Econ homework 2 - OSM Bootcamp 2018

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Proof 1

Suppose by contradiction that $g(\hat{x}) = y \neq \hat{x}$. Then, $\exists \epsilon > 0$ such that $\hat{x} \notin (y - \epsilon, y + \epsilon)$ but $g(\hat{x}) = y \in (y - \epsilon, y + \epsilon)$.
Further, g is continuous at $\hat{x}$ thus $\exists \delta > 0$ such that $\forall x \in (\hat{x} - \delta, \hat{x} + \delta)$ then $g(x) \in (y - \epsilon, y + \epsilon)$.
But, $\{g^t(x)\} \to \hat{x}$ so $\exists T \in \mathbb{N}$ such that $\forall t > T$ then $g^t(x) \in (\hat{x} - \delta, \hat{x} + \delta)$
But by continuity if $g^t(x) \in (\hat{x} - \delta, \hat{x} + \delta)$ then $g(g^t(x)) \in (y - \epsilon, y + \epsilon)$ which violates the assumption that $g^t(x) \to \hat{x}$.
Thus, it must be that $g(\hat{x}) = \hat{x}$

Proof 2

From Gelfand's we have $||A^k||^{1/k} \to r(A)$, so $||A^k|| \to r(A)^k$.
But if $r(A) < 1$ and $k \to \infty$ then $r(A)^k \to 0$
Thus, $||A^k|| \to 0$