t   t - synów   t-1 kluczy (min)

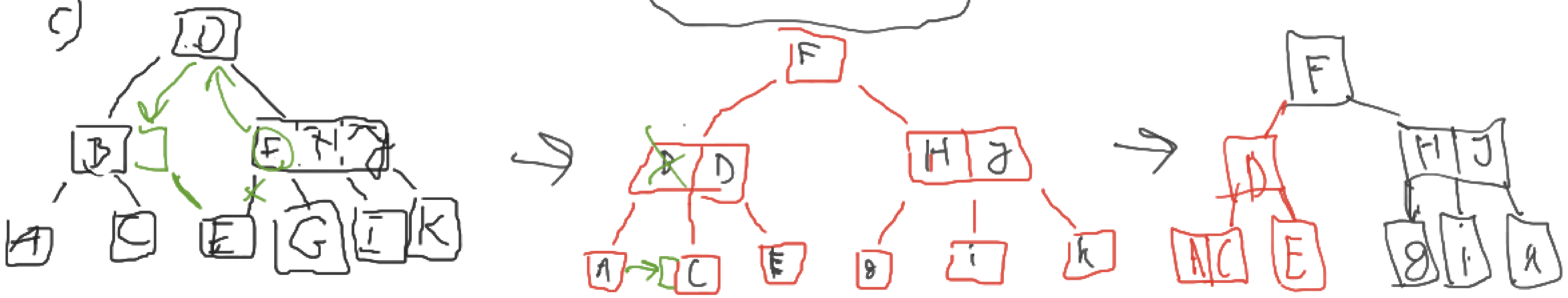
2t   2t-1 kluczy (max)

2) a)

1)

2)

c)



Zad 3

$T = 10$

1) $1 - 19$

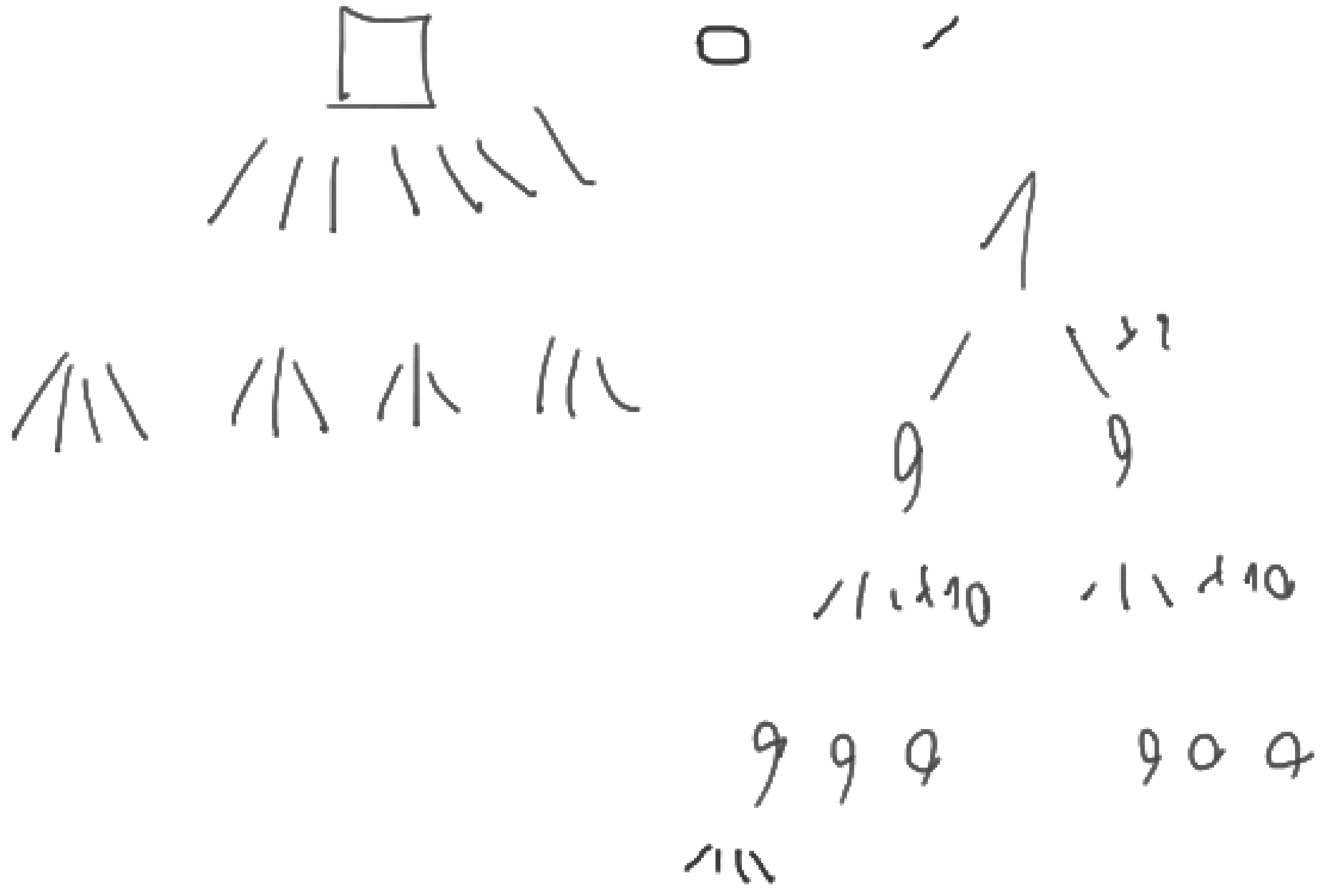
2) $0 - 20$

3) $9 - 19$

4) $10 - 20$

5) $(2T)^k$

6) $\begin{cases} k = 0 \to 1 \\ k \neq 0 \to 2(T-1) \cdot 10^{k-1} \end{cases} - (2t-1)^k$



$2 \cdot 9 \cdot 10^0$

$2 \cdot 9 \cdot 10^1$

$2 \cdot 9 \cdot 10^2$

2 cd 4

$$S = 1 + q + q^2 + \dots + q^k$$

$$S = 1 + q\,(1 + q + \dots + q^{k-1})$$

$$S = 1 + q\,(S - q^k)$$

$$S = 1 + q \cdot S - q^{k+1}$$

$S - qS = \dots$

$$S(1-q) = 1 - q^{k+1}$$

$$S = \frac{1 - q^{k+1}}{1-q} = \frac{q^{k+1} - 1}{q - 1}$$

$$\sum_{i=1}^{n} t^{i-1} = 1 + t^1 + t^2 + \dots + t^{n-1} =$$

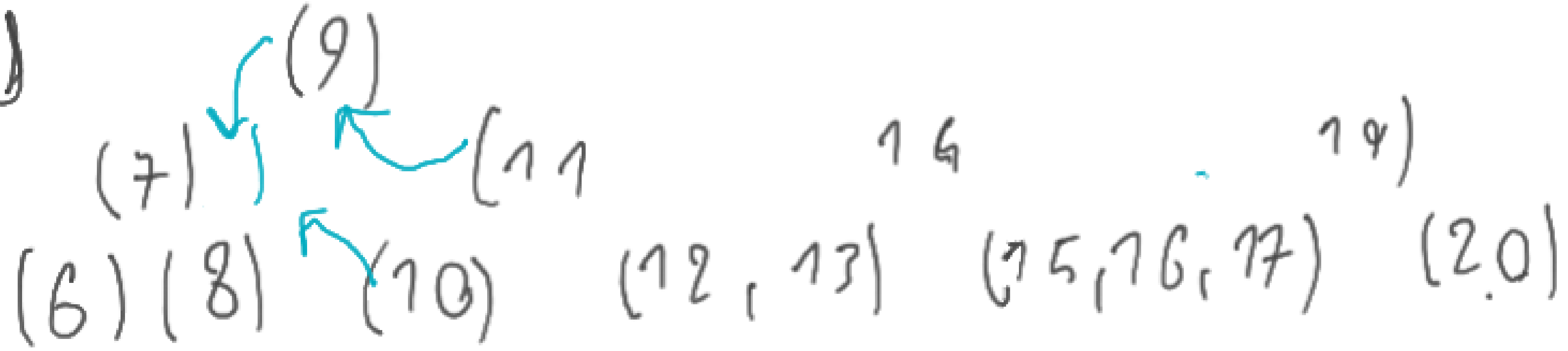
$$\Downarrow$$

$$\frac{t^n - 1}{t - 1}$$

Zad 5 $(2t-1)\sum_{i=0}^{n} t^{i} = (2t-1)\cdot\frac{t^{n+1}-1}{t-1}$

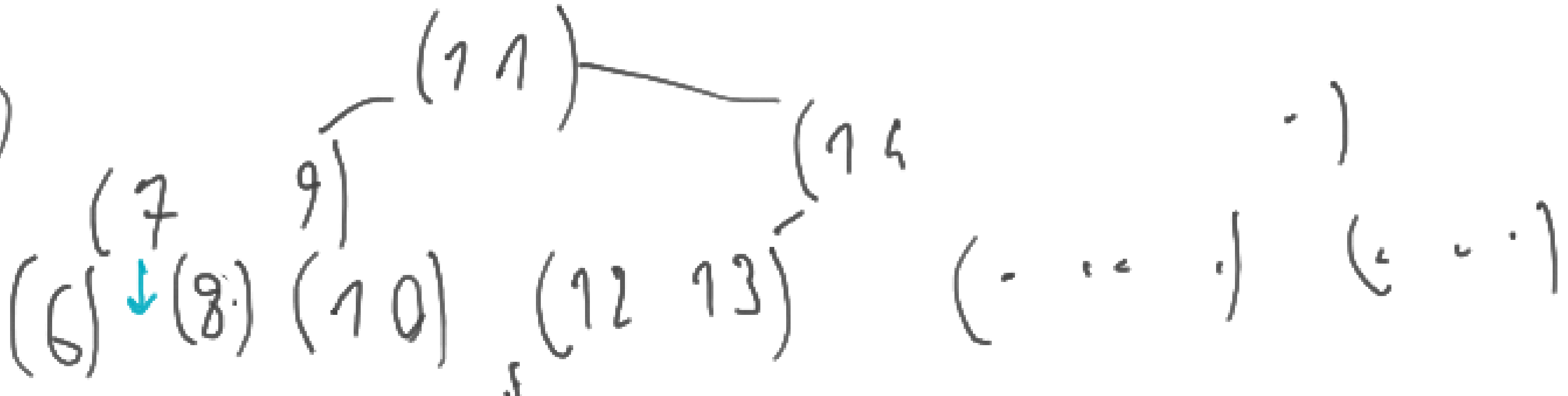
Zad. 5

1)

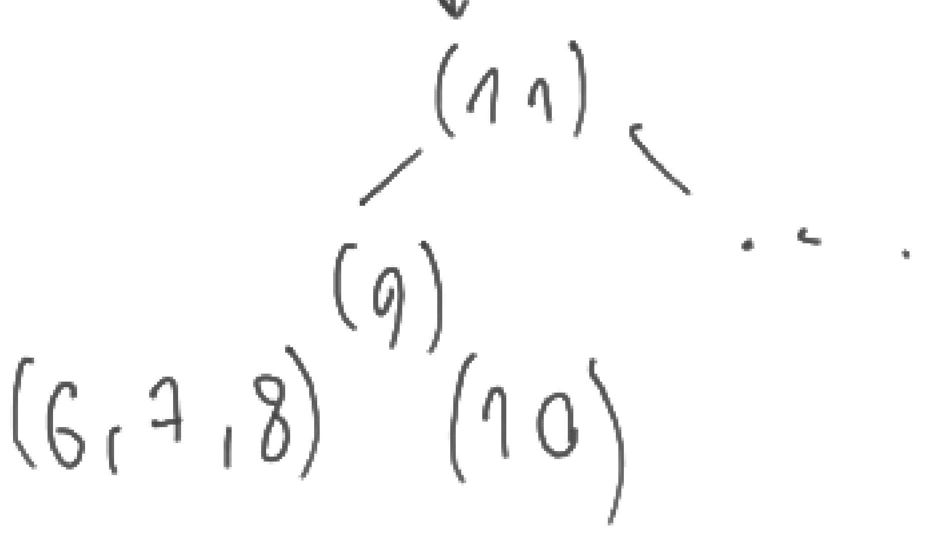


t=2
∧ 2–4
... 1–3

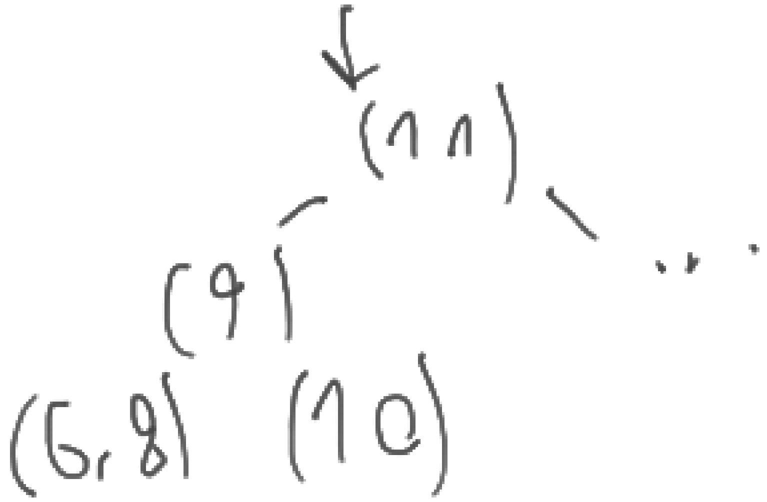
2)



3)



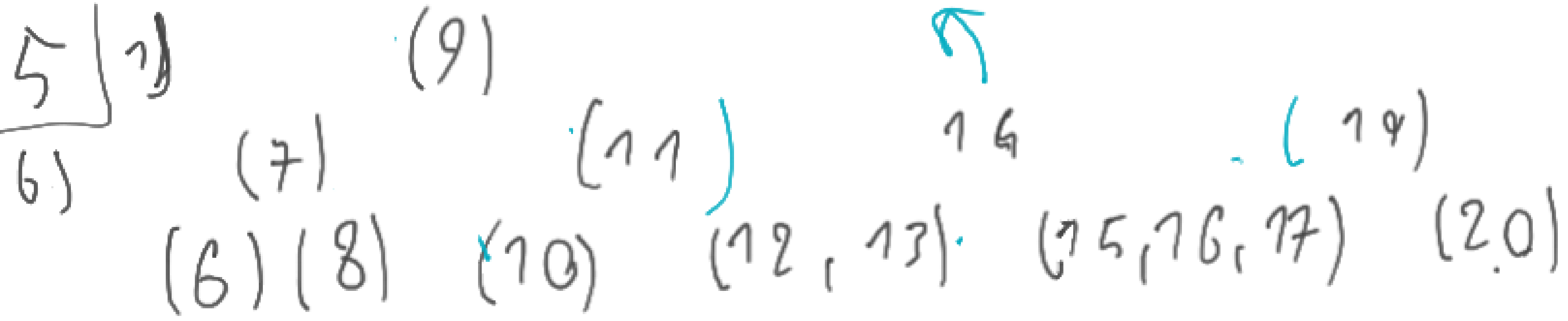
4)



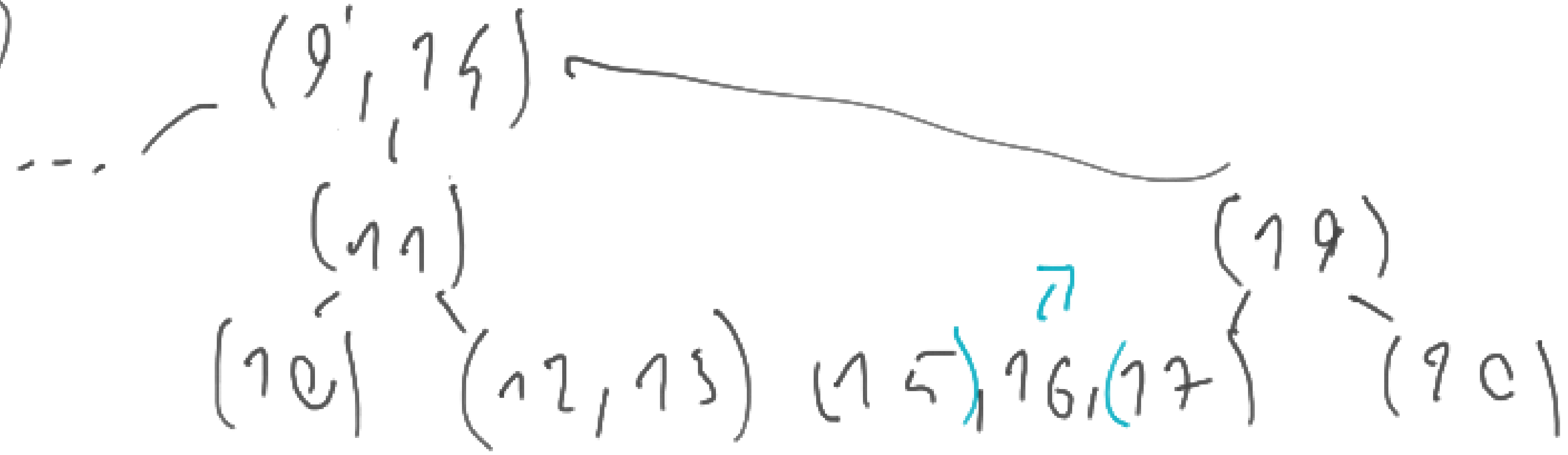
Σ ci! 5

+18

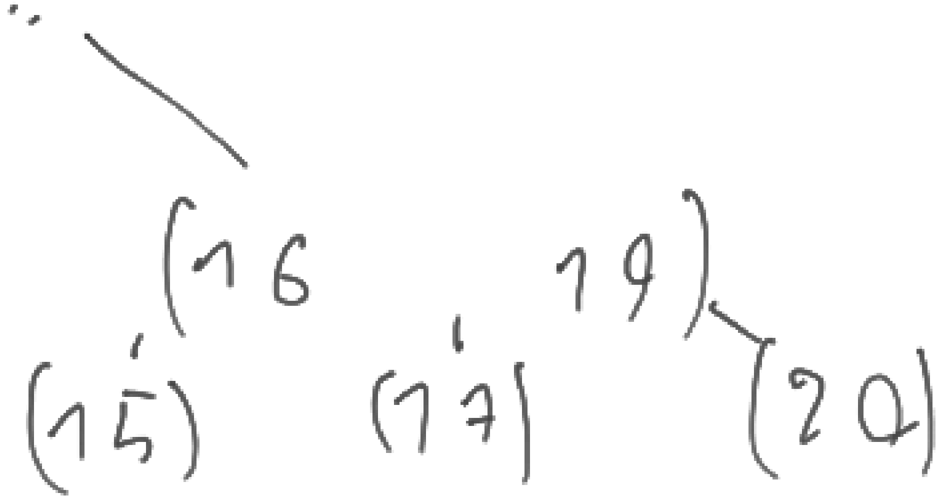
t = 2
∧ 2 – 4
aoo 1 – 3



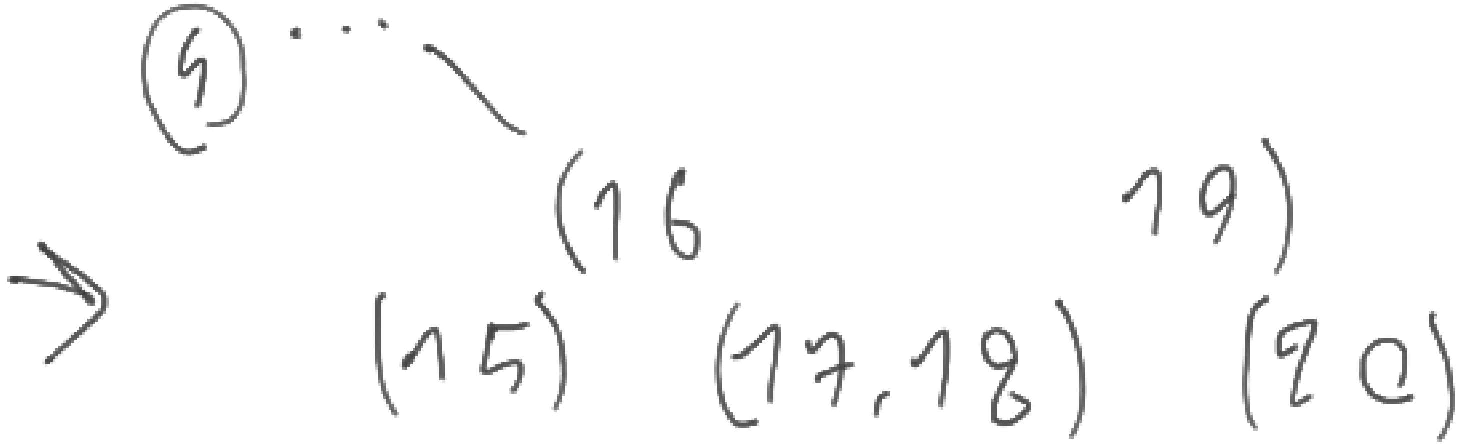
2



3



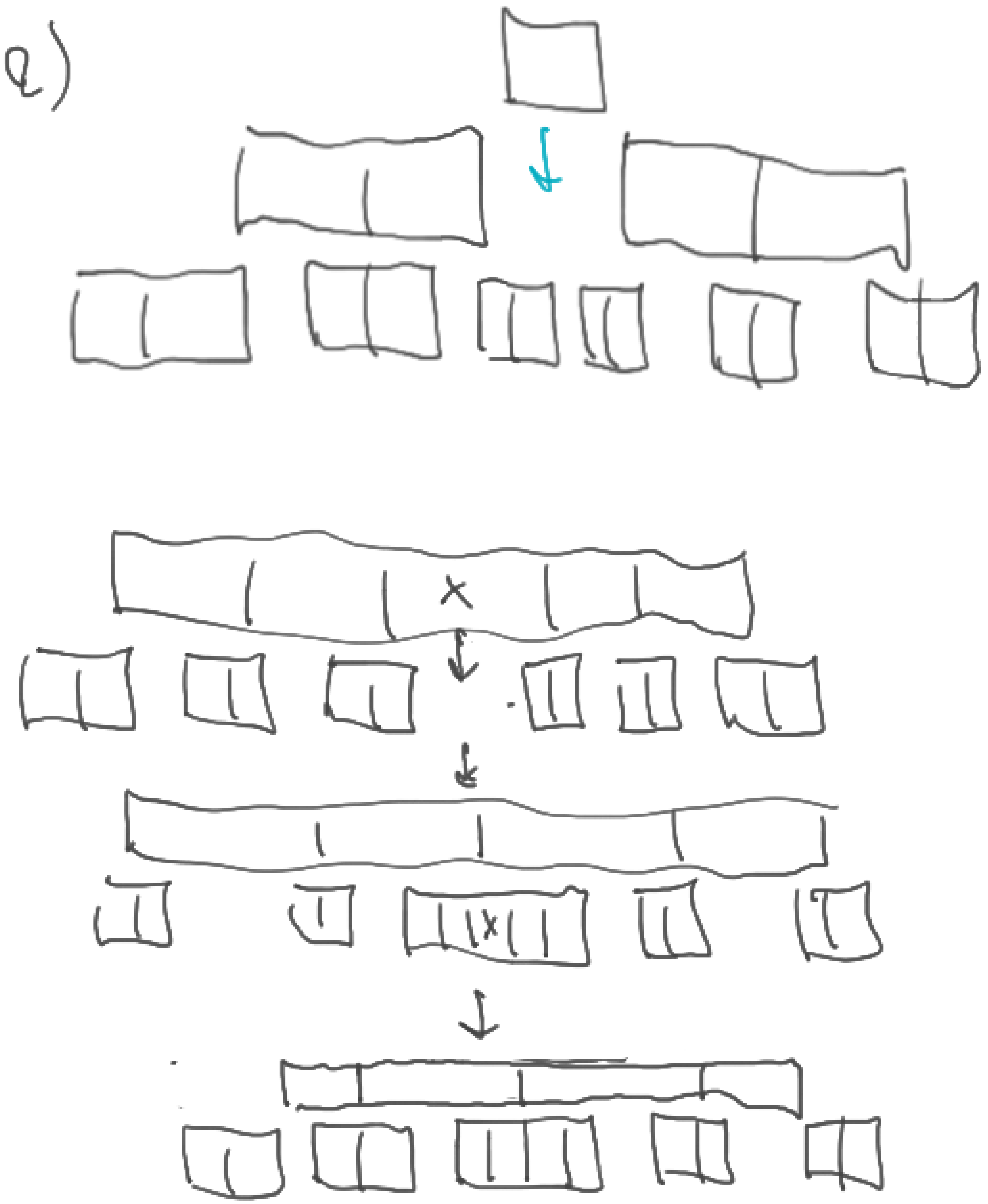
4



2od 7 f=3 л 3 - 6

aoo 2 - 5

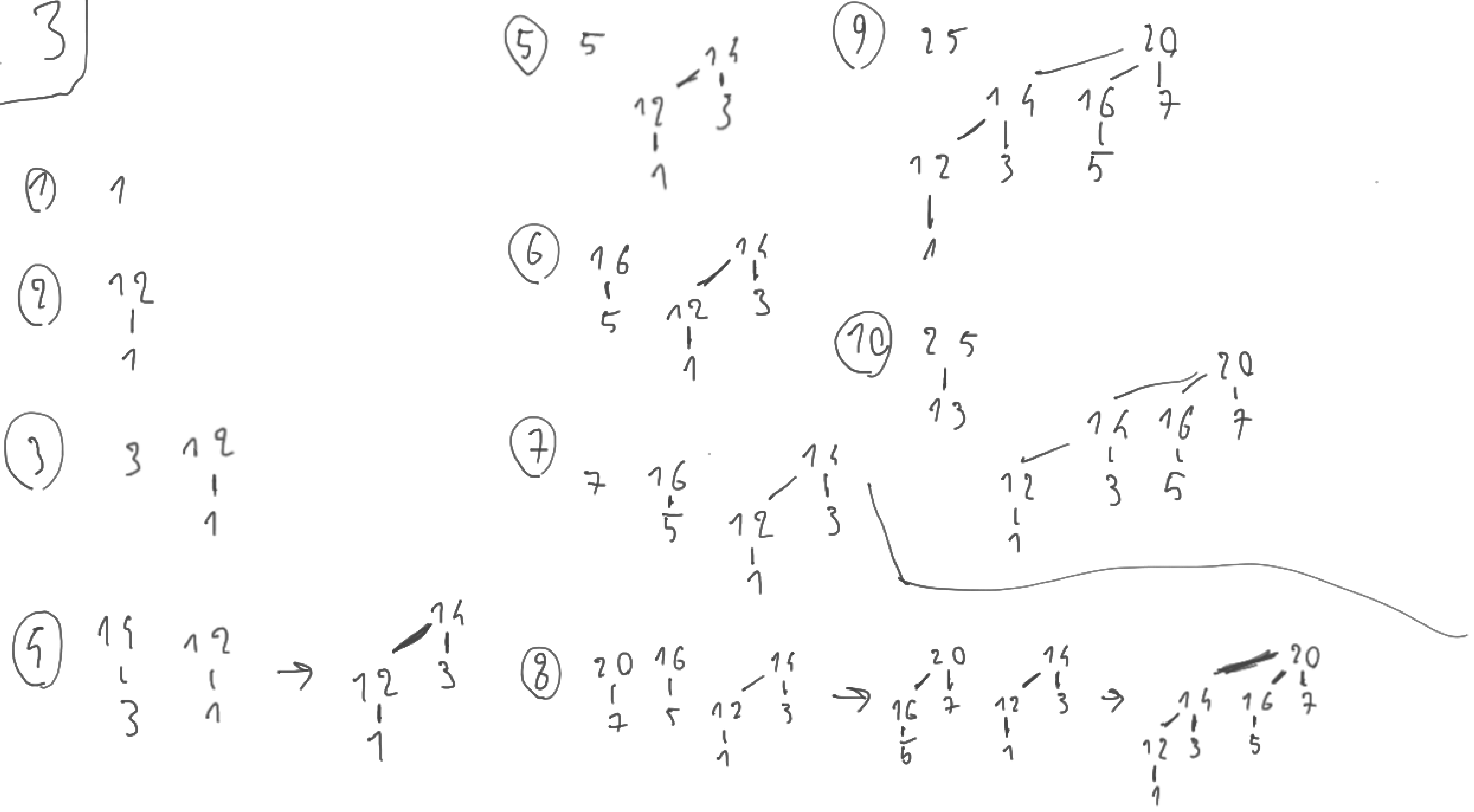
a)



5 (-12)

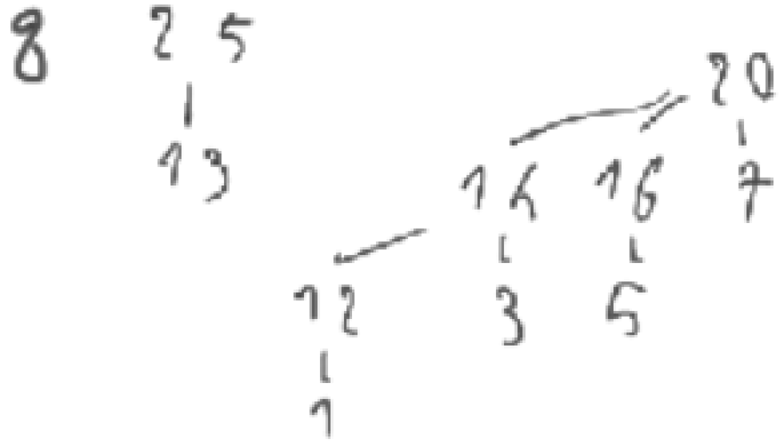
17

$\underbrace{L\ 9\ 2\ 3}_{2}$



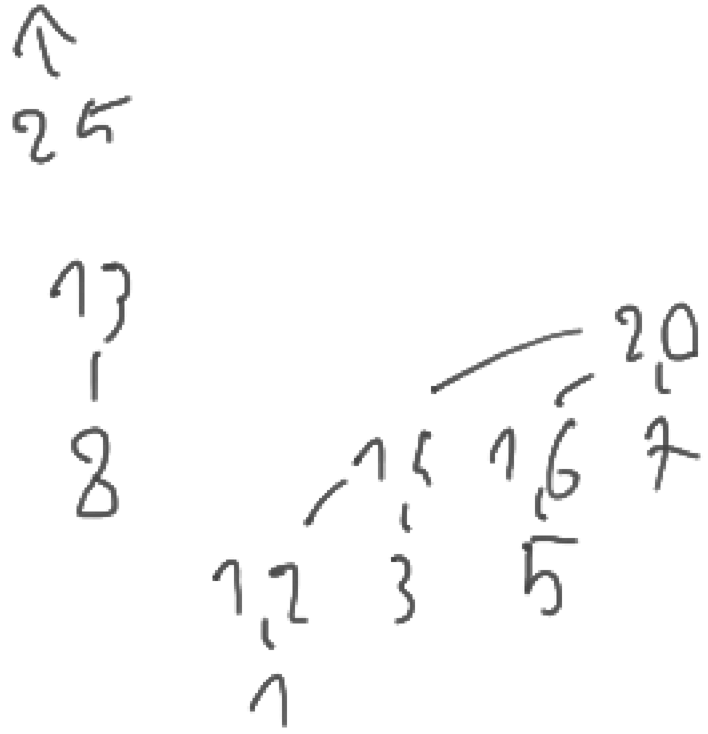
L 9 z 3

11

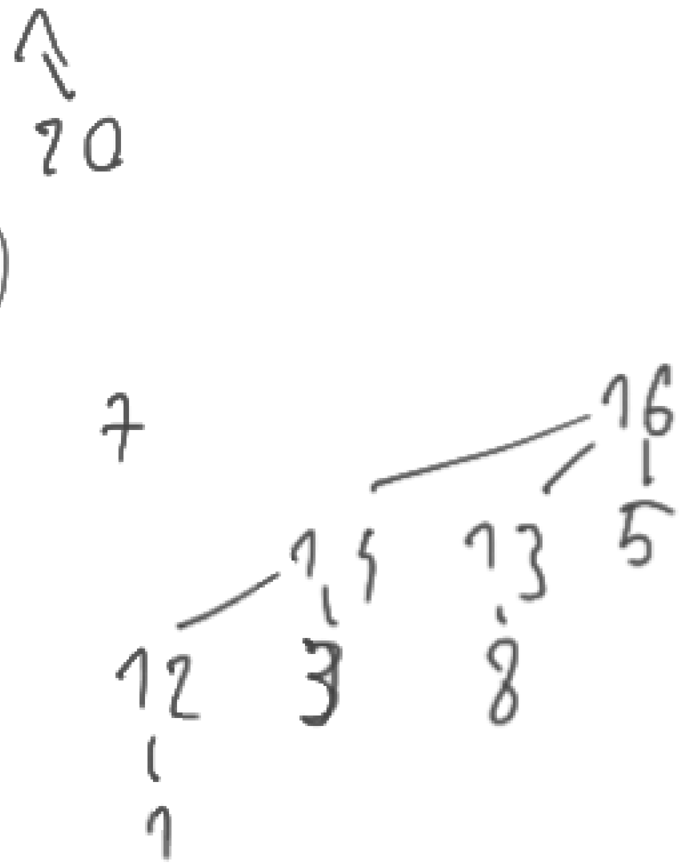


b)

1



2



L10 2 1

$C = A \times B$

$n \times m \cdot m \times k$

$A_{12} = 5 \times 2 \ (100)$

$A_{23} = 10 \times 12 \ (240)$

$A_{34} = 2 \times 5 \ (120)$

$A_{45} = 12 \times 50 \ (3000)$

$A_{56} = 50 \times 6 \ (1500)$

1 | 2 3 4

1 2 | 3 4

1 2 3 | 4

A_1 ~~5x10~~

A_2 10x2

A_3 2x12

A_4 12x5

A_5 5x50

A_6 50x6

$5\times10 \cdot 10\times5 \rightarrow 250 + 220 = 470$

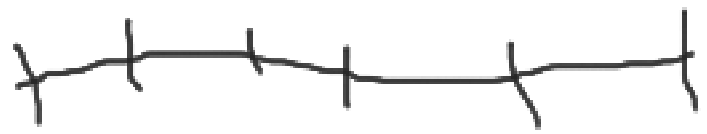
$100 + 120 = \underline{220}$

$5\times12 \cdot 12\times5 \rightarrow 220 + 240 = 460$

A	1	2	3	4	5	6
1		100	220	460		
2			250	220		
3				120	620	
4					3000	
5						1500
6						

z od 5

q & 6



$$\begin{array}{lcccccc} & 0 & 1 & 2 & 3 & 4 & 5 \\ \text{cena} = & [0, & 1, & 3, & 3, & 4, & 5] \\ \text{cieto} = & [0 & 1, & x, & x, & &] \end{array}$$

$$\sum_{i=0}^{n}$$

$$T(n) = \max\left(cena[n];\ \max_{k=1}^{\lfloor n/2 \rfloor}\left(T(k) + T(n-k) - c\right)\right)$$

$\downarrow +$

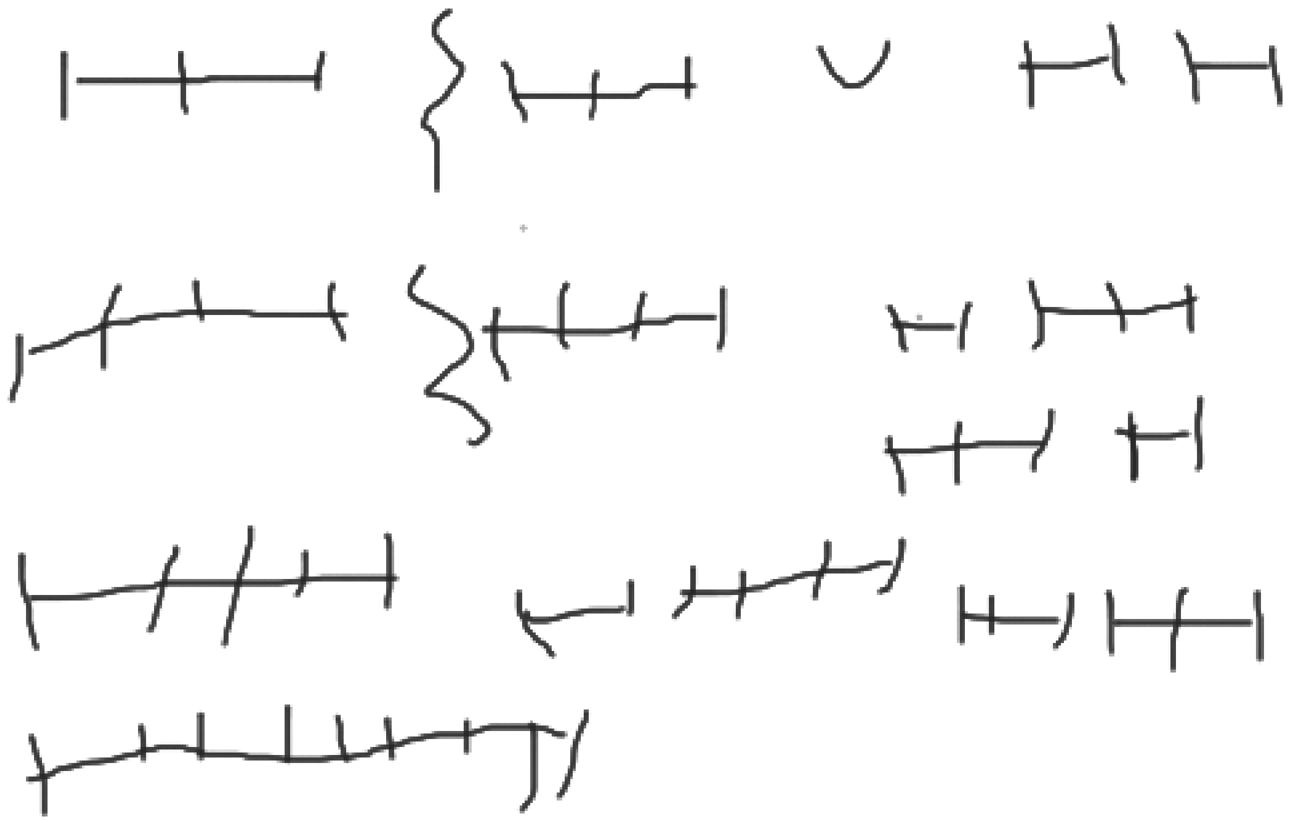
$$T(0) = 0$$

$$T(1) = cena[1]$$

$$T(2) = \max(cena[2];\ T(1) + T(1))\ 1$$

$$T(3) = \max(cena[3];\ T(1) + T(2))\ 1$$

$$T(4) = \max(cena[4];\ T(1) + T(3);\ T(2) + T(2))\ 2$$



L 13 2 1

$$e^{\frac{2\pi i k}{n}(p-q)} \qquad e^{x\cdot y} = \left(e^{x}\right)^{y}$$

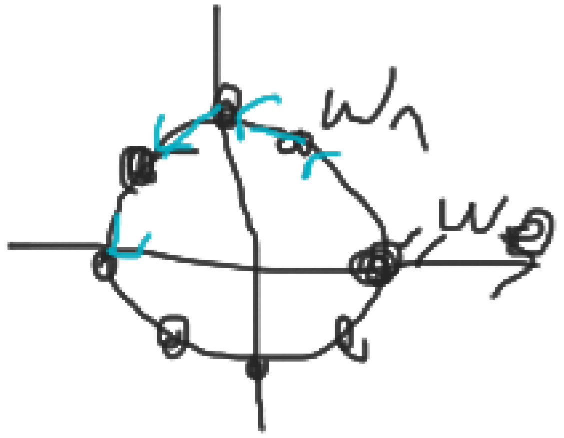
$$b_q = \frac{1}{n}\sum_{k=0}^{n-1}\left(\sum_{p=0}^{n-1} a_p e^{2\pi i k p/n}\right)e^{-2\pi i q k/n} = \frac{1}{n}\sum_{k=0}^{n-1}\left(\sum_{p=0}^{n-1} a_p e^{\left(\frac{2\pi i k p}{n}\right)+\left(-\frac{2\pi i q k}{n}\right)}\right) = \frac{1}{n}\sum_{k=0}^{n-1}\left(\sum_{p=0}^{n-1} a_p e^{\frac{2\pi i k}{n}(p-q)}\right) =$$

$$= \frac{1}{n}\sum_{p=0}^{n-1}\sum_{k=0}^{n-1} a_p e^{\frac{2\pi i k}{n}(p-q)} = \frac{1}{n}\sum_{p=0}^{n-1}\left(a_p\sum_{k=0}^{n-1} e^{\frac{2\pi i k}{n}(p-q)}\right) = \frac{1}{n}\, a_q\cdot n + 0 = a_q$$

$$p == q \Rightarrow a_p\sum_{k=0}^{n-1} e^{0} = a_p\cdot n$$

$$p-q = 1 \qquad \sum e^{\frac{2\pi i k}{n}} = 0$$

$$p-q = 15$$

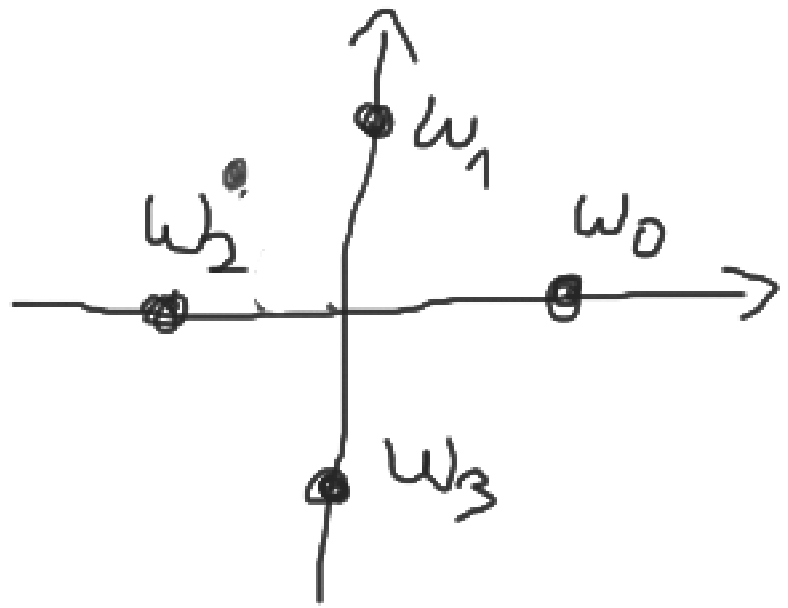


Zad 2

$$A(x) = \sum_{p=0}^{n-1} a_p \cdot x^p = 5x^0 + 3x^1 = 5 + 3x$$

$n = 2$

$$\omega_1 = e^{\frac{2\pi i}{n}} \qquad \omega_1 = -1 \qquad \omega_0 = 1$$



$A_0 = A(\omega_0) = A(1) = 8$
$A_1 = A(\omega_1) = A(-1) = 2$ $\rightarrow (8, 2)$

$$A(x) = 1x^0 + 5x^1 + 3x^2 + 1x^3 = \boxed{1 + 5x + 3x^2 + x^3}$$

$(1, i, -1, -i)$

$$A_0 = A(\omega_0) = A(1) = \underline{10}$$

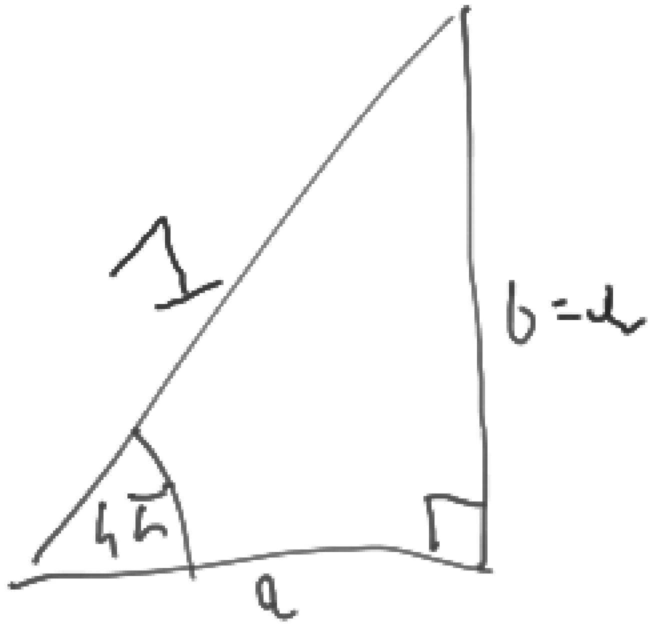
$$A_k = A(\omega_k) \quad A(i) = 1 + 5i - 3 - i = \underline{-2 + 4i}$$

$$A(-1) = 1 - 5 + 3 - 1 = -2$$

$$A(-i) = 1 - 5i - 3 + i = -2 - 4i$$

$A(x) = 1x^0 + 2x^1 + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + 8x^7 =$

$= 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + 8x^7$



$\sin \frac{b}{c}$

$\cos \frac{a}{c}$

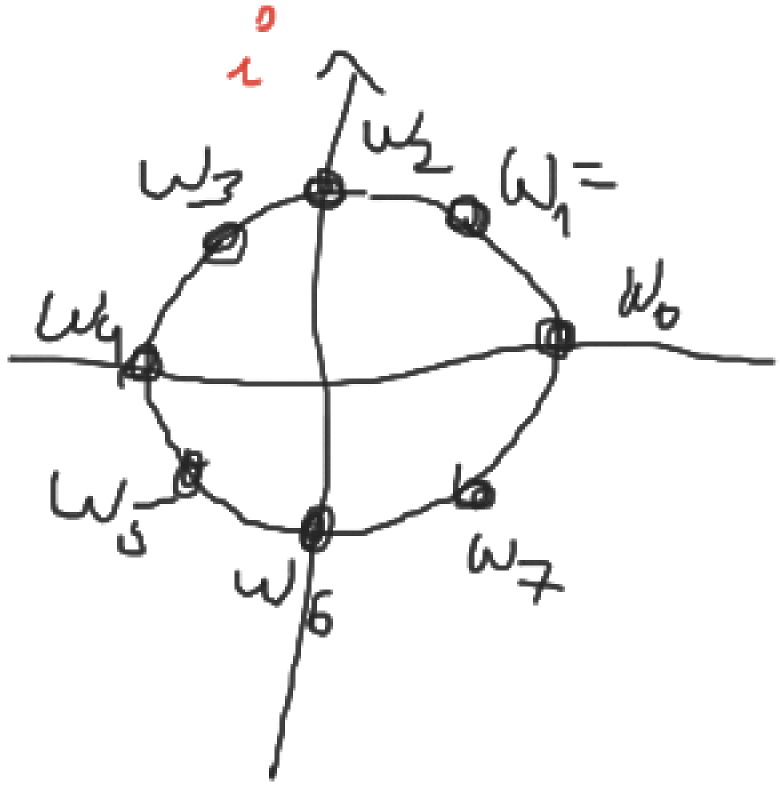
$\sin 45^0 = \frac{\sqrt{2}}{2}$

$a^2 + b^2 = 1$

$2a^2 = 1$

$a^2 = \frac{1}{2}$

$a = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$



$w_0 = 1$

$w_1 = \frac{\sqrt{2} + i\sqrt{2}}{2}$ $\frac{\sqrt{2}}{2}(1+i)$

$w_2 = i$

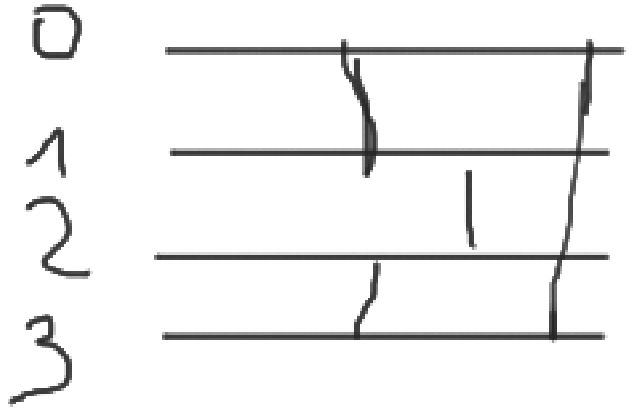
$w_3 = \frac{-\sqrt{2} + i\sqrt{2}}{2}$

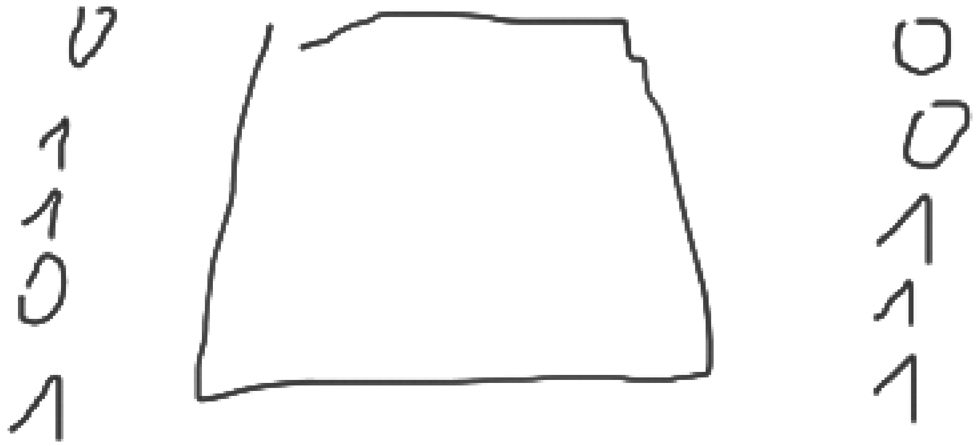
$w_4 = -1$

$w_5 = \frac{-\sqrt{2} - i\sqrt{2}}{2}$

$w_6 = -i$

$w_7 = \frac{\sqrt{2} - i\sqrt{2}}{2}$



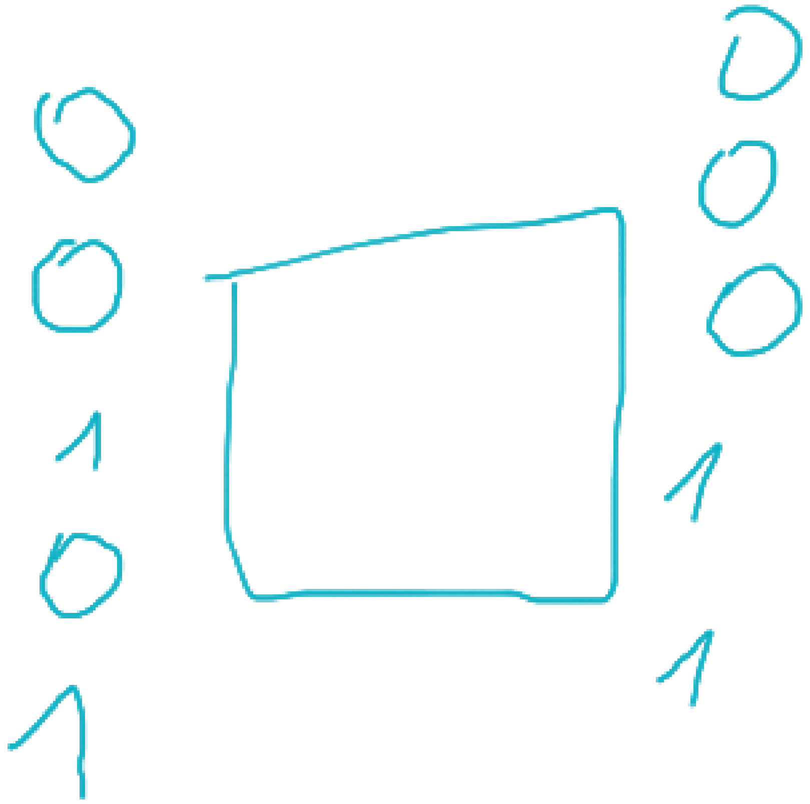


$$2^n$$

$=$

1
3
5
2
4

$\xrightarrow{k=3}$



Lsg 5

9, 22 | 6, 19, 21, 11 | 10, 17 | 3, 5, 60 | 30 | 29 | 1, 8 | 7 | 6, 15 | 12

9, 22　　6, 19, 21　　11　　10, 17　　3, 5, 60　　30　　29　　1, 8　　7　　6, 15　　12

6, 9, 19, 21, 22　　10, 11, 17　　3, 5, 30, 60　　1, 8, 29　　6, 7, 15　　12

6, 9, 10, 11, 17, 19, 21, 22　　1, 3, 5, 8, 29, 30, 60　　6, 7, 12, 15

1, 3, 5, 6, 8, 9, 10, 11, 17, 19, 21, 22, 29, 30, 60

1, 3, 5, 6, 7, 8, 9, 10, 11, 15, 17, 19, 21, 22, 29, 30, 60

L3 2 1

$-\infty$ 1 2 3 4 5

Inw: 0

min n parovan

1 2 3 4 5

$-\infty$ 5 4 3 2 1

$n-1+\ldots+4+3+2+1 = 10$

Inw: $\frac{1+n-1}{2}(n-1) = \frac{n}{2}(n-1)$

max $1+2+\ldots+n = \frac{(n+1)}{2}n$

2 1 5 3 4

1 0 2 0 0 = 3

2 3 4 5 1

1 + 1 + 1 + 1 + 0 = 4

1 2 4 3 5

0 0 1 0 0 = 1

$$\frac{\frac{(n+1)n}{2}+n}{2} = \frac{\frac{(n+1+2)n}{2}}{2} = \frac{(n+3)n}{4} = O(n^2)$$

≈ 2

$n: \quad \frac{n(n+1)}{2} \leftarrow$ Insertion sort

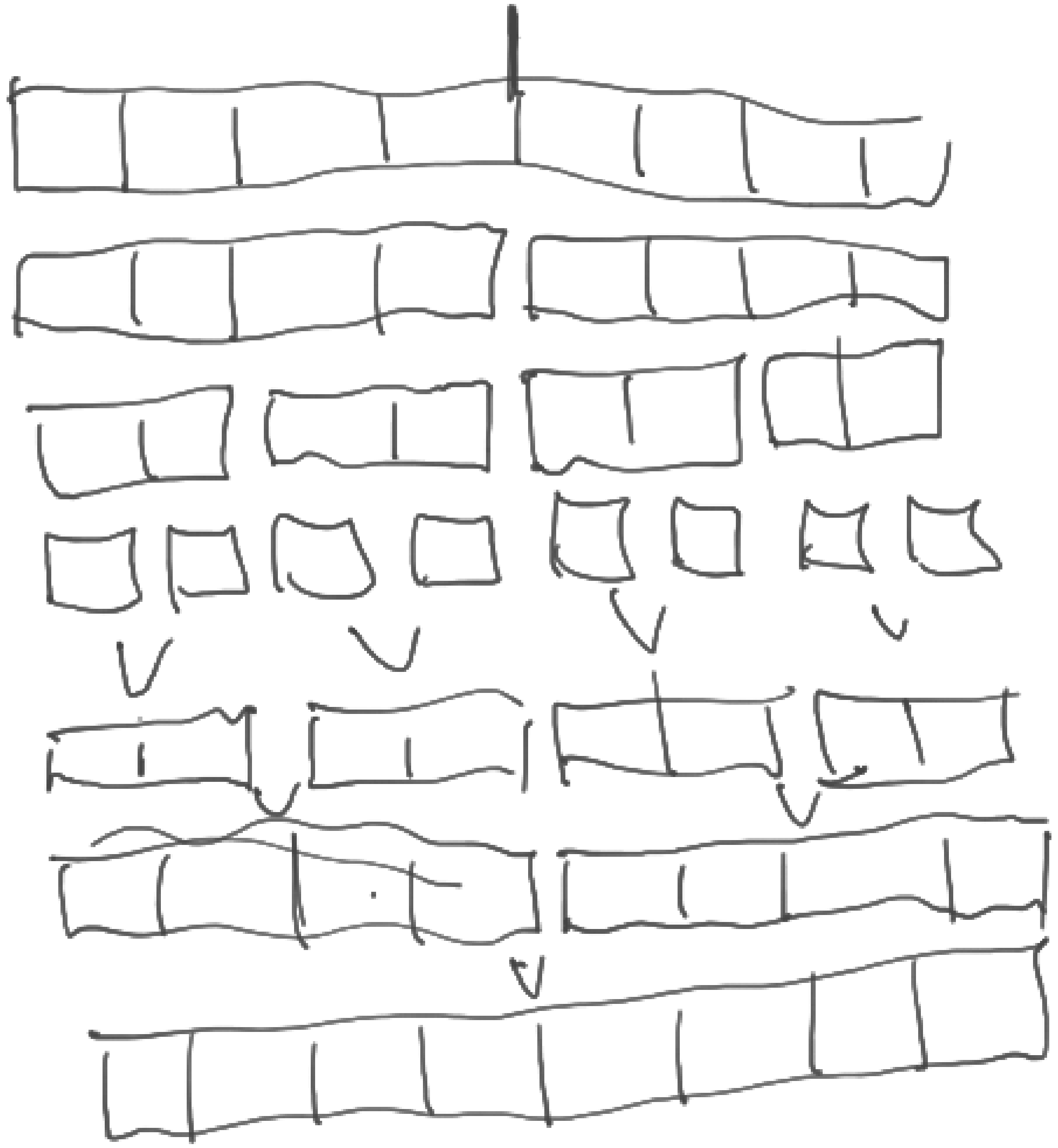
$$\underbrace{\tfrac{n}{k}}\underbrace{\tfrac{n}{k}}\underbrace{\tfrac{n}{k}}$$

$$\left(\frac{\frac{n}{k}\left(\frac{n}{k}+1\right)}{2}\right)\cdot \cancel{k} = \frac{n\left(\boxed{\frac{n}{k}}+1\right)}{2} \qquad k > n \qquad O(n \ )$$

$$O(n\sqrt{n}) = O(n^{\frac{3}{2}})$$

23

a)



$2 \cdot 4$

$+$

$4 \cdot 2$

$+$

$8 \cdot 1$

$\shortparallel$

$3 \cdot 8 = 24$

$3 \cdot 2^3$

$\boxed{k \cdot 2^k}$

b)

4

2

1

0

$2^0 + 2^1 + 2^2$

$\boxed{2^3 - 1}$

$1 \cdot 4$

$3 \cdot 2$

$7 \cdot 1$

$(2^1 - 1) \cdot 2^2 = 2^3 - 2^2$

$(2^2 - 1) \cdot 2^1 = 2^3 - 2^1$

$(2^3 - 1) \cdot 2^0 = 2^3 - 2^0$

$(k \cdot 2^k) - (2^k - 1) = \boxed{2^k(k-1) + 1}$