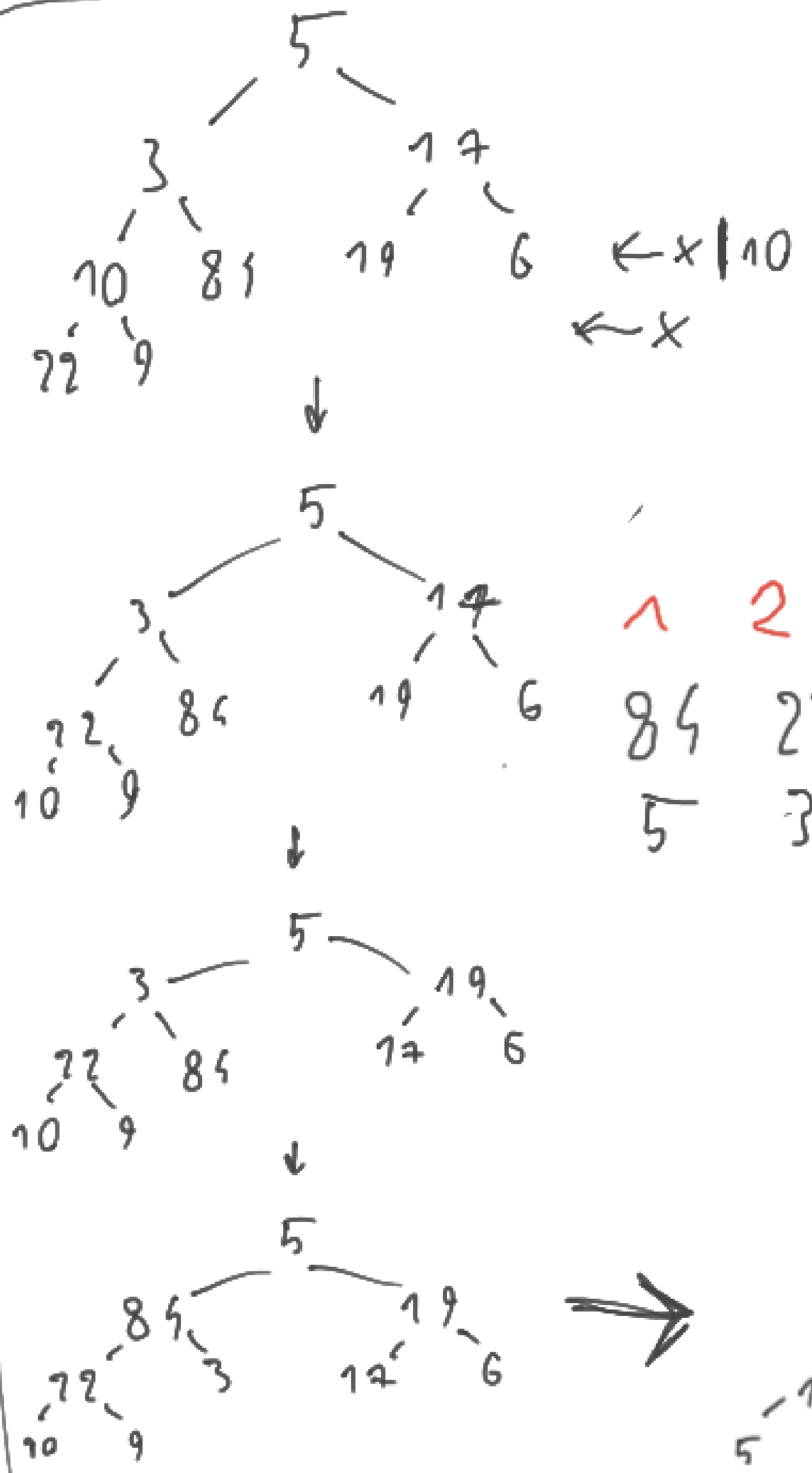


NO

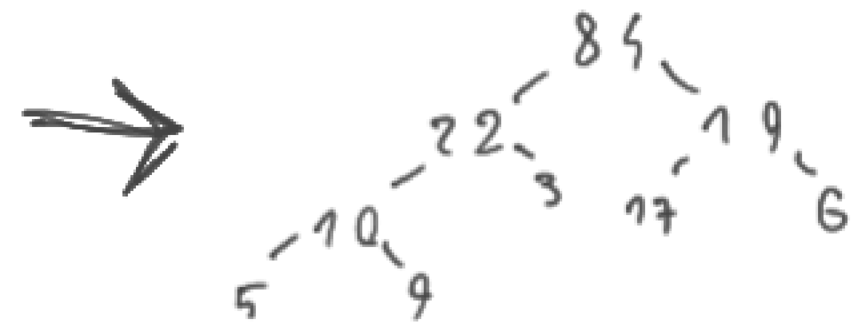
22



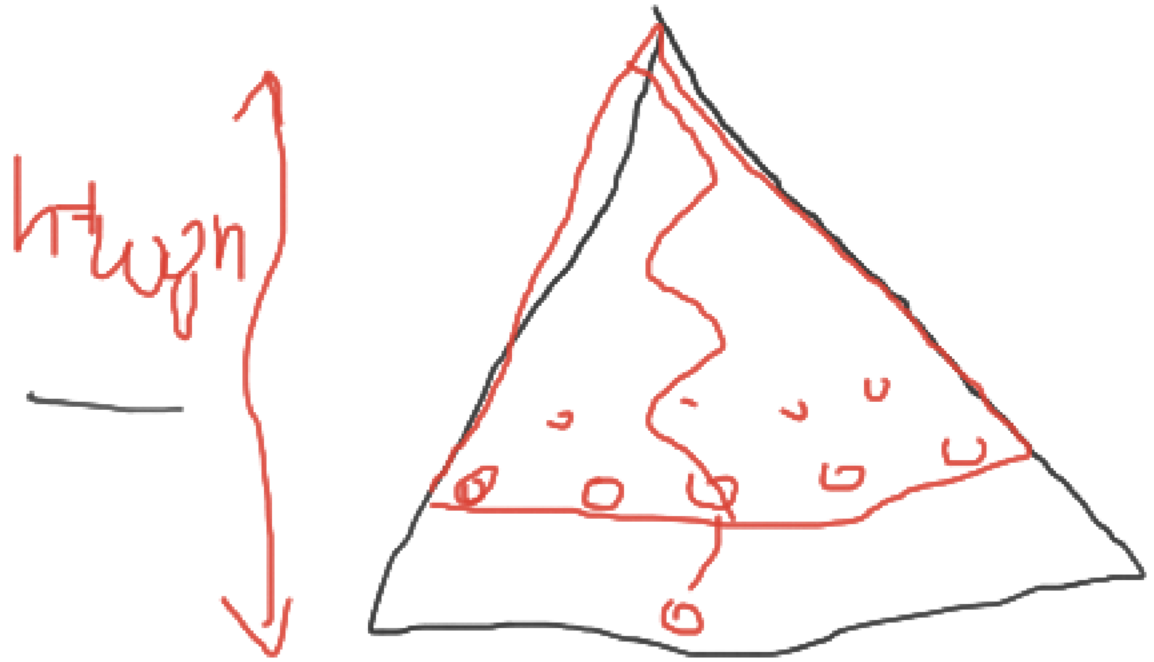
23



1	2	3	4	5	6	7	8	9
84	22	19	10	3	17	6	5	9
5	3	17	10	84	19	6	22	9



25

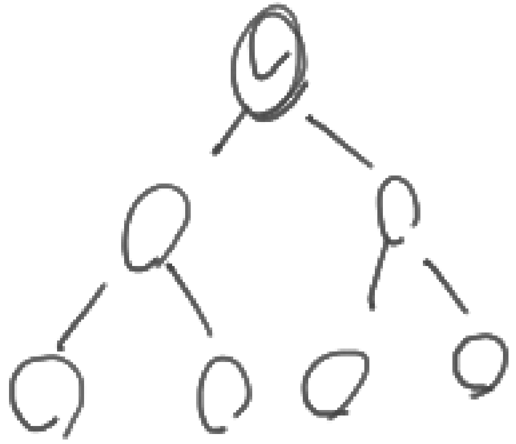


$$\frac{n}{2} \cdot O(\log n) = O(n \log n)$$

$$\text{presiej} = O(h)$$

$\boxed{26}$

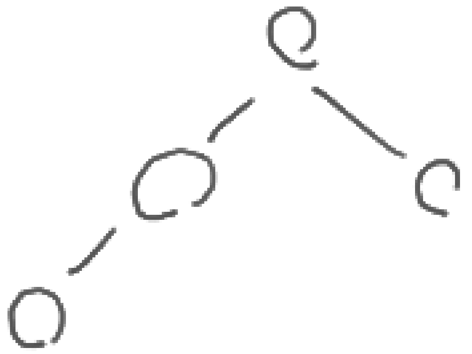
$h=3$



$n=7$

$\log_2 7 = 2.9\ldots$

$\lfloor 2,9 \rfloor + 1 = 3$



$n=5$

$\log_2 5 = 2$

$\lfloor 2,5 \rfloor + 1 = 3$

$2^{h}-1$ max

2^{h-1} min

$k \leq x < k+1$

$\lfloor x \rfloor = k$

$\lfloor 3 \rfloor = 3 \qquad 3 \leq 3 < 4$

$\lfloor 3,5 \rfloor = 3 \qquad 3 \leq 3,5 < 4$

$\lfloor 3,999 \rfloor = 3 \qquad 3 \leq 3,999 < 4$

$1 \qquad 2^1-1$

$3 \qquad 2^2-1$

$7 \qquad 2^3-1$

$h-1 \leq \log_2 n < h \quad | +1$

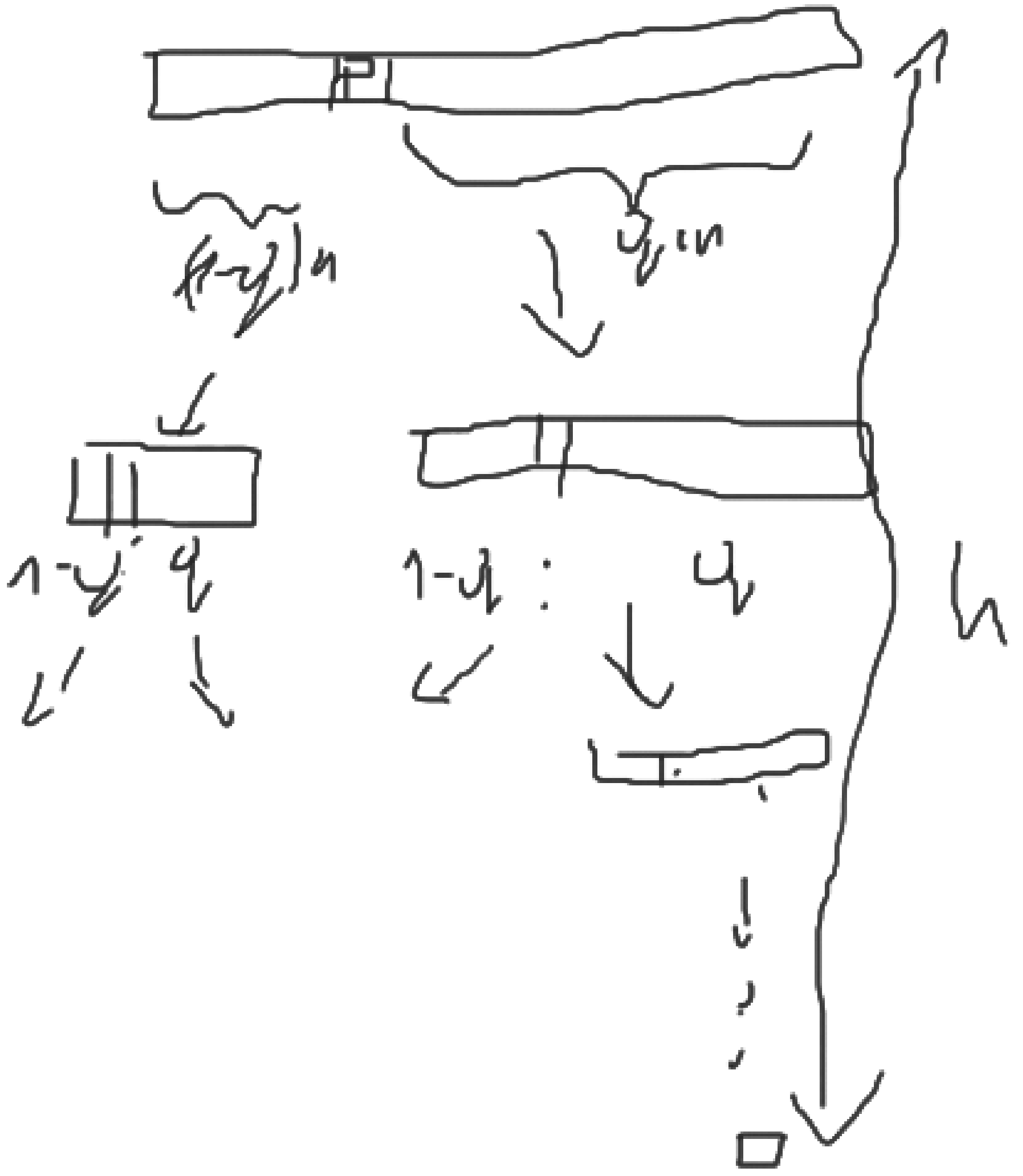
$h \leq \log_2 n + 1 < h+1$

$2^{h-1} \leq n < 2^h$

$\lfloor \log_2 n + 1 \rfloor = \lfloor \log_2 n \rfloor + 1 = h$

$$T(n) = 2T\left(\frac{n}{2}\right) + n = O(n \log n)$$

$$T(n) = T(n-1) + n = \sum_{i=1}^{n} i = O(n^2)$$



n

$q \cdot n$

$q^2 n$

$q^h \cdot n = 1$

$$q^h n = 1$$

$$q^h = \frac{1}{n} \quad \Big| \log_q \qquad a^x = b$$

$$\log_q q^h = \log_q \frac{1}{n} \qquad \log_a b = x$$

$$h = \log_q \frac{1}{n} = \log_q n^{-1} =$$

$$= -1 \cdot \log_q n = -\log_q(n) =$$

$$= \boxed{-\frac{\log_2(n)}{\log_2(q)}} = O(\log(n))$$

$$\log x^k = k \cdot \log x$$

$$\log_a b \cdot \log_b c = \log_a c$$

$$\log_a b \cdot \log_b c = \log_a c$$

$$\log_2(q) \cdot \log_q(n) = \log_2(n) \quad \Big| : \log_2(q)$$

$$\log_q(n) = \frac{\log_2(n)}{\log_2(q)}$$

2 mod 2

$f(a) \to 0$

$-1 \; 0 \; 1 \quad 2 \; 3 \; 4 \quad 5$

$f(b) \to \underline{1}$

$$f(x) = \frac{x-a}{b-a}$$

$k=3$

$$\left\lfloor n \frac{x-a}{b-a} \right\rfloor$$

$n \quad n-1 \quad n-2 \; \ldots \ldots \; \underline{1}$

$[n, n-1, n-2, n-3, \ldots, 1]$

$$\left(\frac{n}{k}\right)^2 \cdot k = \frac{n^2}{k} = O(n^2)$$

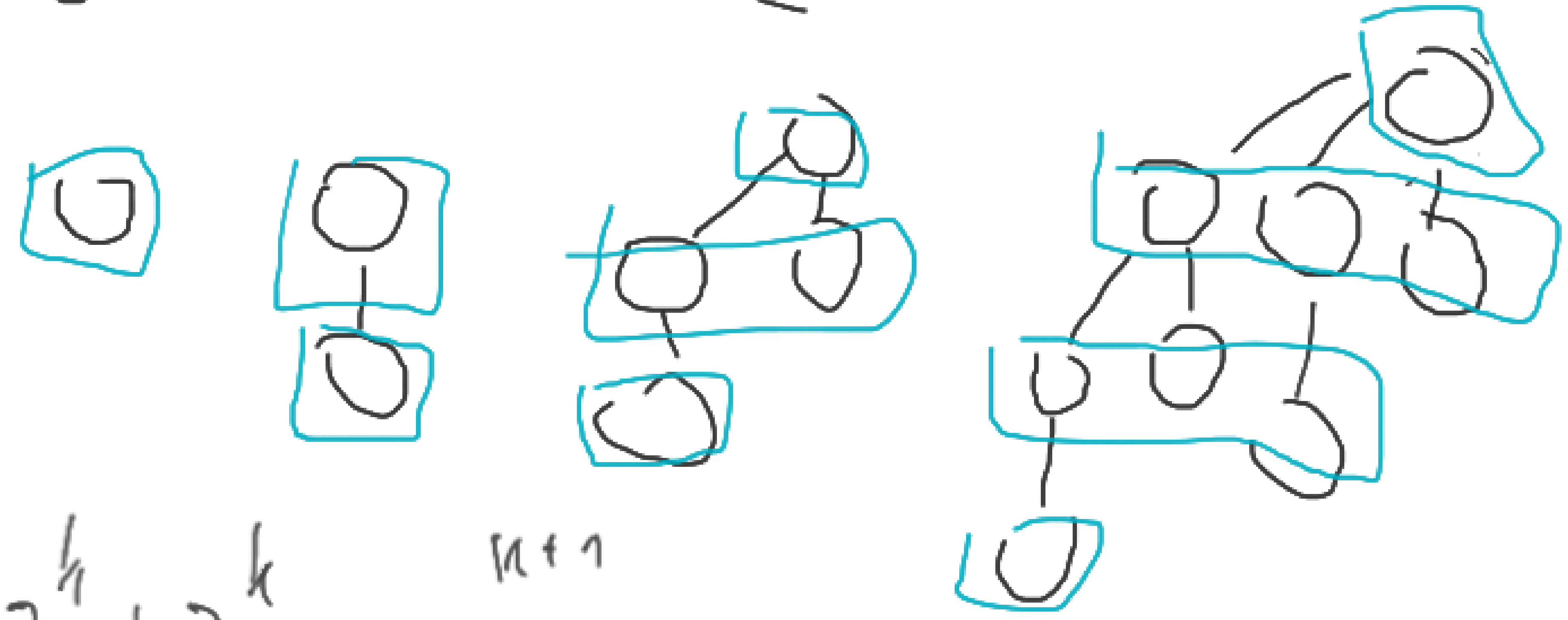
Lista 8

zad 1

- t
- tablica pointerów$[k+1]$ $(2t+1)$ $\begin{bmatrix} \text{min } t \text{ synów} \\ \text{max } 2t \text{ synów} \end{bmatrix}$
- tablica kluczy$[k]$ $(2t)$ default
- Rodzic
- k - liczba kluczy = $(t, 2t)$

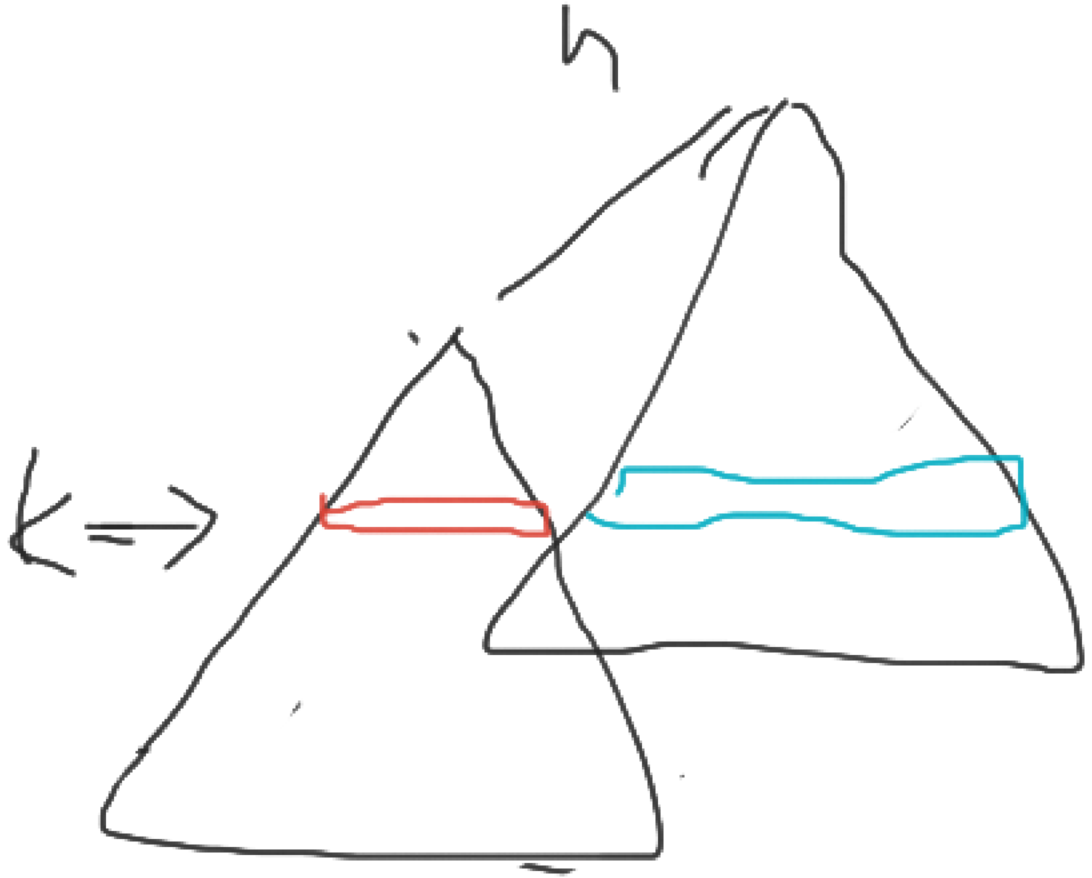
L921

0 1 2 3



$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$2^k + 2^k = 2^{n+1}$$



$$T(n,k) = T(n-1,k) + T(n-1,k-1)$$

$$\binom{n-1}{k} + \binom{n-1}{k-1} = \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-1-k+1)!} =$$

$$= \left(\frac{(n-1)!}{(k-1)!\cdot(n-k-1)!}\right)\left(\frac{1}{k}+\frac{1}{n-k}\right) = (\ldots)\left(\frac{n-k+k}{k\cdot(n-k)}\right) = (\ldots)\left(\frac{n}{k\cdot(n-k)}\right) \text{ FBC}$$

L921

$$= \left(\frac{(n-1)!}{(k-1)! \cdot (n-k-1)!}\right)\left(\frac{1}{k} + \frac{1}{n-k}\right) = (\ldots)\left(\frac{n-k+k}{k \cdot (n-k)}\right) = (\ldots)\left(\frac{n}{k \cdot (n-k)}\right) =$$

$$= \frac{n!}{k!\,(n-k)!} = \binom{n}{k}$$