CS310 Natural Language Processing
自然语言处理
Lecture 01 - Word Vectors and Neural Networks

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Motivation

- How do we represent word **meanings**?
- by *Webster* dictionary - the **idea** that is represented by a word, phrase, etc.
- by 《现代汉语词典》- 意义：语言文字或其他信号所表示的内容
- by 《说文解字》- 意：志也。从心，察言而知意也。

- From dictionary, we can get a sequence of *symbols*.
- $Sym_A \longrightarrow Sym_B \longrightarrow Sym_C \longrightarrow Sym_D$ …

Symbolic meanings

- Common linguistic way: use symbols as signifier

signifier (symbol) ⟺ signified (idea or thing)

= denotational semantics

tree ⟺ { , , , …}

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Computable meaning

- Example, **WordNet**, a thesaurus containing lists of synonym sets and hypernyms

```
>>> from nltk.corpus import wordnet as wn
```

```
>>> wn.synset('dog.n.01')
Synset('dog.n.01')
>>> print(wn.synset('dog.n.01').definition())
a member of the genus Canis (probably descended from the
```

```
>>> dog = wn.synset('dog.n.01')
>>> dog.hypernyms()
[Synset('canine.n.02'), Synset('domestic_animal.n.01')]
```

source: https://www.nltk.org/howto/wordnet.html

Words are nodes in a tree/graph structure

```
>>> dog = wn.synset('dog.n.01')
>>> cat = wn.synset('cat.n.01')
```

```
>>> dog.path_similarity(cat)
0.2...
```

How similar two word senses are, based on the shortest path

Limits of WordNet (-like) resources

- Missing nuance
  - E.g., "proficient" is listed as a synonym for "good"
  - This is only correct in some contexts
- Missing new meanings of words
  - E.g., wicked, badass, nifty, wizard, genius, ninja, bombast
  - Impossible to keep up-to-date
- Subjective
- Requires human labor to create and adapt
- Cannot be used to accurately compute word sense similarity

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Words as one-hot vectors

- Words as discrete symbols ⇔ (equivalent to) localist representations
- **One-hot** vectors

Vocabulary (10k) = $\begin{bmatrix} a \\ about \\ all \\ \vdots \\ zoo \end{bmatrix}$

Apple [0 0 0 0 0 0 0 0 **1** 0 0 0 0 0 0 0 0 … 0]

Orange [0 0 0 0 0 0 0 0 0 0 0 0 0 **1** 0 0 0 … 0]

I would like some apple juice

I would like some orange _____

One-hot vector is not helpful

Distance between any pair of words is **constant:**

Euclidean distance = $\sqrt[2]{(1-0)^2+(1-0)^2}$

Cosine distance = 0

Words as real-valued vectors

	Man	Woman	King	Queen	Apple	Orange
Gender	-1	1	-0.98	0.97	0.00	-0.01
Royal	0.01	0.02	0.93	0.98	-0.01	0.00
Age	0.03	0.02	0.72	0.68	0.03	0.02
Food	0.00	0.00	0.01	0.02	0.95	0.97

main difference: gender

main difference: gender

$$e_{Man} = \begin{bmatrix} -1 \\ 0.01 \\ 0.03 \\ 0.0 \end{bmatrix} \quad e_{Woman} = \begin{bmatrix} 1 \\ 0.02 \\ 0.02 \\ 0.0 \end{bmatrix} \quad e_{Man} - e_{Woman} = \begin{bmatrix} -2 \\ -0.01 \\ 0.01 \\ 0.00 \end{bmatrix} \quad e_{King} - e_{Queen} = \begin{bmatrix} -1.95 \\ -0.05 \\ 0.04 \\ -0.01 \end{bmatrix}$$

With real-valued dense vectors, word similarity can be computed more accurately

What use of word vectors?

- More accurate semantic representation than WordNet-like method
- Better performance in almost all areas of supervised learning tasks, such as
  - Text classification
  - Named entity recognition
  - Sentiment analysis
  - Information retrieval
  - Question answering
- One of the most useful supervised learning model: **Neural Networks**

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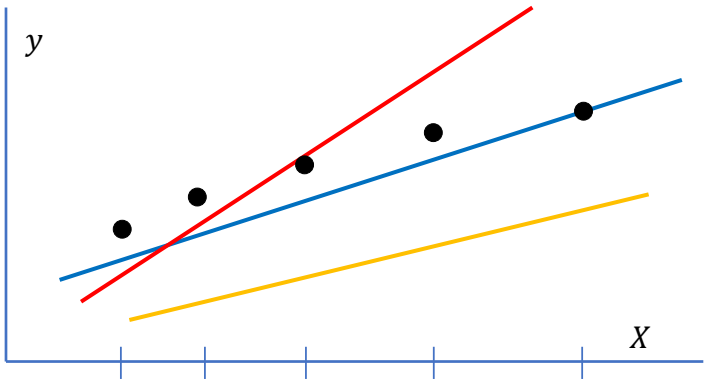
Logistic regression (binary classification)

Linear regregression $X \Rightarrow$ continuous y

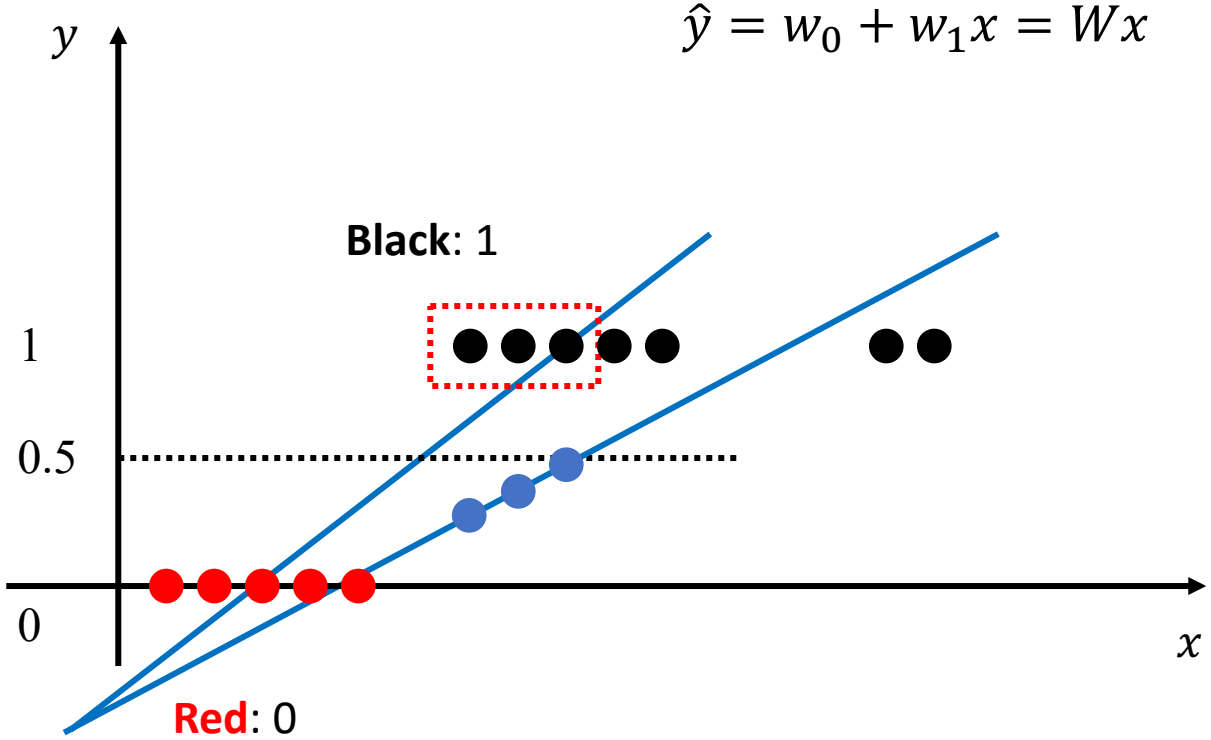


Solution: Sigmoid function

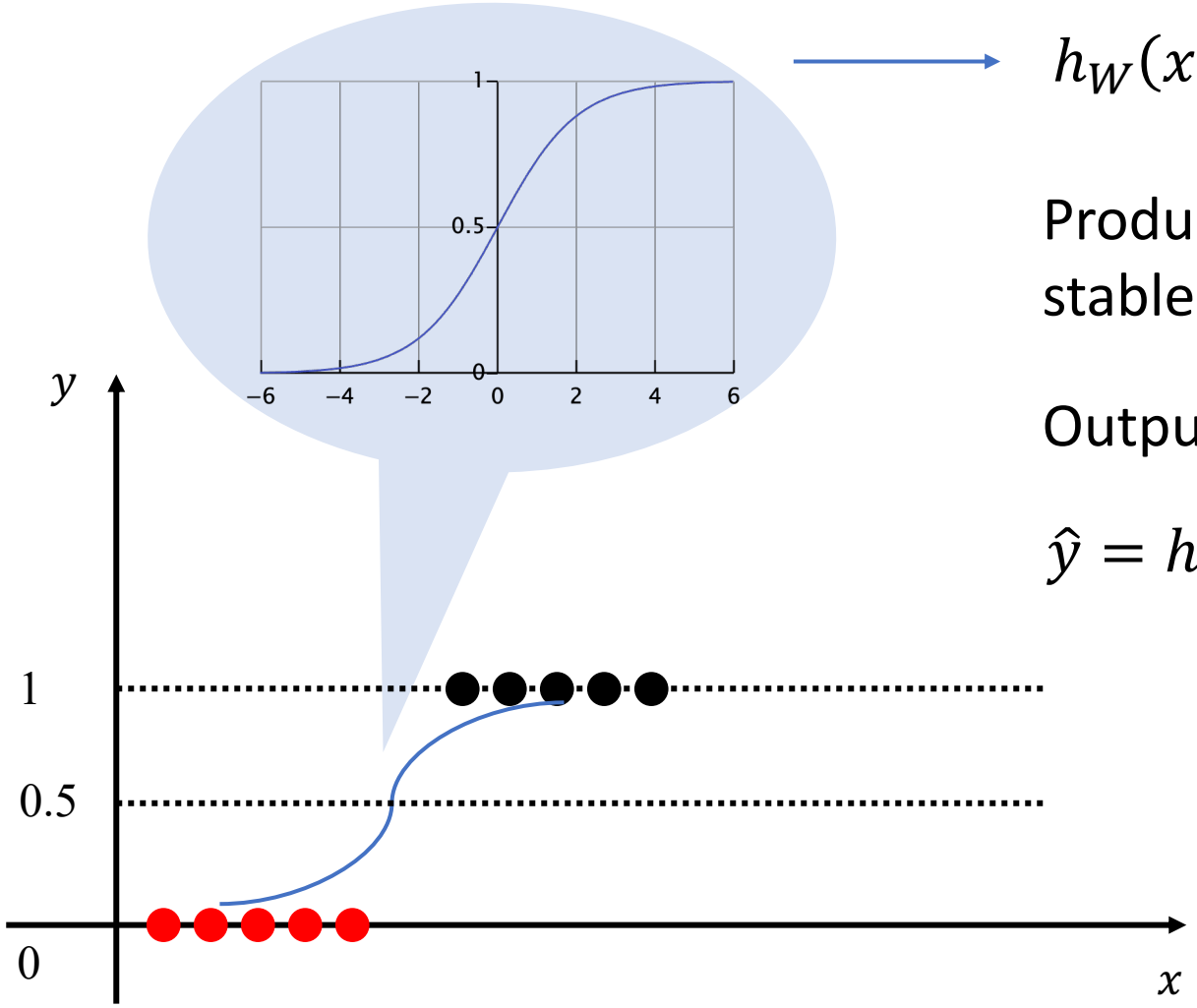
$$\hat{y} = h_W(x) = \frac{1}{1 + e^{-Wx}}$$

Linear function fails when y is binary, i.e., $y \in \{0,1\}$

$$\hat{y} = w_0 + w_1 x = Wx$$



Sigmoid function



$$h_W(x) = \frac{1}{1 + e^{-(wx+b)}}$$

Produces a smoother curve;
stable against outliers

Output $\hat{y}$ is bounded to [0, 1]

$\hat{y} = h_W(x)$ estimates the **probability** of x is class 1

$$\hat{y} = P(y = 1|x)$$

$$1 - \hat{y} = P(y = 0|x)$$

Question: How to learn the parameter W?

- Need to have an objective function to optimize
- **Likelihood**: $P(y=1|x) = h_{w,b}(x)$, $P(y=0|x) = 1 - h_{w,b}(x)$
- A compact way: $P(y|x) = \left(h_{w,b}(x)\right)^{y} \left(1 - h_{w,b}(x)\right)^{1-y}$
- **Goal**: Learn the parameter W to maximize the likelihood of the given data, $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \cdots, (x^{(m)}, y^{(m)})\}$

$$\mathcal{L}(w,b) = \prod_i \left(h_{w,b}\left(x^{(i)}\right)\right)^{y^{(i)}} \left(1 - h_{w,b}\left(x^{(i)}\right)\right)^{1-y^{(i)}}$$

Hence, log-likelihood $\log\left(\mathcal{L}(w,b)\right) = \sum_i \left[y^{(i)} \log h_{w,b}(x^{(i)}) + \left(1 - y^{(i)}\right) \log(1 - h_{w,b}(x^{(i)}))\right]$

Log-Likelihood

Question: How to?
⇒ stochastic gradient descent

- To maximize log-likelihood

$$\mathcal{LL}(w,b) = \sum_i \left[y^{(i)} \log h_{w,b}(x^{(i)}) + \left(1 - y^{(i)}\right) \log(1 - h_{w,b}(x^{(i)}))\right]$$

- Equivalent to minimize **negative** log-likelihood

$$\mathcal{NLL}(w,b) = -\sum_i \left[y^{(i)} \log h_{w,b}(x^{(i)}) + \left(1 - y^{(i)}\right) \log(1 - h_{w,b}(x^{(i)}))\right]$$

$$= -\sum_i \left[y^{(i)} \log \hat{y}^{(i)} + \left(1 - y^{(i)}\right) \log(1 - \hat{y}^{(i)})\right]$$

only exists when $y^{(i)} = 1$, where $\hat{y}^{(i)} = P(y^{(i)} = 1|x^{(i)})$

only exists when $y^{(i)} = 0$, where $\hat{y}^{(i)} = P(y^{(i)} = 0|x^{(i)})$

Gradient descent (GD) algorithm

- Gradient points in the direction of the *fastest increase* of function
- ***Opposite*** of gradient points to the direction of ***fastest decrease***

- Goal: Minimize objective function $J(\theta)$ θ are parameters just like w, b
  i. Start with $\theta^{(0)}$.
  ii. Change $\theta^{(0)}$ a bit to $\theta^{(1)}$, so that $J(\theta^{(1)})$ decreases a bit from $J(\theta^{(0)})$.

$$\theta^{(1)} \leftarrow \theta^{(0)} - \alpha \frac{\partial J(\theta^{(0)})}{\partial \theta}$$

  iii. Repeat until $J(\theta)$ no longer decreases significantly.

Gradients

- A function with 1 output and 1 input
- $f(x) = x^3$
- Its gradient = its derivative
- $\frac{df}{dx} = 3x^2$
- How much will the output change if we change the input a bit?
- $\frac{d}{dx} f(1) = 3$, output changes 3 times as much as input: $1.01^3 \approx 1.03$
- $\frac{d}{dx} f(2) = 12$, output changes 12 times as much as input: $2.01^3 \approx 8.12$

Gradients of multivariate functions

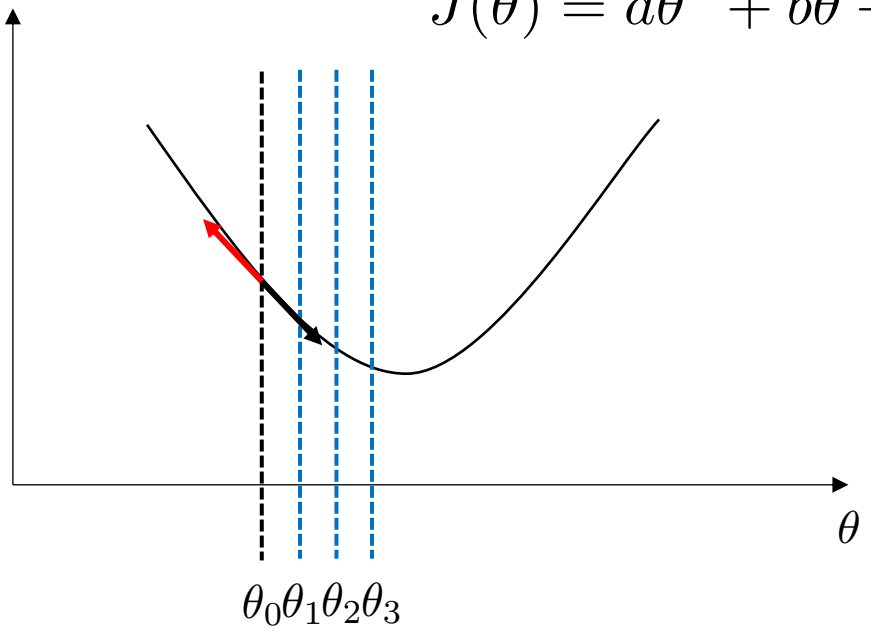
- A function of 1 output and n inputs
- $f(\boldsymbol{x}) = f(x_1, x_2, \dots, x_n)$

- The gradients is a vector of partial derivatives with respect to each input
- $\frac{\partial f}{\partial \boldsymbol{x}} = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}\right]$

1-D demo of GD

- How do we change θ to minimize $J(\theta)$

$$J(\theta) = a\theta^2 + b\theta + c$$



$$\frac{\partial J}{\partial \theta_0} < 0$$

Move towards the opposite direction of gradient by a step (determined by *learning rate* α):

$$\theta_1 = \theta_0 - \alpha \frac{\partial J}{\partial \theta_0}$$

Therefore, $\theta_1 > \theta_0$

If α is small enough, θ_1 will be closer to minimum than θ_0

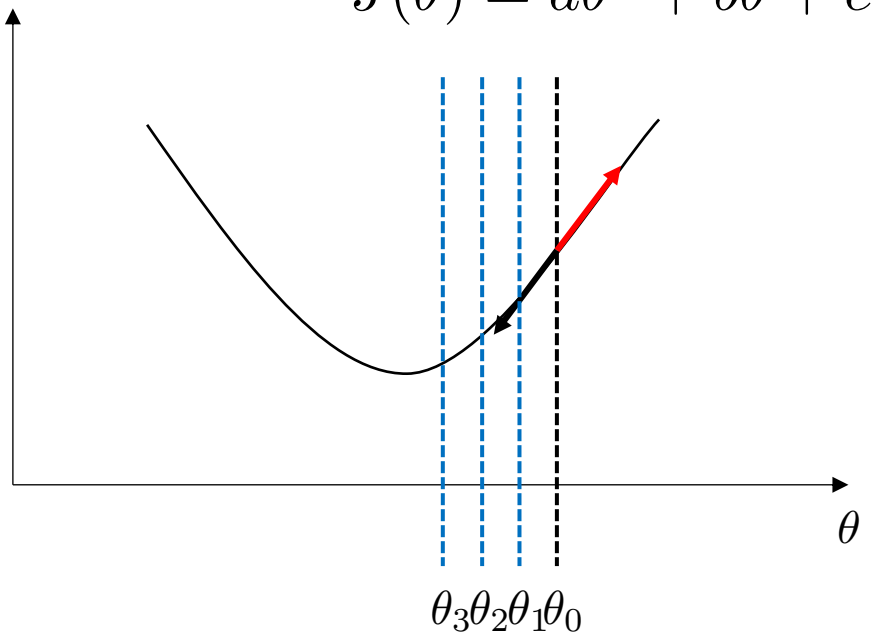
$$J(\theta_1) < J(\theta_0)$$

Repeat until a satisfactory θ_n is reached

1-D demo of GD (cont.)

- θ initialized to a different value

$$J(\theta) = a\theta^2 + b\theta + c$$



$$\frac{\partial J}{\partial \theta_0} > 0$$

Move towards the opposite direction of gradient:

$$\theta_1 = \theta_0 - \alpha \frac{\partial J}{\partial \theta_0}$$

Therefore $\theta_1 < \theta_0$

If α is small $J(\theta_1) < J(\theta_0)$

Repeat until satisfied.

The principle holds: **move along the opposite direction of gradient**

Gradient descent for logistic regression

- **Goal**: Minimize the **cost function** $J(W)$, i.e., the negative log-likelihood

$$J(W) = -\sum_i \left[y^{(i)} \log h_W(x^{(i)}) + \left(1 - y^{(i)}\right) \log(1 - h_W(x^{(i)}))\right]$$

- Need to compute the gradient of $J(W)$

$$\frac{\partial J}{\partial W} = \frac{\partial\left[-y^{(i)} \log h_W\left(x^{(i)}\right) - \left(1 - y^{(i)}\right) \log(1 - h_W(x^{(i)}))\right]}{\partial W}$$

(summation omitted)

$$= -\frac{y^{(i)}}{h_W(x^{(i)})} \frac{\partial h_W\left(x^{(i)}\right)}{\partial W} - \frac{1 - y^{(i)}}{1 - h_W(x^{(i)})} \frac{-\partial h_W\left(x^{(i)}\right)}{\partial W} \quad = -\frac{y^{(i)}}{\hat{y}^{(i)}} \frac{\partial h_W\left(x^{(i)}\right)}{\partial W} - \frac{1 - y^{(i)}}{1 - \hat{y}^{(i)}} \frac{-\partial h_W\left(x^{(i)}\right)}{\partial W}$$

Key: $\dfrac{\partial h_W\left(x^{(i)}\right)}{\partial W}$

Gradient descent for logistic regression

$$h_W(x) = \frac{1}{1+e^{-Wx}}$$

let $z = Wx$, $g(z) = \frac{1}{1+e^{-z}}$, and then apply the chain rule of calculus

$$\frac{\partial h_W(x)}{\partial W} = \frac{\partial g}{\partial z}\frac{\partial z}{\partial W}$$

$$\frac{\partial g}{\partial z} = \frac{\partial}{\partial z}\left(\frac{1}{1+e^{-z}}\right) = \frac{e^{-z}}{(1+e^{-z})^2}$$

$$\frac{\partial z}{\partial W} = x$$

$$= \frac{e^{-z}}{(1+e^{-z})^2}x = \frac{1}{1+e^{-z}}\left(1-\frac{1}{1+e^{-z}}\right)x$$

$$= \frac{1}{1+e^{-z}}\left(1-\frac{1}{1+e^{-z}}\right)x$$

$$= h_W(x)(1-h_W(x))x$$

$$= \hat{y}\,(1-\hat{y})x$$

Gradient descent for logistic regression

Plugin $\frac{\partial h_W(x^{(i)})}{\partial W} = \hat{y}^{(i)}\left(1-\hat{y}^{(i)}\right)x^{(i)}$ to $\frac{\partial J(W)}{\partial W} = -\frac{y^{(i)}}{\hat{y}^{(i)}}\frac{\partial h_W(x^{(i)})}{\partial W} - \frac{1-y^{(i)}}{1-\hat{y}^{(i)}}\frac{-\partial h_W(x^{(i)})}{\partial W}$

$$\frac{\partial J(W)}{\partial W} = -\frac{y^{(i)}}{\hat{y}^{(i)}}\hat{y}^{(i)}\left(1-\hat{y}^{(i)}\right)x^{(i)} - \frac{1-y^{(i)}}{1-\hat{y}^{(i)}}\hat{y}^{(i)}\left(1-\hat{y}^{(i)}\right)x^{(i)} = \left(\hat{y}^{(i)}-y^{(i)}\right)x^{(i)}$$

Place back the summation and average over all samples:

$$\frac{\partial J(W)}{\partial W} = \frac{1}{m}\sum_i\left(\hat{y}^{(i)}-y^{(i)}\right)x^{(i)}$$

Plugin to

Gradient descent

Initialize W

Repeat until satisfied {

Compute $\frac{\partial J(W)}{\partial W_t}$ using

Update $W_{t+1} = W_t - \alpha\frac{\partial J(W)}{\partial W_t}$

}

Computational graph

$z = w^T x + b$

$\hat{y} = \underline{a} = \sigma(z) = \dfrac{1}{1 + e^{-z}}$

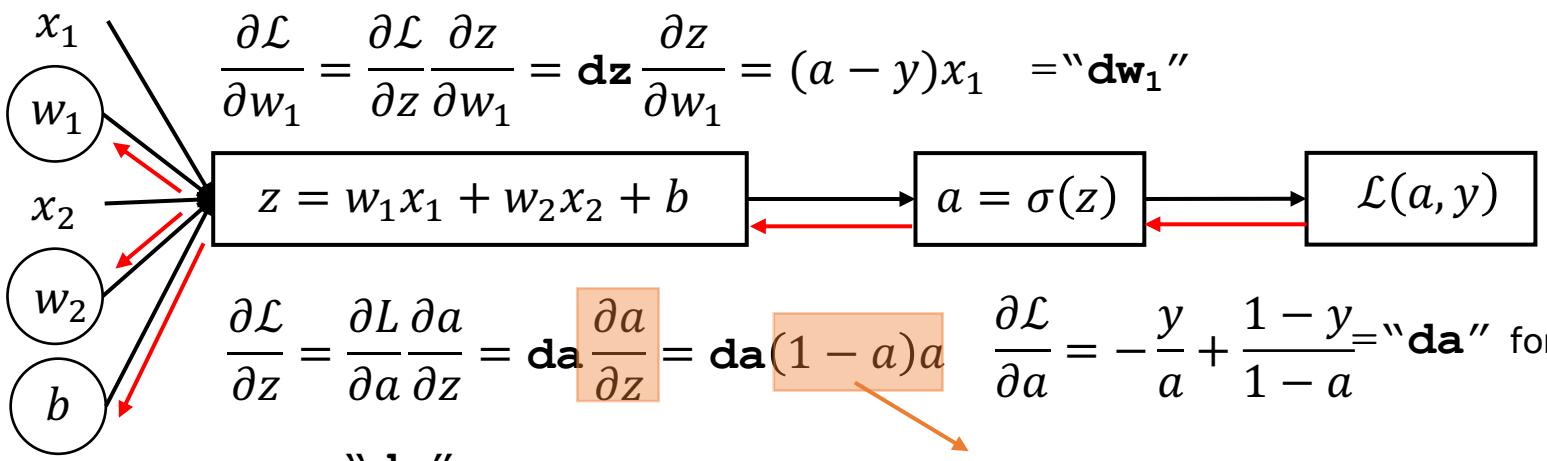
$\underline{\mathcal{L}(a, y)} = -(y \log(a) + (1 - y) \log(1 - a))$

a is for "activation"

$\mathcal{L}$ is for "loss", similar "cost"

Chain rule:

$$\frac{\partial \mathcal{L}}{\partial c} = \frac{\partial \mathcal{L}}{\partial a} \frac{\partial a}{\partial b} \frac{\partial b}{\partial c}$$



$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial \mathcal{L}}{\partial z} \frac{\partial z}{\partial w_1} = \mathbf{dz} \frac{\partial z}{\partial w_1} = (a - y) x_1 \quad = \text{"}\mathbf{dw_1}\text{"}$$

$$\frac{\partial \mathcal{L}}{\partial z} = \frac{\partial L}{\partial a} \frac{\partial a}{\partial z} = \mathbf{da} \frac{\partial a}{\partial z} = \mathbf{da}(1 - a)a$$

$$= \text{"}\mathbf{dz}\text{"} \quad = a - y$$

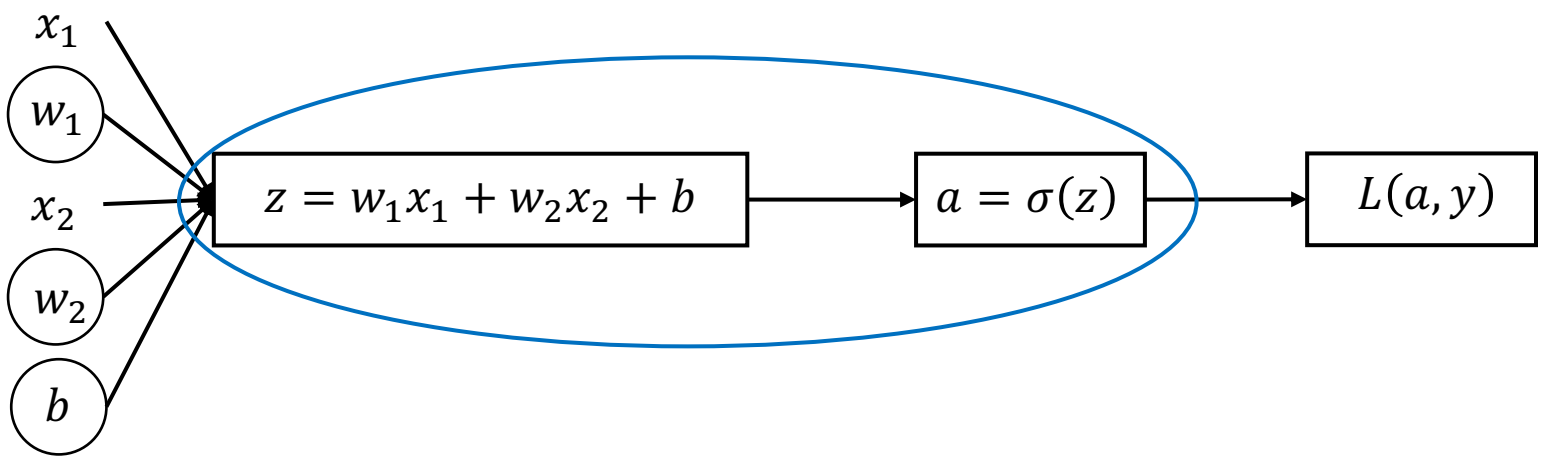
$$\frac{\partial \mathcal{L}}{\partial a} = -\frac{y}{a} + \frac{1 - y}{1 - a} = \text{"}\mathbf{da}\text{"} \text{ for short}$$

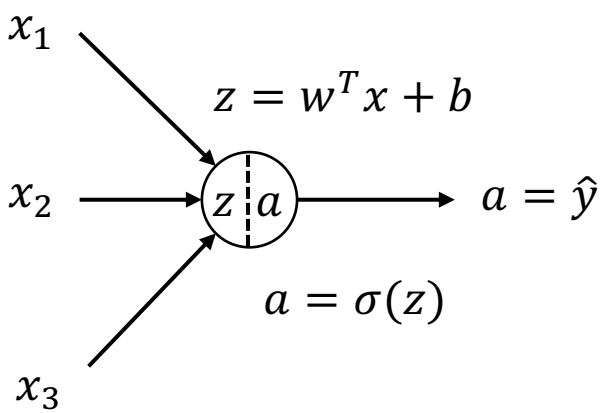
The derivative of sigmoid function

$\mathbf{dw_2} = (a - y) x_2$

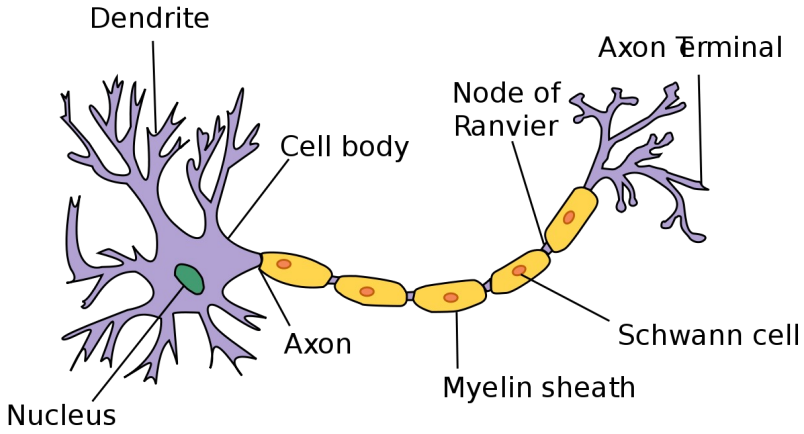
$\mathbf{db} = (a - y)$

Wrap logistic regression into a single unit



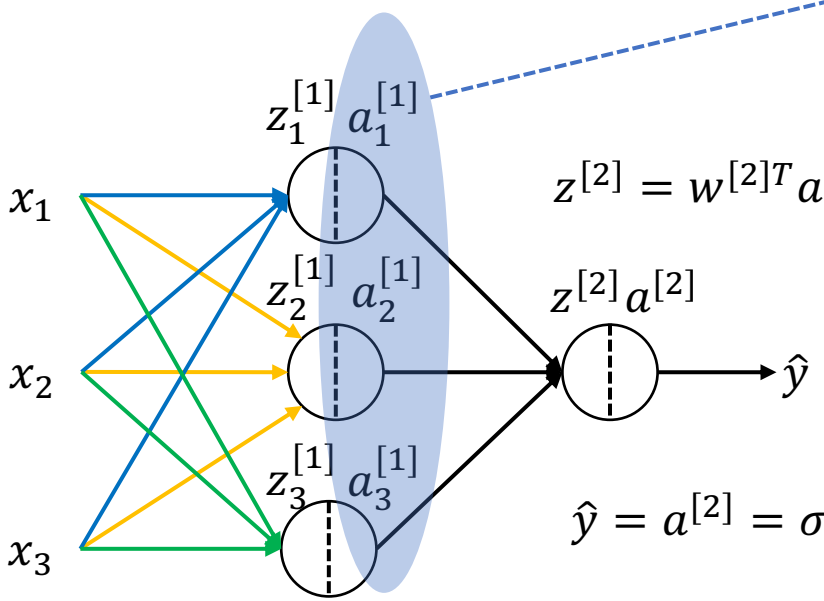


Neuron



Neural Networks

- Combine multiple neurons in one **layer**
- Put multiple layers in one network



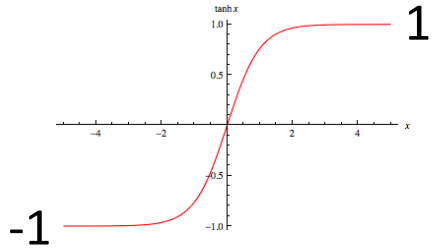
$$z^{[2]} = w^{[2]T} a^{[1]} + b^{[2]}$$

$$\hat{y} = a^{[2]} = \sigma(z^{[2]})$$

$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]} \qquad a_1^{[1]} = \sigma(z_1^{[1]})$$

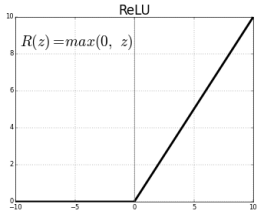
Other options for activation functions

- **Hyperbolic tangent**: Almost always works better than sigmoid.



$$g(z) = \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

- **Rectified linear unit (ReLU)**

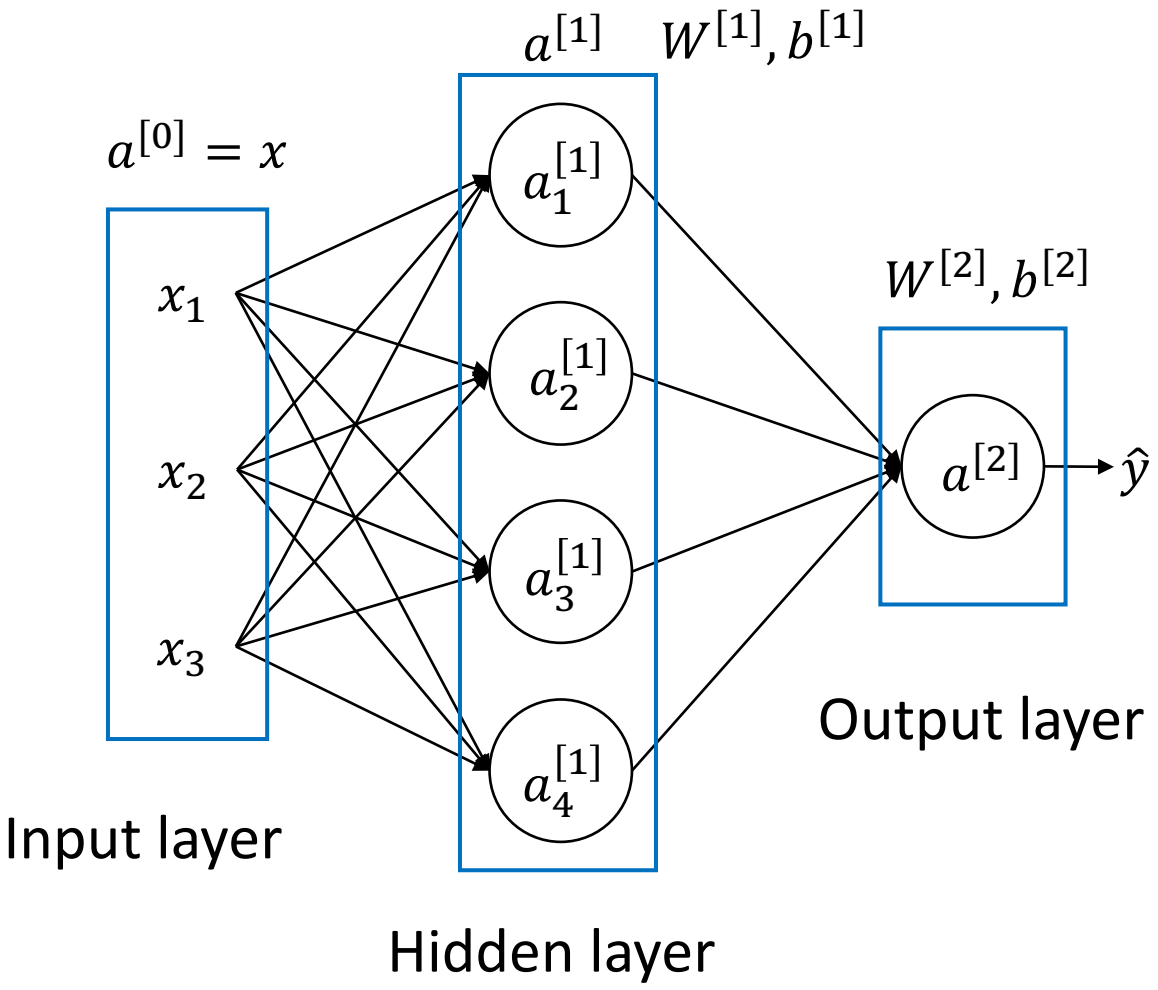


$$g(z) = \max(0, z)$$

$$g'(z) \in \{0,1\}$$

Neural Networks - Matrix & vector repr.

A two-layer neural network



Parameters of layer 1 (input to hidden)

$$W^{[1]} = \begin{bmatrix} W_{11}^{[1]} W_{12}^{[1]} W_{13}^{[1]} \\ W_{21}^{[1]} W_{22}^{[1]} W_{23}^{[1]} \\ W_{31}^{[1]} W_{32}^{[1]} W_{33}^{[1]} \\ W_{41}^{[1]} W_{42}^{[1]} W_{43}^{[1]} \end{bmatrix} \in \mathbb{R}^{4\times 3} \qquad b^{[1]} = \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix} \in \mathbb{R}^{4\times 1}$$

Parameters of layer 2 (hidden to output)

$$W^{[2]} = [W_{11}^{[2]}, W_{12}^{[2]}, W_{13}^{[2]}, W_{14}^{[2]}] \in \mathbb{R}^{1\times 4} \qquad b^{[2]} \in \mathbb{R}^{1\times 1}$$

General principle

$$\underset{n^{[l]}\times 1}{z^{[l]}} = \underset{n^{[l]}\times n^{[l-1]}}{W^{[l]}} \underset{n^{[l-1]}\times 1}{a^{[l-1]}} + \underset{n^{[l]}\times 1}{b^{[l]}}$$

$$a^{[l]} = g(z^{[l]})$$

$n^{[l]}$: number of units in layer l

g is the activation function

Backpropagation of neural networks

$$\mathbf{d}\boldsymbol{W}^{[1]} = \mathbf{dz}^{[1]}\frac{\partial z^{[1]}}{\partial W^{[1]}} = \mathbf{dz}^{[1]}\boldsymbol{x}^{T}$$

$$\mathbf{db}^{[1]} = \mathbf{dz}^{[1]}\frac{\partial z^{[1]}}{\partial b^{[1]}} = \mathbf{dz}^{[1]}$$

$$\mathbf{da}^{[1]} = \mathbf{dz}^{[2]}\frac{\partial z^{[2]}}{\partial a^{[1]}} = W^{[2]T}\mathbf{dz}^{[2]}$$

$W^{[2]}$, $b^{[2]}$

$\mathbf{d}\boldsymbol{W}^{[2]} = ?$ ----- $\mathbf{d}\boldsymbol{W}^{[2]} = \mathbf{dz}^{[2]}\mathbf{a}^{[1]T}$

$\mathbf{db}^{[2]} = \mathbf{dz}^{[2]}$

X, $W^{[1]}$, $b^{[1]}$ → $z^{[1]} = W^{[1]}x + b^{[1]}$ → $a^{[1]} = g(z^{[1]})$ → $z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$ → $a^{[2]} = \sigma(z^{[2]})$ → $L(a^{[2]}, y)$

$$\mathbf{dz}^{[2]} = a^{[2]} - y$$

$$\mathbf{da}^{[2]} = \frac{\partial L}{\partial a^{[2]}} = -\frac{y}{a^{[2]}} + \frac{1-y}{1-a^{[2]}}$$

$$\mathbf{dz}^{[1]} = \mathbf{da}^{[1]} * \frac{\partial a^{[1]}}{\partial z^{[1]}}$$

$$= W^{[2]T}\mathbf{dz}^{[2]} * g'(z^{[1]})$$

element-wise multiplication

$$\frac{\partial z^{[2]}}{\partial a^{[1]}} = W^{[2]T}$$

$\mathbf{da}^{[1]}$ is the same dimension as $a^{[1]}$

$$\frac{\partial z^{[2]}}{\partial W^{[2]}} = \frac{\partial z^{[2]}}{\partial w^{[2]}} = a^{[1]T}$$

$\mathbf{d}\boldsymbol{W}^{[2]}$ is the same dimension as $\boldsymbol{W}^{[2]}$

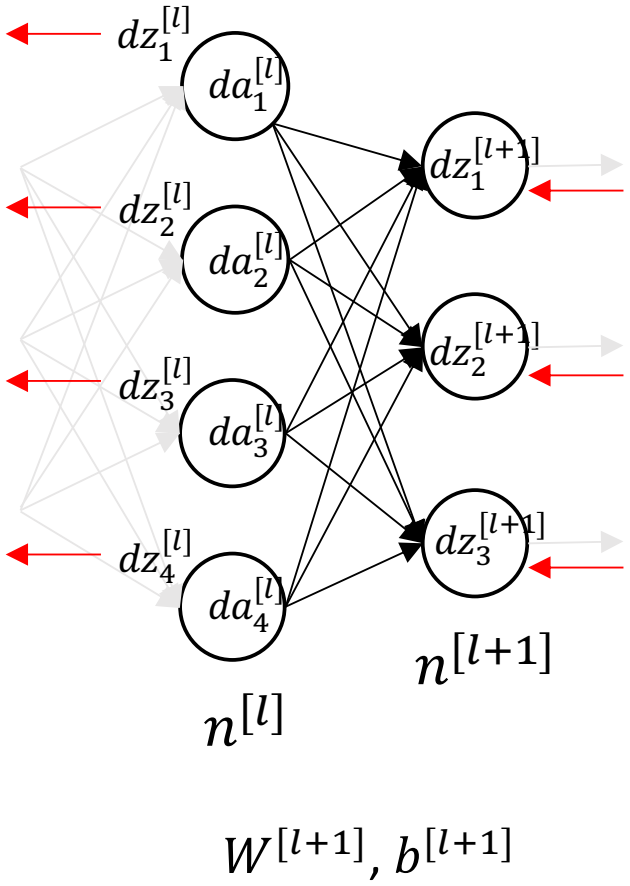
$\frac{\partial a^{[1]}}{\partial z^{[1]}} = g'(z^{[1]})$, depending on the activation function

Backprop in general

$$z^{[l+1]} = W^{[l+1]}a^{[l]} + b^{[l+1]}$$

$$a^{[l+1]} = g(a^{[l+1]})$$

Forward propagation



$$dW^{[l+1]} = dz^{[l+1]}a^{[l]T}$$
$$db^{[l+1]} = dz^{[l+1]}$$

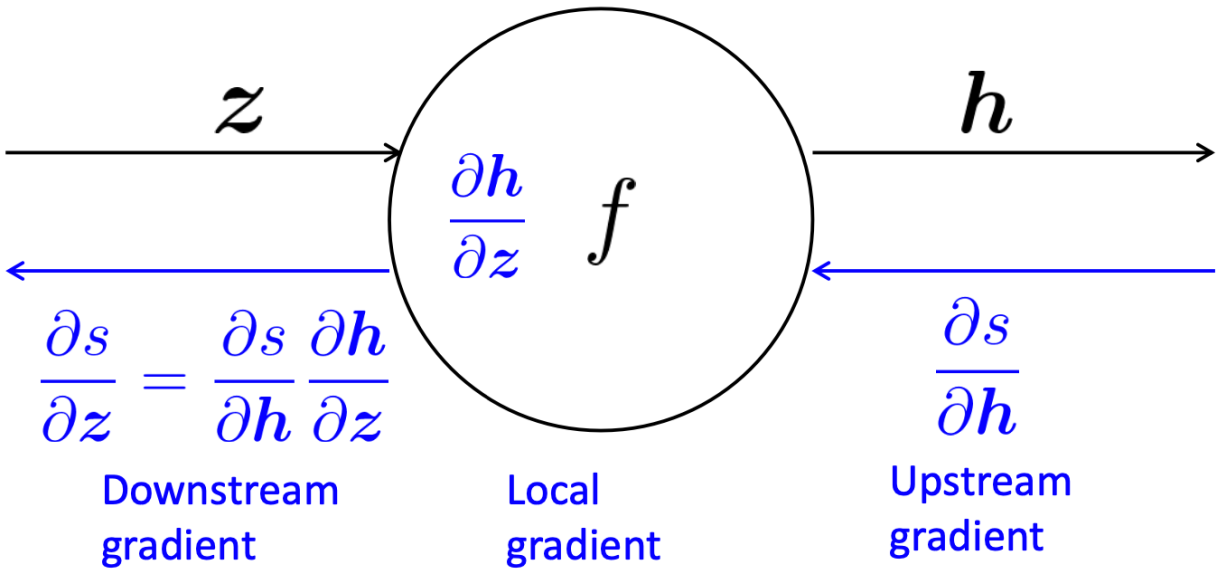
Backward propagation

$$da^{[l]} = W^{[l+1]T}dz^{[l+1]}$$
$$dz^{[l]} = da^{[l]} * g'(z^{[l]}) = W^{[l+1]T}dz^{[l+1]} * g'(z^{[l]})$$

$dz^{[l]}$ is then passed to layer $l-1$ to compute $dW^{[l]}$ and $db^{[l]}$

Backprop: F-Prop + B-Prop

- F-Prop: Compute results and save intermediate values
- B-Prop: Apply chain rule to compute gradients



Each node

- Receives an "upstream gradient"
- Goal is to pass on the correct "downstream gradient"

[downstream gradient] =
[upstream gradient] x [local gradient]

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Basic neural network in PyTorch

- torch.nn.Linear

1. Use the torch.nn module

```
import torch
import torch.nn as nn
import numpy as np
torch.manual_seed(0)
```

2. Define a Linear layer

```
m = nn.Linear(4, 3)
# Equivalent to nn.Linear(in_features = 4, out_features = 3, bias = True
```

3. Let's look at the initial weight and bias:

```
print(m.weight)
print(m.bias)
```

```
Parameter containing:
tensor([[ 0.3742, -0.0806,  0.0529,  0.4527],
        [-0.4638, -0.3148, -0.1266, -0.1949],
        [ 0.4320, -0.3241, -0.2302, -0.3493]], requires_grad=True)
Parameter containing:
tensor([-0.4683, -0.2919,  0.4298], requires_grad=True)
```

Basic neural network in PyTorch

4. Set some values we like

```
# Manually set weight and bias
m.weight.data = torch.tensor(np.arange(1,13).reshape((3,4))).float()
print(m.weight)
m.bias.data = torch.ones_like(m.bias.data).float()
print(m.bias)
```

```
Parameter containing:
tensor([[ 1.,  2.,  3.,  4.],
        [ 5.,  6.,  7.,  8.],
        [ 9., 10., 11., 12.]], requires_grad=True)
Parameter containing:
tensor([1., 1., 1.], requires_grad=True)
```

Basic neural network in PyTorch

5. Prepare a single input data example

```
x = torch.randn(1, 4)
print(x.shape)
```

```
torch.Size([1, 4])
```

Get the output by ***calling*** the model

```
out = m(x)
print(out.shape)
```

```
torch.Size([1, 3])
```

x is 1×4:
By default, the first dimension of a tensor stands for batch size

out is 1×3:
As the result of m.weight (4×3) times x (1×4)

Basic neural network in PyTorch

6. Prepare a batch of data examples

```
x1 = torch.randn(100, 4)
out1 = m(x1)
print(out1.shape)
```

```
torch.Size([100, 3])
```

x is 100×4:

By default, the first dimension of a tensor stands for batch size

out is 100×3:

As the result of m.weight (4×3) times x (100×4)

Basic neural network in PyTorch

7. Let's manually check what computation is done

Input vector: [1,1,1,1]

```
x2 = torch.ones(1,4)
print(x2)
out2 = m(x2)
print(out2)
```

```
tensor([[1., 1., 1., 1.]])
tensor([[11., 27., 43.]], grad_fn=<AddmmBackward>)
```

Manual computation:

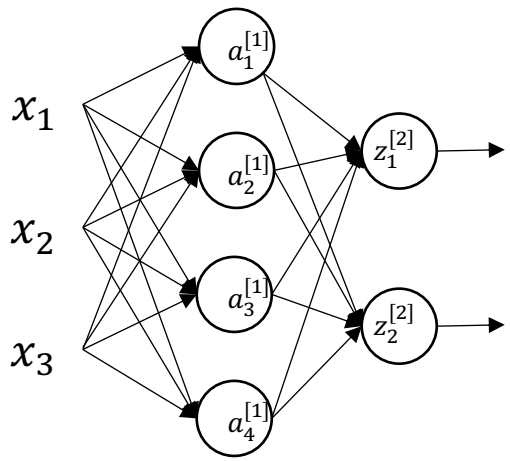
$$\text{out2} = \underbrace{\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}}_{\text{Weight}} \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{\text{Bias}}$$

Backprop in PyTorch

Necessary imports

```
import torch
import torch.nn as nn
import numpy as np
torch.manual_seed(0)
```

```
<torch._C.Generator at 0x111b7ddb0>
```



Define linear and activation layers

```
linear1 = nn.Linear(3, 4)
act1 = nn.ReLU()
linear2 = nn.Linear(4, 2)
act2 = nn.Sigmoid()
```

Initialize to values easier to read:

```
linear1.weight.data = torch.tensor(np.arange(1,13).reshape((4,3))).float()
print(linear1.weight)
linear1.bias.data = torch.zeros_like(linear1.bias.data).float()
print(linear1.bias)
```

```
Parameter containing:
tensor([[ 1.,  2.,  3.],
        [ 4.,  5.,  6.],
        [ 7.,  8.,  9.],
        [10., 11., 12.]], requires_grad=True)
Parameter containing:
tensor([0., 0., 0., 0.], requires_grad=True)
```

Backprop in PyTorch (layer 1)

Prepare some toy data, and watch the output from layer 1

```
x = torch.tensor([[1.,2.,3.]], requires_grad=True)
print('x: ', x)
z1 = linear1(x)
a1 = act1(z1)

print(z1)
print(a1)
print(a1.shape)
```

```
x:  tensor([[1., 2., 3.]], requires_grad=True)
tensor([[14., 32., 50., 68.]], grad_fn=<AddmmBackward>)
tensor([[14., 32., 50., 68.]], grad_fn=<ReluBackward0>)
torch.Size([1, 4])
```

Forward:

$$z1 = \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}}_{\text{Weight}} \underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\text{Bias}}$$

Backprop in PyTorch (layer 1)

Create some pseudo gradients that are in the same shape as a1

```
external_grad = torch.ones_like(a1) * 0.5
a1.backward(gradient=external_grad)
```

$$dz = da = \begin{pmatrix} .5 \\ .5 \\ .5 \\ .5 \end{pmatrix} \qquad W = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

Gradients computed:

```
print(x.grad)
print(linear1.weight.grad)
print(linear1.bias.grad)
```

```
tensor(
tensor(


tensor(                                    )
```

$$dx = W^T dz = \begin{bmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{bmatrix} \begin{pmatrix} .5 \\ .5 \\ .5 \\ .5 \end{pmatrix} = \begin{pmatrix} 11 \\ 13 \\ 15 \end{pmatrix}$$

$$dW = dz \cdot x^T = \begin{pmatrix} .5 \\ .5 \\ .5 \\ .5 \end{pmatrix} (1., 2., 3.) =$$

Backprop in PyTorch (layer 2)

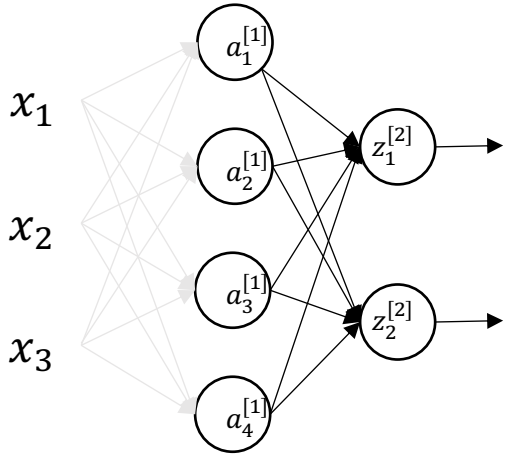
Initialize the weight of linear2 to zeros (0), because sigmoid function saturates very fast

```
linear2.weight.data = torch.zeros_like(linear2.weight.data).float()
linear2.bias.data = torch.zeros_like(linear2.bias.data).float()
print(linear2.weight)
print(linear2.bias)
```

```
Parameter containing:
tensor([[0., 0., 0., 0.],
        [0., 0., 0., 0.]], requires_grad=True)
Parameter containing:
tensor([0., 0.], requires_grad=True)
```

Forward pass:

```
x = torch.tensor([[1.,2.,3.]])
z1 = linear1(x)
a1 = act1(z1)
z2 = linear2(a1)
a2 = act2(z2)
```



Output:

```
z2:  tensor([[0., 0.]], grad_fn=<AddmmBackward>)
a2:  tensor([[0.5000, 0.5000]], grad_fn=<SigmoidBackward>)
```

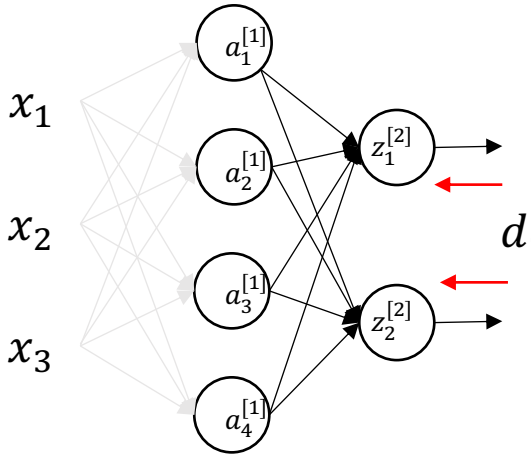
$$\sigma(0) = \frac{1}{1+e^0} = 0.5$$

Backprop in PyTorch (layer 2)

Backward pass:

Tell PyTorch to save the gradients for intermediate variables

```
z2.retain_grad()
a1.retain_grad()
external_grad = torch.ones_like(a2) * 0.5
a2.backward(gradient=external_grad)
```



$$da^{[2]} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

Manually check dz_2

```
# dz = a*(1 - a) * da
a2_np = a2.data.numpy()
dz2_manual = a2_np * (1 - a2_np) * external_grad.numpy()
dz2 = z2.grad.numpy()
print(np.equal(dz2_manual, dz2))
print('dz2: ', dz2)
```

Output:

```
[[ True  True]]
dz2:  [[0.125 0.125]]
```

$$dz^{[2]} = da^2 * \frac{\partial a^{[2]}}{\partial z^{[2]}}$$

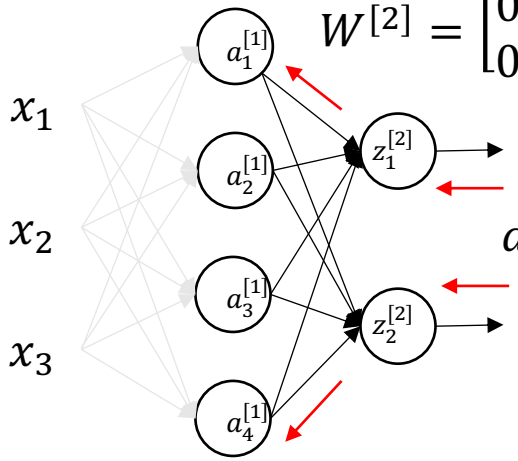
$$= da^2 * g'(z^{[2]})$$

$$= da^2 * \sigma'(z^{[2]})$$

Derivatives of sigmoid function:

$$\sigma'(z^{[2]})$$
$$= \sigma(z^{[2]}) \cdot (1 - \sigma(z^{[2]}))$$
$$= a^{[2]} \cdot (1 - a^{[2]}))$$
$$= \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \cdot (1 - \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}))$$

Backprop in PyTorch (layer 2)



$$W^{[2]} = \begin{bmatrix} 0000 \\ 0000 \end{bmatrix}$$

$$da^{[2]} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

$$da^{[1]} = ?$$

We know: $dz^{[2]} = \begin{pmatrix} 0.125 \\ 0.125 \end{pmatrix}$ so $db^{[2]} = da^{[2]} = \begin{pmatrix} 0.125 \\ 0.125 \end{pmatrix}$

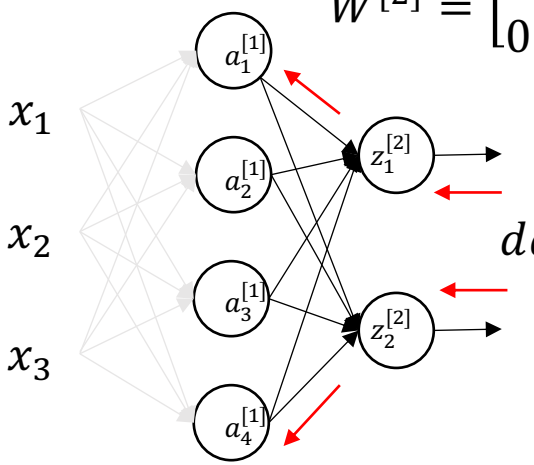
$$da^{[1]} = W^T dz^{[2]} = \begin{bmatrix} 0,0 \\ 0,0 \\ 0,0 \\ 0,0 \end{bmatrix} \begin{pmatrix} 0.125 \\ 0.125 \end{pmatrix} = \begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{pmatrix}$$

```python
print('dW2: ', linear2.weight.grad)
print('db2: ', linear2.bias.grad)
print('dz2: ', z2.grad)
print('da1: ', a1.grad)
```

```
dW2:  tensor([[1.7500, 4.0000, 6.2500, 8.5000],
              [1.7500, 4.0000, 6.2500, 8.5000]])
db2:  tensor([0.1250, 0.1250])
dz2:  tensor([[0.1250, 0.1250]])
da1:  tensor([[0., 0., 0., 0.]])
```

Backprop in PyTorch (layer 2)

$$W^{[2]} = \begin{bmatrix} 0000 \\ 0000 \end{bmatrix}$$



$$da^{[2]} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

$$a^{[1]} = \begin{pmatrix} 14 \\ 32 \\ 52 \\ 68 \end{pmatrix}$$

$$dW^{[2]} = ?$$

$$dW_2 = dz_2 a_1^T = \begin{pmatrix} 0.125 \\ 0.125 \end{pmatrix} (14, 32, 50, 68)$$

Manually check dW_2

```
a1_np = a1.data.numpy()
print('a1: ', a1_np)
dW2 = np.dot(dz2.T, a1_np)
print('dW2: ', dW2)
```

Output:

```
dW2:  [[1.75 4.   6.25 8.5 ]
 [1.75 4.   6.25 8.5 ]]
```

Backprop in PyTorch - nn.module

```
class Net(nn.Module):
    def __init__(self):
        super(Net, self).__init__()
        …
    def forward(self, x):
        …
        return xx
```

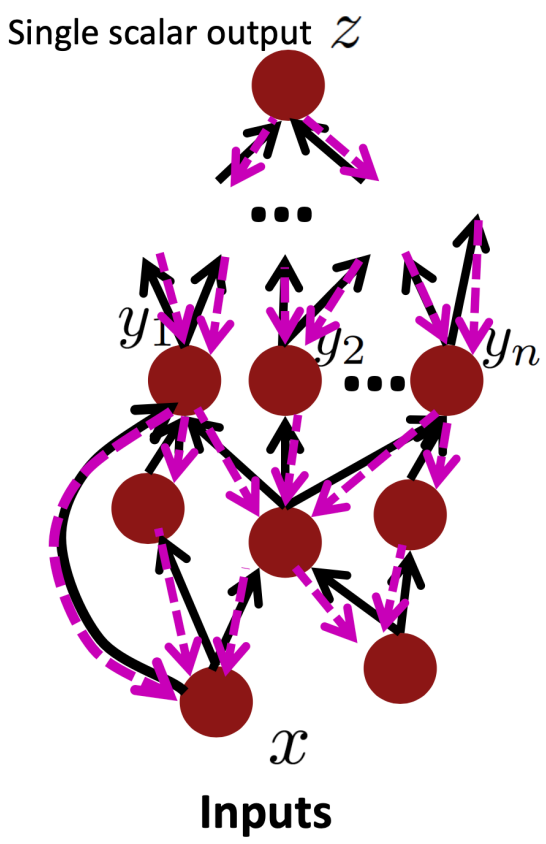
Define model architecture in __init__()

Define forward propagation in forward()

General procedure of gradient descent:

```
# Build the model
net = Net()
# Execute F-prop
output = net(input)
# Compute loss
loss = criterion(output, target)
# B-prop
loss.backward()
optimizer.step()
```

Backprop in General



1. F-prop: visit nodes in topological sort order
- Compute value of node given predecessors

2. B-prop:
- initialize output gradient = 1
- visit nodes in *reverse* order: Compute gradient w.r.t. each node using gradient w.r.t. successors

$\{y_1, y_2, \dots, y_n\}$ = successors of x

$$\frac{\partial z}{\partial x} = \sum_{i=1}^{n} \frac{\partial z}{\partial y_i} \frac{\partial y_i}{\partial x}$$

Slide credit to: https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/

Table of Content

- Motivation
- Word Vectors
- Neural Networks
- **Neural Text Classification**
  - **Bag-of-words; evaluate**

Text Classification

- Library-like subjects (e.g., the Dewey decimal system)
- News stories: politics vs. sports vs. business vs. technology ...
- Reviews of films, restaurants, products: positive vs. negative
- Author attributes: identity, political stance, gender, age, ...
- Email, arXiv submissions, etc.: spam vs. not
- What is the reading level of a piece of text?
- How influential will a scientific paper be?
- Will a piece of proposed legislation pass?
- What dialect is a text written in?
- Does the text contain content that will likely offend people?

Slide credit to: https://nasmith.github.io/NLP-winter23/calendar/

Traditional way of text classification

- Traditional = feature engineering. Question: What features?

Running example:

$\boxed{\boldsymbol{x} = \text{“The vodka was great, but don't touch the hamburgers.”}}$

A different representation of the text sequences: features.

- Often, these are term (word or word sequence) frequencies.
  E.g., $\phi^{freq.}_{\text{hamburgers}}(\boldsymbol{x}) = 1$, $\phi^{freq.}_{\text{the}}(\boldsymbol{x}) = 2$, $\phi^{freq.}_{\text{delicious}}(\boldsymbol{x}) = 0$, $\phi^{freq.}_{\text{don't touch}}(\boldsymbol{x}) = 1$.

  Term frequency

- Can also be binary word “presence” features.
  E.g., $\phi^{presence}_{\text{hamburgers}}(\boldsymbol{x}) = 1$, $\phi^{presence}_{\text{the}}(\boldsymbol{x}) = 1$, $\phi^{presence}_{\text{delicious}}(\boldsymbol{x}) = 0$, $\phi^{presence}_{\text{don't touch}}(\boldsymbol{x}) = 1$.

  Term presence

- Transformations on word frequencies: logarithm, idf weighting

  Term freq. * inverse document freq. (TFIDF)

$$\forall v \in \mathcal{V}, \text{idf}(v) = \log \frac{n}{|i : \text{count}_{\boldsymbol{x}_i}(v) > 0|}$$

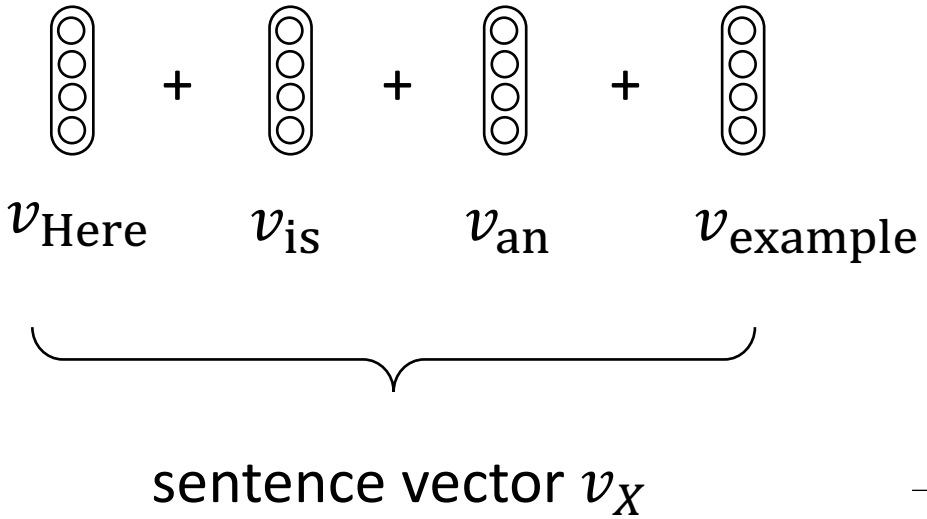
$$\phi^{tfidf}_v(\boldsymbol{x}) = \phi^{freq.}_v(\boldsymbol{x}) \cdot \text{idf}(v)$$

Slide credit to: https://nasmith.github.io/NLP-winter23/calendar/

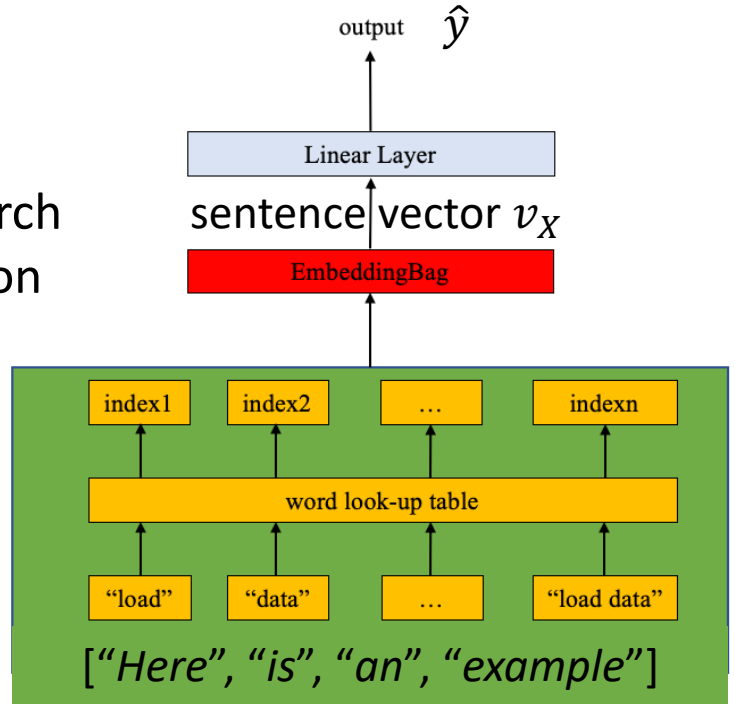
Bag-of-Words Neural Net

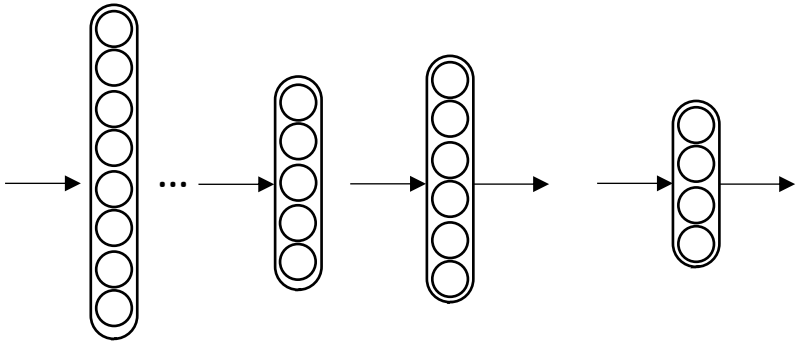
Task: News text classification

X: [*"Here"*, *"is"*, *"an"*, *"example"*]



A typical PyTorch implementation

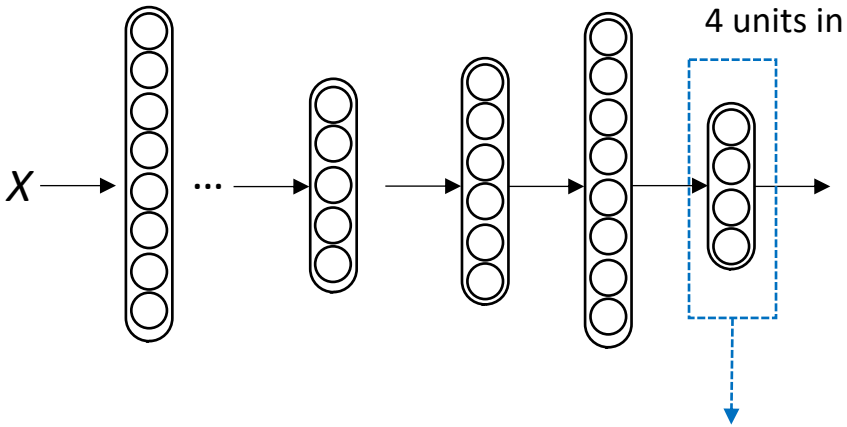




$$\hat{y} = \begin{bmatrix} P(\text{World}|X) \\ P(\text{Sports}|X) \\ P(\text{Business}|X) \\ P(\text{Sci/Tec}|X) \end{bmatrix}$$

Extended to multiple classes: Softmax layer

Size of output layer = number of classes = 4 (World, Sports, Business, Sci/Tec)



$$\hat{y} = \begin{bmatrix} P(\text{World}|X) \\ P(\text{Sports}|X) \\ P(\text{Business}|X) \\ P(\text{Sci/Tec}|X) \end{bmatrix}$$

Softmax activation function

Linear part: $z^{[L]} = W^{[L]}a^{[L-1]} + b^{[L]}$ $\quad 4\times1$

Activation part: $t = e^{z^{[L]}}$ $\quad 4\times1$

Input: vector $z^{[L]}$

Output: vector $a^{[L]}$

$$a^{[L]} = \frac{e^{z^{[L]}}}{\sum_{j=1}^{4} t_j} \qquad 4\times1$$

In which, $a_i^{[L]} = \dfrac{t_i}{\sum_{j=1}^{4} t_j}$

$$z^{[L]} = \begin{bmatrix} 4 \\ 1 \\ -1 \\ 2 \end{bmatrix} \quad t = e^{z^{[L]}} = \begin{bmatrix} e^4 \\ e^1 \\ e^{-1} \\ e^2 \end{bmatrix} = \begin{bmatrix} 54.60 \\ 2.72 \\ 0.37 \\ 7.39 \end{bmatrix}$$

$$\sum_{j=1}^{4} t_j = 54.60 + 2.72 + 0.37 + 7.390 = 65.07$$

$$a^{[L]} = \begin{bmatrix} 54.60/65.07 \\ 2.72/65.07 \\ 0.37/65.07 \\ 7.39/65.07 \end{bmatrix} = \begin{bmatrix} 0.84 \\ 0.04 \\ 0.01 \\ 0.11 \end{bmatrix}$$

P(World |X) = 0.84
P(Sports |X) = 0.04
P(Business |X) = 0.01
P(Sci/Tec |X) = 0.11

Softmax layer: loss function

1 $\hat{y} = \begin{bmatrix} 0.84 \\ 0.04 \\ 0.01 \\ 0.11 \end{bmatrix}, y = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ 😊

2 $\hat{y} = \begin{bmatrix} 0.84 \\ 0.04 \\ 0.01 \\ 0.11 \end{bmatrix}, y = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ 🙁

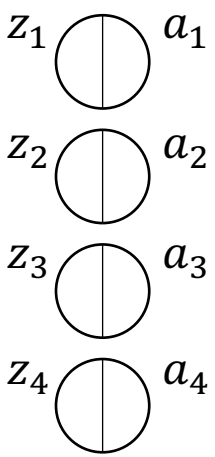
Cross entropy loss: $L = -\sum_i y_i \log(\hat{y}_i)$

Measures the distance between output distribution ($\hat{y}$) and the actual distribution (y)

In 1, $L = -\log(0.84) = 0.174$

In 2, $L = -\log(0.04) = 3.22$

Gradient in the same form as that of logistic regression



$$L = -\sum_i y_i \log(a_i)$$

$$\frac{\partial L}{\partial a_i} = -\frac{y_i}{a_i}$$

$$\Rightarrow \frac{\partial L}{\partial z_i} = \sum_k \frac{\partial L}{\partial a_k}\frac{\partial a_k}{\partial z_i} = -\sum_k \frac{y_k}{a_k}\frac{\partial a_k}{\partial z_i}$$

$$\frac{\partial a_k}{\partial z_i} = \begin{cases} k = i, \frac{\partial}{\partial z_i}\left(\frac{e^{(z_i)}}{\sum_j e^{(z_j)}}\right) = a_i(1 - a_i) \\ k \neq i, \frac{\partial}{\partial z_i}\left(\frac{e^{(z_k)}}{\sum_j e^{(z_j)}}\right) = -a_k a_i \end{cases}$$

$$\Rightarrow \frac{\partial L}{\partial z_i} = -\frac{y_i}{a_i} a_i(1 - a_i) - \sum_{k \neq i} \frac{y_k}{a_k}(-a_k a_i) = -y_i + y_i a_i + \sum_{k \neq i} y_k a_i = \boxed{a_i - y_i}$$

Evaluate a Classifier

Accuracy:

$$\begin{aligned}
\mathrm{A}(\text{classify}) &= p(\text{classify}(\boldsymbol{X}) = Y) \\
&= \sum_{\boldsymbol{x} \in \mathcal{V}^*, \ell \in \mathcal{L}} p(\boldsymbol{X} = \boldsymbol{x}, Y = \ell) \cdot \begin{cases} 1 & \text{if classify}(\boldsymbol{x}) = \ell \\ 0 & \text{otherwise} \end{cases} \\
&= \sum_{\boldsymbol{x} \in \mathcal{V}^*, \ell \in \mathcal{L}} p(\boldsymbol{X} = \boldsymbol{x}, Y = \ell) \cdot \mathbf{1}\left\{\text{classify}(\boldsymbol{x}) = \ell\right\}
\end{aligned}$$

Training accuracy

where p is the *true* distribution over data. Error is $1 - \mathrm{A}$.

This is *estimated* using a test dataset $\langle \bar{\boldsymbol{x}}_1, \bar{y}_1 \rangle, \ldots \langle \bar{\boldsymbol{x}}_m, \bar{y}_m \rangle$:

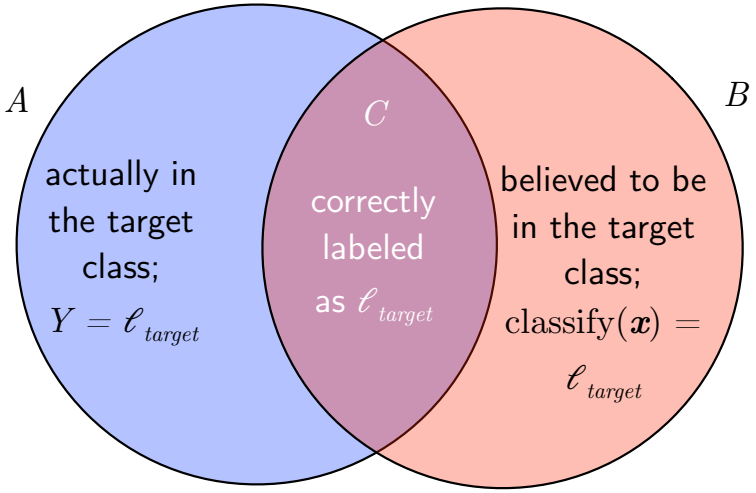
$$\hat{\mathrm{A}}(\text{classify}) = \frac{1}{m} \sum_{i=1}^{m} \mathbf{1}\left\{\text{classify}(\bar{\boldsymbol{x}}_i) = \bar{y}_i\right\}$$

Testing accuracy (unbiased)

Slide credit to: https://nasmith.github.io/NLP-winter23/calendar/

Evaluation in the "Needle in a Haystack" Case

Suppose one label $\ell_{target} \in \mathcal{L}$ is a "target."
Precision and **recall** encode the goals of returning a "pure" set of targeted instances and capturing *all* of them.



$$\hat{\mathrm{P}}(\text{classify}) = \frac{|C|}{|B|} = \frac{|A \cap B|}{|B|}$$

$$\hat{\mathrm{R}}(\text{classify}) = \frac{|C|}{|A|} = \frac{|A \cap B|}{|A|}$$

$$\hat{F}_1(\text{classify}) = 2 \cdot \frac{\hat{\mathrm{P}} \cdot \hat{\mathrm{R}}}{\hat{\mathrm{P}} + \hat{\mathrm{R}}}$$

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To-Do List

- Start working on A1
- Continue reading Ch 6 of SLP3
- Attend Lab2