

Machine Learning for Correcting Inaccurate DFT Calculations

By: Cooper Lorsung
AP 275 Final Project Spring 2020

Introduction

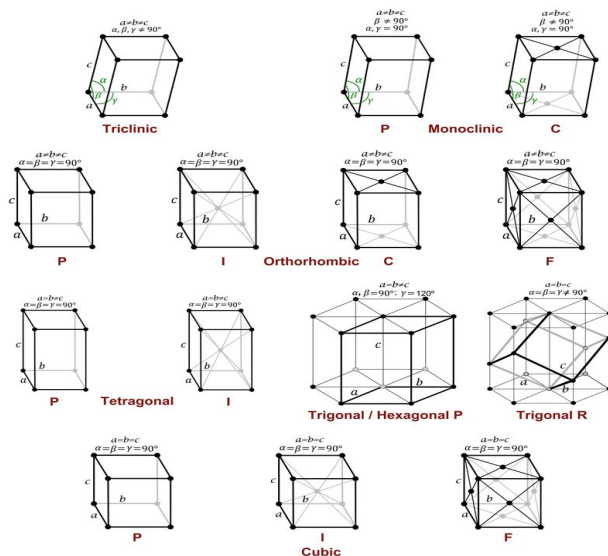
- Formation energy of Cu-Sn alloys
 - CuSn, CuSn₃, Cu₃Sn, Cu₅Sn₄, Cu₈₁Sn₂₂, Cu₁₀Sn₃, Cu₆Sn₅, Cu₄₁Sn₁₁, Cu₁₉Sn₁₃
 - LDA functional, ultra soft pseudopotential
- Cheap and inaccurate calculations
- Very small data sets (15 points)
- Use machine learning to correct for errors (Y. Zhang and Z. Lin)
- Target energies come from MaterialsProject.org
 - GGA functional, PAW potential



Source: <http://conexsprings.com/subproduct/bronze-helical-springs.html>

Crude Estimate of Property

$$E_{\text{Formation}} = \frac{E - N_{\text{Cu}} E_{\text{Cu}} - N_{\text{Sn}} E_{\text{Sn}}}{N_{\text{Cu}} + N_{\text{Sn}}}$$

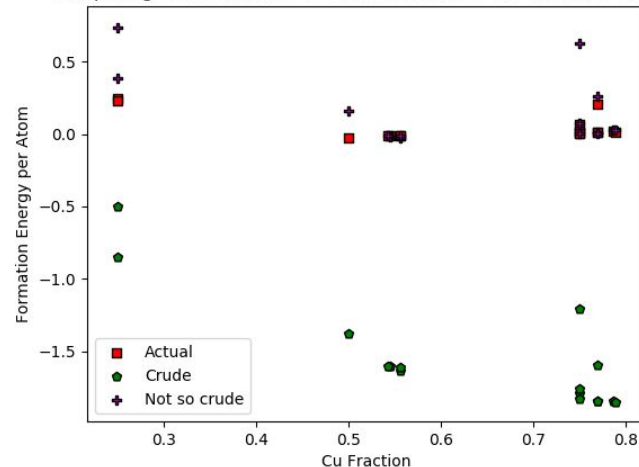


- Crude Estimate of property
 - Use crude estimate of target property (i.e. fast, inaccurate calculations)
 - Reduces degrees of freedom
- Feature Selection
 - Carefully select features to give insights into relevant physics
 - Use readily available features such as density, lattice parameters, etc.
 - I found no clear trends in with any single feature and formation energy

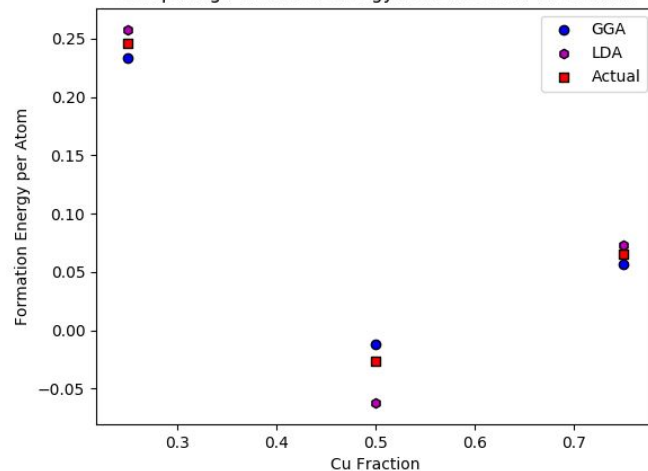
Sources of Error

- K-point error
 - Primary source of error
 - Mostly from non-converged reference material energies
 - Can compare crude estimate and 'not-so-crude' estimates
- Functional error
 - With k-point error converged, GGA performed slightly better than LDA
 - I was unable to perfectly replicate MaterialsProject data

Comparing Crude and Not-So-Crude Estimates of Formation Energy



Comparing Formation Energy For Different Functionals



Models

Kernel Ridge Regression

$$\hat{f}_{\text{KRR}}(\mathbf{x}) = \sum_{i=1}^n \alpha_i K(\mathbf{x}, \mathbf{x}_i)$$

$$C(\alpha_1, \dots, \alpha_n) = \frac{1}{2} \sum_{i=1}^n \left(y_i - \hat{f}_{\text{KRR}}(\mathbf{x}_i) \right)^2 + \eta \sum_{ij} \alpha_i K(\mathbf{x}, \mathbf{x}_i) \alpha_j$$

Source: Y. Zhang and Z. Lin

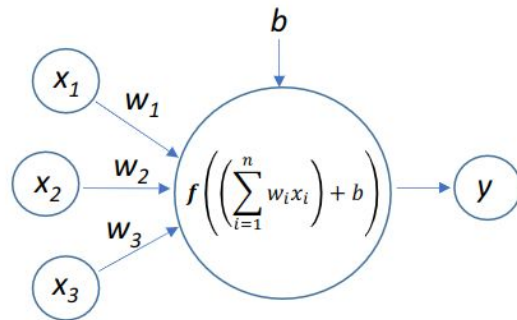
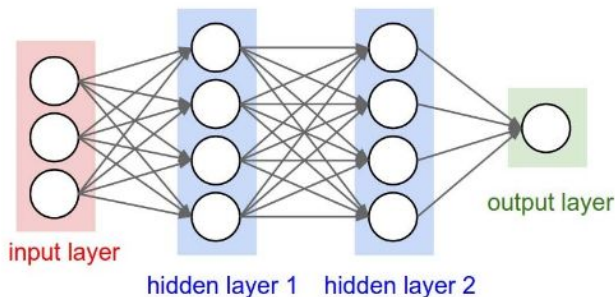
$$K_{\text{RBF}}(\mathbf{x}, \mathbf{x}') = \exp \left(\frac{\|\mathbf{x} - \mathbf{x}'\|^2}{2\sigma^2} \right)$$

$$K_{\text{CS}}(\mathbf{x}, \mathbf{x}') = \frac{\mathbf{x}\mathbf{x}'^T}{\|\mathbf{x}\| \|\mathbf{x}'\|}$$

Source: https://en.wikipedia.org/wiki/Radial_basis_function_kernel

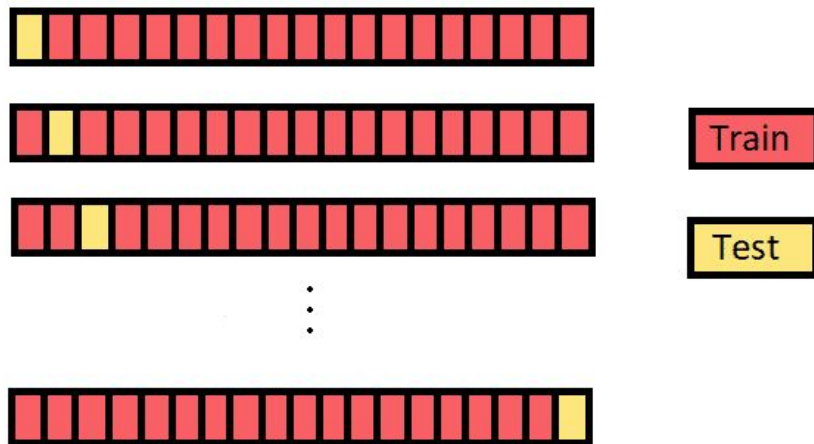
Source: <https://scikit-learn.org/stable/modules/metrics.html>

Multilayer Perceptron



Source: AP275 SP20 Lecture 16 Slide 12

Model Selection

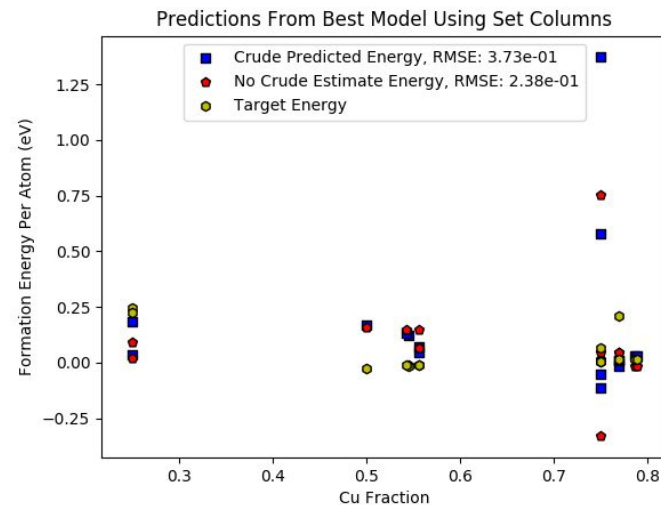
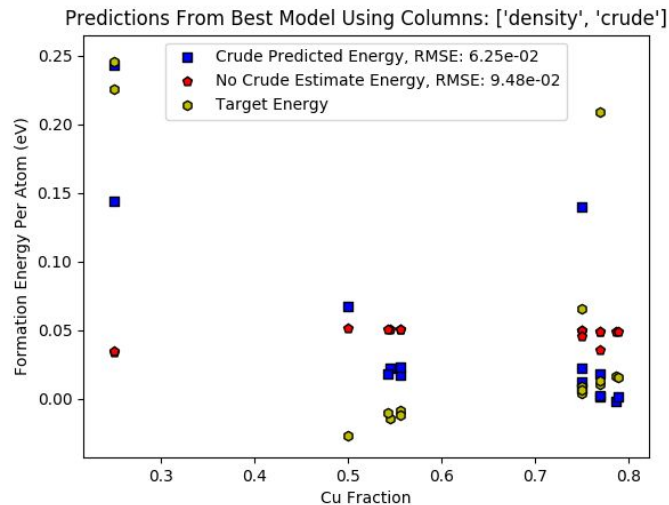


- Leave one out Cross Validation
 - Excludes one datapoint, trains on the rest, predicts that datapoint
 - Average error after all data points have been used as test point
- All possible combinations of features were used
- Selected model with lowest average RMSE

Source: <https://aiaspirant.com/cross-validation/>

Results

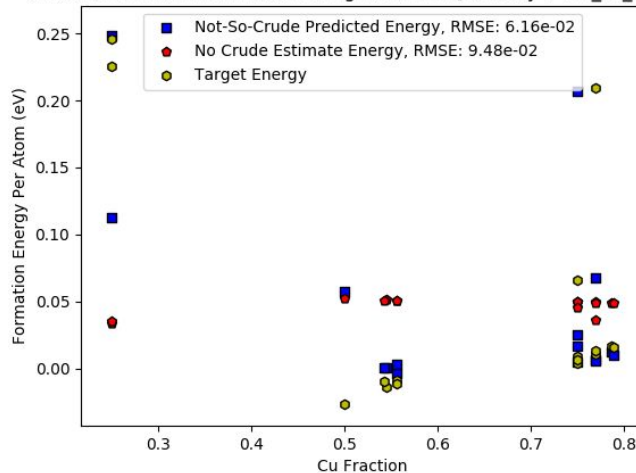
- Using the crude estimate gives much better results than not using it.
- Kernel Ridge Regression with cosine similarity is the best model in both cases
- Using additional physical features seems to qualitatively improve the no crude estimate predictions



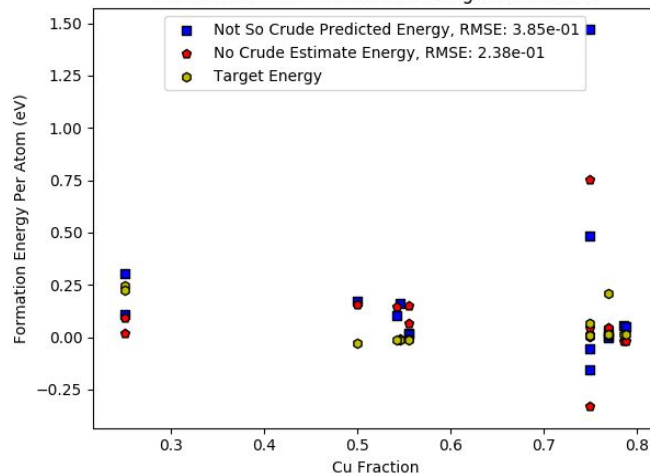
Results Cont.

- Best columns agree with cruder estimate
- Best model with set columns is Kernel Ridge Regression
- Best model with adaptive columns is the Neural Network
- With set columns an outlier seems to be causing issues

Predictions From Best Model Using Columns: ['density', 'not_so_crude']



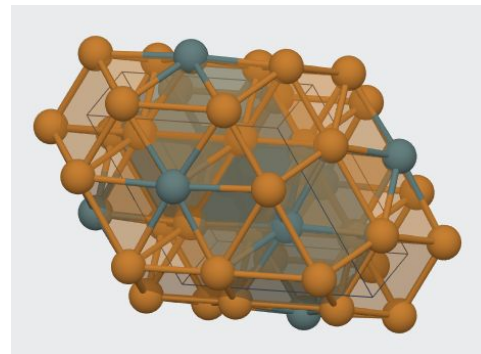
Predictions From Best Model Using Set Columns



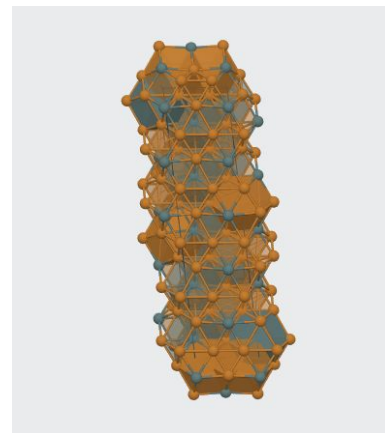
Conclusion

- Using crude estimates improves predictive quality
- Using more accurate crude estimates increases predictive quality further
- CuSn₃ with spacegroup 63 was an outlier for most calculations.
 - Could be due to unusual shape
 - Is very oblong compared to other materials
- This idea shows promise but is not ready to make high-stakes predictions

P63/mmc Cu₃Sn



Cmcm Cu₃Sn



References

- [1] Anubhav Jain et al. “The Materials Project: A materials genome approach to accelerating materials innovation”. In: *APL Materials* 1.1 (2013), p. 011002. ISSN: 2166532X. DOI: 10.1063/1.4812323. URL: <http://link.aip.org/link/AMPADS/v1/i1/p011002/s1%5C&Agg=doi>.
- [2] Kevin P. Murphy. *Machine Learning: A Probabilistic Perspective*. The MIT Press, 2012. ISBN: 0262018020.
- [3] Shyue Ping Ong et al. “Li-Fe-P-O₂ Phase Diagram from First Principles Calculations”. In: *Chemistry of Materials* 20.5 (Mar. 2008), pp. 1798–1807. ISSN: 0021-9606. DOI: 10.1021/cm702327g.
- [4] Gaegun You et al. “Machine learning-based prediction models for formation energies of interstitial atoms in HCP crystals”. In: *Scripta Materialia* 183 (July 2020), pp. 1–5.
- [5] Y. Zhang and C. Ling. “A strategy to apply machine learning to small datasets in materials science”. In: *npj Computational Materials* 4 (1 May 2018).