Cold atoms experiment : ^{133}Cs

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?| Why ^{133}Cs ?

→ Alkali (cold atoms ✓)

→ Bosons ! (No pauli exclus°, we can perform the exp w/ 1000 non-int. atoms in each micro well).

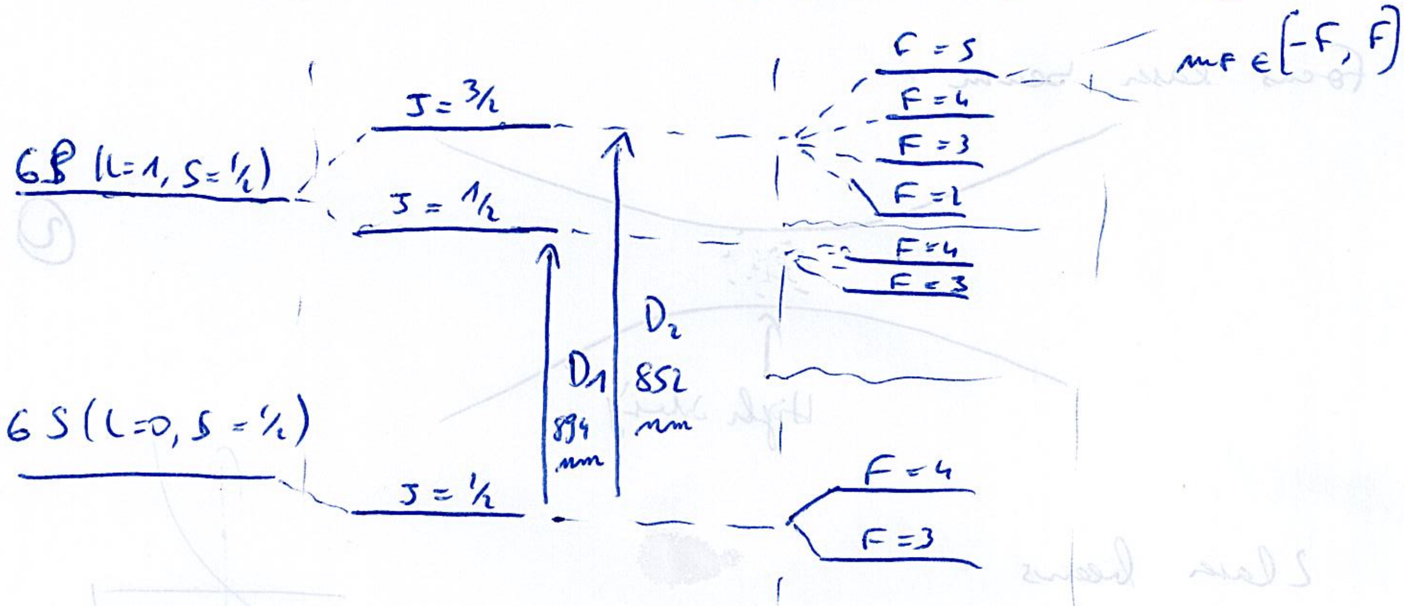
→ N at : 55 ⇝ 133 − 55 = 78 neutrons pair !

- seul isotope à l'état nat, non radioactif.

?| Electronic structure?

Notat° spectro : $n\,^{2S+1}L_J$

For ^{133}Cs : $n = 6$, $S = 1/2$ (alkali), $J = 7/2$



Fine struct

$\propto \vec{L}\cdot\vec{S}$

$J = \vec{L} + \vec{S}$

$\Rightarrow J \in [|L-S|, L+S]$

$m_J \in -J, J$

$\vec{L}\cdot\vec{S} = \frac{1}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$

HF struct

$\propto \vec{I}\cdot\vec{J}$

$\vec{F} = \vec{I} + \vec{J}$

idem

Zeeman

1 Dipole trap?

Polarizability of the external e^- :

$$\vec{d} = \alpha \vec{E}$$

$\vec{E}$ oscillating : $\vec{d}(\omega) = \alpha(\omega)\, \vec{E}(\omega)$

$$\mathcal{E}_{dipole} = -\frac{1}{2}\, \alpha(\omega) \underbrace{\langle \vec{E}^2 \rangle}_{I(\vec{r})} \cancel{t}$$

LRT (lin resp th)

can be >0

laser intensity

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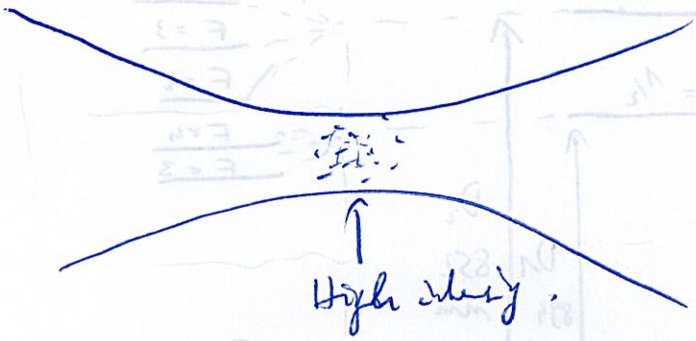
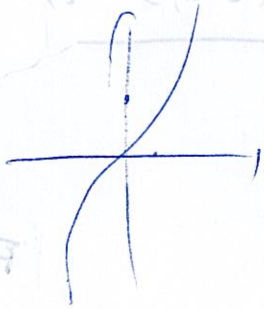
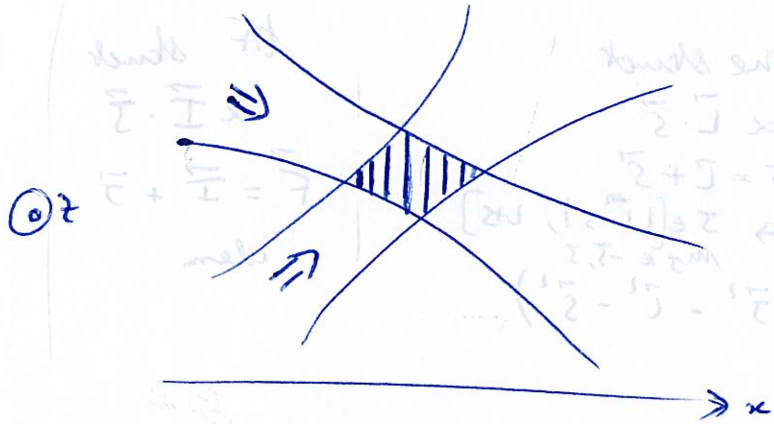
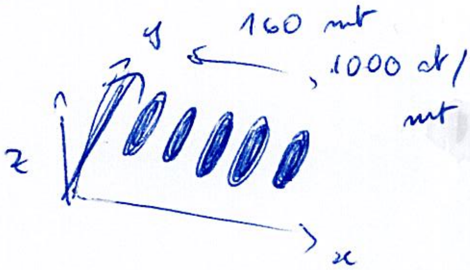
Focus laser beam :

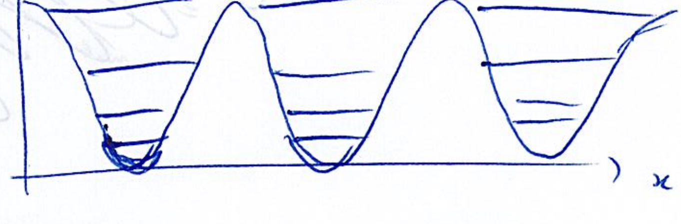




2 laser beams







?| For squeezed / coherent states, what do we need to have?

→ Harmonic oscillator! Mandatory. → Harmonicity of the trap → equal spacing of the levels

→ Non interacting atoms

↳ each of them in the same state

↳ we do single-atom φ w/ a large n° of single atoms.

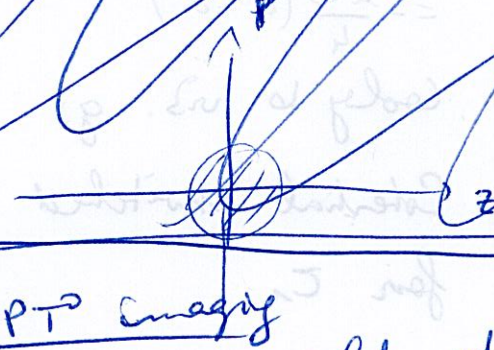
T low = HO regime.

Sideband cooling?

HO 3D: we consider oscillat° along z.

~~Cooling to the g.s. : 95% atoms in the gs.~~

~~| What is the gs?~~



~~Gaussian state:~~

~~$$\psi(z) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega z^2/2\hbar}$$~~

ABSORPT° imaging

Now: no more cold atoms, just QHO physics.

2 relevant Hamiltonians in the problem:

$$H_{OFF} = \frac{p^2}{2m}$$

$$H_{ON} = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 z^2$$

Abs img.
Pot switched off:

$$n(\vec{r}) \propto \left|\tilde{\Psi}\left(\vec{p} = \frac{\vec{r} - \vec{r}_{trap}}{t_{BE}}\right)\right|^2$$

Using ⊕ convenient units:

$\vec{p} = m\vec{v}$ $\quad \vec{v}t = \vec{r}$

$$\tilde{H}_{ON} = \frac{H_{ON}}{\hbar\omega} = \frac{1}{2m\hbar\omega} p^2 + \frac{1}{2}\frac{m\omega}{\hbar} z^2$$ (4)

$$= \frac{1}{2}\left(\tilde{p}^2 + \tilde{z}^2\right)$$

w/ $\tilde{p} = \frac{1}{\sqrt{m\hbar\omega}} p$, $\tilde{z} = \sqrt{\frac{m\omega}{\hbar}} z$.

Omit the ~ in the following.

Def $a = \frac{1}{\sqrt{2}}(\tilde{x} + i\tilde{p})$, $a^+ = \frac{1}{\sqrt{2}}(\tilde{x} - i\tilde{p})$

$\hookrightarrow$ $\tilde{H}_{ON} = a^+a + \frac{1}{2}$

$H_{ON} = \hbar\omega.\left(a^+a + \frac{1}{2}\right)$

$\tilde{p} = \frac{1}{i\sqrt{2}}\left(a - a^+\right)$

$\tilde{p}^2 = -\frac{1}{2}\left(a - a^+\right)^2$

$$H_{OFF} = \frac{p^2}{2m} = \hbar\omega\frac{\tilde{p}^2}{2} = \cancel{\frac{\hbar\omega}{\sqrt{2}i}(a - a^+)}$$

$$= -\frac{\hbar\omega}{4}\left(a - a^+\right)^2$$

Experimental procedure : a $\rightarrow$ Cooling to vib. gs

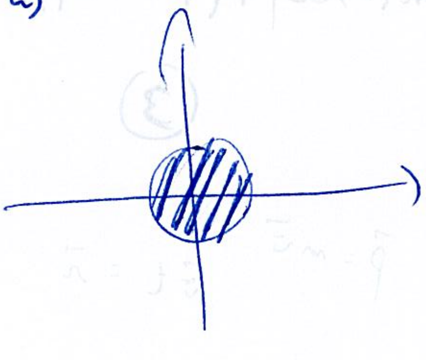
b $\rightarrow$ Potential switched off for τ_1

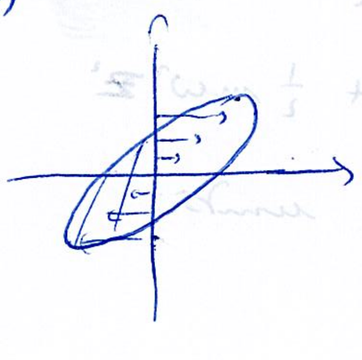
c $\rightarrow$ Potential switched back on for τ_2.

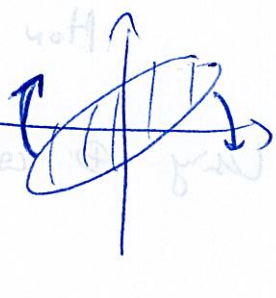
a) H_{ON}, eig state

b) H_{OFF}, evol:

c) H_{ON}







@ $t=0$: $\quad a|0\rangle = 0 \Rightarrow$ ~~$(z + z_0^2 \frac{d}{dz})\psi_0(z) = 0$~~

$$a = \frac{\tilde{z} + i\tilde{p}}{\sqrt{2}} \quad \text{so: } \cancel{\left(z + \frac{\hbar}{m\omega}\frac{d}{dz}\right)\psi_0(z) = 0}$$

~~Solut:~~

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So $\quad a = \xi \tilde{z} + \varsigma \tilde{p} \quad$ w/ $\xi = \frac{1}{\sqrt{2}}$, $\varsigma = \frac{i}{\sqrt{2}}$

Hence: $\quad (i \xi \partial_{\tilde{p}} + \varsigma \tilde{p}) \psi_0(\tilde{p}) = 0$

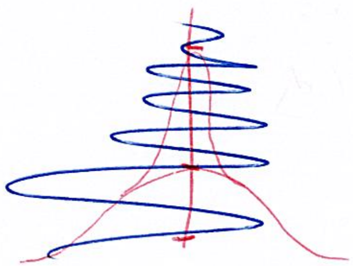
Good start

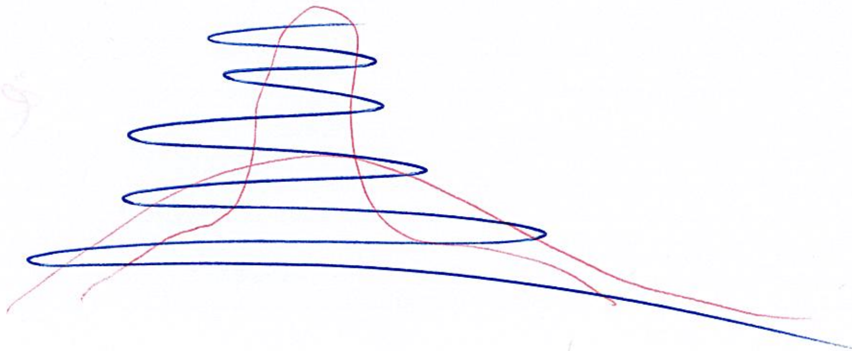
Solut: $\quad \psi_0(p) = \cancel{\gamma e^{i \varsigma/\xi}}$

$$\gamma e^{i \frac{\varsigma}{\xi} \tilde{p}^2/2}$$

$$= \gamma e^{-\tilde{p}^2/2}$$

Gaussian w/ variance $\sigma^2 = 1$ in $\tilde{p}$





We want to compute $\Psi_g(p)$.

Introduce $a' = U a U^+$

We have $a' |\Psi_g\rangle = U a \underbrace{U^+ U}_{\mathbb{1}} |0\rangle$

$$= U \underbrace{a |0\rangle}_{0}$$

$$= 0$$

So if we compute a' we have a differential equat° for $\Psi_g(p)$! $(a'|\Psi_g\rangle = 0)$!

$U = U_2 U_1$ with: $U_1 = e^{-i \frac{H_{OFF}}{\hbar} \tau_1}$

$$U_2 = e^{-i \frac{H_{ON}}{\hbar} \tau_2}$$

$$a' = U_2 U_1 \, a \, U_1^+ U_2^+$$

$$U_1 a U_1^+ = e^{-i \frac{\omega}{2} \tilde{p}^2 \tau_1} \, a \, e^{i \frac{\omega}{2} \tilde{p}^2 \tau_2}$$

2opt° eq: $a = x + ip$, $p^2 = \left(\frac{a - a^+}{i}\right)^2$

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$$\tilde{p}^2 = \cancel{A}\left(\frac{1}{\sqrt{2}i}(a-a^+)\right)^2$$

$$= -\frac{1}{2}(a-a^+)^2$$

⑦

$$U_1 a U_1^+ = e^{i\frac{\omega}{4}\cancel{\hbar}(a-a^+)^2\tau_1}\; a\; e^{-i\frac{\omega}{4}(a-a^+)^2\tau_1}$$

$$\overset{\underline{\text{BH lemma}}}{=} a + i\frac{\omega\tau_1}{4}\left[(a-a^+)^2, a\right] + \frac{\left(\frac{i\omega\tau_1}{4}\right)^2}{2!}\left[(a-a^+)^2,\left[(a-a^+)^2, a\right]\right] + \dots$$

$$\left[(a-a^+)^2, a\right] = (a-a^+)\left[a-a^+, a\right] + \left[a-a^+, a\right](a-a^+)$$

$$= 2\underbrace{(a-a^+)}\;\cancel{\hbar}$$

$$U_1 a U_1^+ = a + i\frac{\omega\tau_1}{\cancel{4}\,2}(a-a^+)$$

$$= a\left(1 + i\frac{\omega\tau_1}{2}\right) - i\frac{\omega\tau_1}{2}a^+$$

BH lemma:

$$e^B A e^{-B} = A + [B,A] + \frac{1}{2!}[B,[B,A]] + \dots$$

$$U a U^+ = U_2 \left(U_1 a U_1^+\right) U_2^+$$

$$= e^{-i\omega a^+ a \tau_2} \left[a \left(1 + i \frac{\omega \tau_1}{2}\right) - i \frac{\omega \tau_1}{2} a^+ \right] e^{i\omega a^+ a \tau_2}$$

Pour calculer $e^{-i\omega a^+ a \tau_2} \, a \, e^{i\omega a^+ a \tau_2}$

↳ BH lemma ! ~~possible~~ !

~~On va faire autrement~~ :

$$e^{-i\omega a^+ a \tau_2} a e^{i\omega a^+ a \tau_2} = a - i\omega \tau_2 [a^+ a, a] + \frac{(-i\omega\tau_2)^2}{2!} [a^+ a, [a^+ a, a]] + \dots$$

$$[a^+ a, a] = a^+ \cancel{[a, a]} + [a^+, a] a$$

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$$= -a$$

$$[a^+ a, [a^+ a, a]] = [a^+ a, -a] = a .$$

$$e^{-} a e^{-} = a - (i\omega\tau_1) a + \frac{(-i\omega\tau_1)^2}{2!} a - \frac{(-i\omega\tau_2)^3}{3!} a + \dots$$

$$= a \left[\left(1 - \frac{(\omega\tau_1)^2}{2} + \frac{(\omega\tau_2)^4}{4} - \dots \right) + i\left(\omega\tau_2 - \frac{(\omega\tau_2)^3}{3!} + \dots \right) \right]$$

$$= a \left(\cos(\omega\tau_1) + i \sin(\omega\tau_2) \right)$$

$$= a \, e^{i\omega\tau_1}$$

Naturally, $e^{-i\dots}\, a^+ e^{i\dots} = a^+ e^{-i\omega\tau_2}$

Hence:

$$a' = \cancel{a\, e^{i\omega\tau_2}} \left(1 + i\frac{\omega\tau_1}{2}\right) - i\frac{\omega\tau_1}{2} a^+ e^{-i\omega\tau_2}$$

$a e^{i\omega\tau_2}$

Replace a, a^+ by $\frac{\tilde{z} \pm i\tilde{p}}{\sqrt{2}}$:

$$a' = \frac{1}{\sqrt{2}}\left(\left(1 + i\frac{\omega\tau_1}{2}\right)e^{i\omega\tau_2} \cancel{+} - i\frac{\omega\tau_1}{2} e^{-i\omega\tau_2}\right)\tilde{z}$$

$$+ \frac{i}{\sqrt{2}}\left(\left(1 + i\frac{\omega\tau_1}{2}\right)e^{i\omega\tau_2} + i\frac{\omega\tau_1}{2} e^{-i\omega\tau_2}\right)\tilde{p}$$

$$= \frac{1}{\sqrt{2}}\left(e^{i\omega\tau_2} + i\frac{\omega\tau_1}{2}\left(e^{i\omega\tau_2} - e^{-i\omega\tau_2}\right)\right)\tilde{z}$$

$$+ \frac{i}{\sqrt{2}}\left(e^{i\omega\tau_2} + i\frac{\omega\tau_1}{2}\left(e^{i\omega\tau_2} + e^{-i\omega\tau_2}\right)\right)\tilde{p}$$

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$$= \frac{1}{\sqrt{2}}\left(e^{i\omega\tau_2} \cancel{+} - \omega\tau_1 \sin(\omega\tau_2)\right)\tilde{z}$$

$$+ \frac{i}{\sqrt{2}}\left(e^{i\omega\tau_2} \cancel{+} - \omega\tau_1 \cos(\omega\tau_2)\right)\tilde{p}$$

$$= \overbrace{\frac{1}{\sqrt{2}}\left(\cos(\omega\tau_1) + (i - \omega\tau_1)\sin(\omega\tau_1)\right)}^{\xi}\tilde{z}$$

$$+ \underbrace{\frac{1}{\sqrt{2}}\left(-\sin(\omega\tau_1) + (i - \omega\tau_1)\cos(\omega\tau_2)\right)}_{\xi}\tilde{p}.$$

Remember $\hat{a}' |\psi_g\rangle = 0$

So $\psi_g(p) = \gamma e^{-\alpha \tilde{p}^2/2}$ $\qquad$ w/ $\alpha = -i \, \dot{\xi}/\xi$

Momentum uncertainty:

$$|\psi_g(p)|^2 = \gamma \left| e^{-\alpha \tilde{p}^2/2} \right|^2$$

$$= \gamma \, e^{-Re(\alpha) \tilde{p}^2} \cancel{\left| e^{-i Im(\alpha) \tilde{p}^2/2} \right|^2}$$

Gaussian w/ width: $\sigma' = \dfrac{1}{2 Re(\alpha)}$

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$$\cancel{\sigma' = \frac{i}{2} \frac{1}{2}}$$

$$Re(\alpha) = Re\left(-i \frac{\dot{\xi}}{\xi} \right)$$

$$\cancel{= Re\left(-i \frac{-\sin(\omega\tau_1) + (i - \omega\tau_1)\cos(\omega\tau_1)}{} \right.}$$

$$= Re\left(-i \frac{-\sin(\omega\tau_2) + (i - \omega\tau_1)\cos(\omega\tau_2)}{\cos(\omega\tau_2) + (i - \omega\tau_1)\sin(\omega\tau_2)} \right)$$

$\cancel{i \sin(\omega\tau_2)}$

$$\mathrm{Re}\left(\frac{i\sin(\omega\tau_2)+\cos(\omega\tau_2)+i\omega\tau_1\cos(\omega\tau_2)}{\cos(\omega\tau_2)+i\sin(\omega\tau_2)-\omega\tau_1\sin(\omega\tau_2)}\right)$$

$$=\mathrm{Re}\left(\frac{(i\sin(\omega\tau_2)+\cos(\omega\tau_2)+i\omega\tau_1\cos(\omega\tau_2))(\cos(\omega\tau_2)-i\sin(\omega\tau_2)-\omega\tau_1\sin(\omega\tau_2))}{(\cos(\omega\tau_2)-\omega\tau_1\sin(\omega\tau_2))^2+\sin^2(\omega\tau_2)}\right)$$

$$=\frac{\sin^2(\omega\tau_2)+\cos^2(\omega\tau_2)-\omega\tau_1\cos(\omega\tau_2)\sin(\omega\tau_2)+\omega\tau_1\cos(\omega\tau_2)\sin(\omega\tau_2)}{(\cos(\omega\tau_2)-\omega\tau_1\sin(\omega\tau_2))^2+\sin^2(\omega\tau_2)}$$

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$$=\cancel{\cos(\omega\tau_2)-\omega\tau_1}\;\frac{1}{\cos^2(\omega\tau_2)+(\omega\tau_1)^2\sin^2(\omega\tau_2)-2\omega\tau_1(\cos\omega\tau_2)\sin(\omega\tau_2)+\sin^2(\omega\tau_2)}$$

$$\frac{1}{\mathrm{Re}(\alpha)}=1+(\omega\tau_1)^2\sin^2(\omega\tau_2)-2\omega\tau_1\cos(\omega\tau_2)\sin(\omega\tau_2)$$

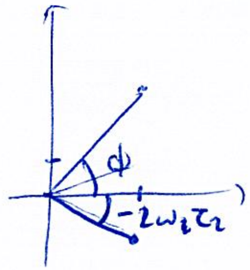
$$\cancel{=1+\omega\tau_1\left[\omega\tau_1\sin^2(\omega\tau_2)-2\cos(\omega\tau_2)\sin(\omega\tau_2)\right]}$$

$\cos x \sin x = \frac{1}{2}\sin(2x)$

$\sin^2(x) = \frac{1-\cos 2x}{2}$

$$\frac{1}{\mathrm{Re}(x)} = 1 + (\omega\tau_1)^2 \, \frac{1-\cos(2\omega\tau_2)}{2} - \omega\tau_1 \sin(2\omega\tau_2)$$

$$= 1 + \frac{(\omega\tau_1)^2}{2} - \omega\tau_1 \left[\frac{\omega\tau_1}{2} \cos(2\omega\tau_2) - \sin(2\omega\tau_2) \right]$$



w/ $\cancel{\sin(\omega\tau_2) =}$

$$\frac{\omega\tau_1}{2}\cos(2\omega\tau_2) - \sin(2\omega\tau_2) = \begin{pmatrix} \frac{\omega\tau_1}{2} \\ 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(2\omega\tau_2) \\ -\sin(2\omega\tau_2) \end{pmatrix}$$

$$= \sqrt{1 + \left(\frac{\omega\tau_1}{2}\right)^2} \, \cos(\underbrace{2\omega\tau_2 - \phi}_{\text{of angles}})$$