Quantum Optics – M1 ICFP

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September 18th, 2024

TD3: Motional squeezing of ultracold atoms

This exercise sheet is inspired by the following article: `https://doi.org/10.1103/PhysRevLett.83.4037`. The objective is to dicover a way to produce motional squeezing on an assembly of cold caesium atoms in an optical dipole trap.

1 Cold atoms in a harmonic trap

We consider an assembly of ultracold caesium atoms trapped in an optical dipole trap produced by far-detuned crossed laser beams.

1. What qualitative condition can we give on the gas temperature and inter-atomic interactions to consider that the motional atomic Hamiltonian of each atom is that of a harmonic oscillator?
2. Recall the expression of the annihilation and creation operators a and $a^\dagger$ for a harmonic oscillator of mass m and frequency ω as functions of the position and momentum operators x and p.
3. Give the expression of the Hamiltonian when the trap is on H_{on} and when the trap is off H_{off}, first as a function of $\tilde{x} = \sqrt{\frac{m\omega}{\hbar}}x$ and $\tilde{p} = \frac{1}{\sqrt{m\omega\hbar}}p$, and then as a function of a and $a^\dagger$. The $\tilde{}$ will be omitted in the following, and x and p will represent the dimensionless position and momentum operators.
4. Propose an experimental method to measure the momentum distribution of the atoms, i.e. make a projective measurement on the momentum operator of each atom.
5. We want to start the experiment with a ensemble of atoms all initiated in the motional ground state of the harmonic trap $|0\rangle$, and then produce a squeezed state. Could this experiment be done with a single atom ? Explain the choice of ^{133}Cs to perform this experiment.

2 Production of a motional squeezed state

We now want to put all the atoms in a squeezed state of the harmonic trap. The following experimental protocol is followed:

- We cool down the atoms until all atoms reach the motional ground state $|0\rangle$.
- We turn off the trap for a time τ_1 by switching off the trapping lasers.
- We turn the trap back on and wait for a time τ_2.
- We make a ballistic expansion and measure the momentum distribution of the cloud.

This experiment is repeated for different waiting times τ_2. We want to measure a momentum probability distribution below the ground state quantum fluctuations, which is the signature of a squeezed state. We introduce the time evolution operator U:

$$U = e^{-iH_{\mathrm{on}}\tau_2/\hbar}\, e^{-iH_{\mathrm{off}}\tau_1/\hbar} \tag{1}$$

and the time-evolved annihilation operator:

$$a' = UaU^\dagger = \xi x + \zeta p \tag{2}$$

where ξ and ζ are complex number to be computed.

1. Show that a' and the final state of the evolution $|\psi_f\rangle$ obey:

$$a' |\psi_f\rangle = 0 \tag{3}$$

2. Deduce from this that the differential equation verified by the final state in momentum representation is the following:

$$(i\xi\partial_p + \zeta p)\psi_f(p) = 0 \tag{4}$$

We recall the solution of this differential equation :

$$\psi_f(p) = \gamma e^{-\alpha p^2/2} \text{ where } \alpha = -i\frac{\zeta}{\xi} \tag{5}$$

3. At $t = 0$ (i.e. $\tau_1 = \tau_2 = 0$) give the value of ζ and ξ, and show that the variance of the momentum distribution is:

$$\Delta p^2 = \frac{1}{2\,\mathrm{Re}(\alpha)} = \frac{1}{2} \tag{6}$$

4. Now for an arbitrary value of τ_1 and τ_2, show that we have:

$$\begin{cases} \xi = \frac{1}{\sqrt{2}}[\cos(\omega\tau_2) + (i - \omega\tau_1)\sin(\omega\tau_2)] \\ \zeta = \frac{1}{\sqrt{2}}[-\sin(\omega\tau_2) + (i - \omega\tau_1)\cos(\omega\tau_2)] \end{cases} \tag{7}$$

5. Conclude that at the end of the experiment, the variance of the momentum distribution is given by:

$$\Delta p^2 = \frac{1}{2}\left[1 + \frac{(\omega\tau_1)^2}{2} - \omega\tau_1\sqrt{1 + \frac{(\omega\tau_1)^2}{4}}\cos(2\omega\tau_2 - \phi)\right] \text{ where } \phi = \tan^{-1}\left(\frac{2}{\omega\tau_1}\right) \tag{8}$$

Did we succeed at producing momentum squeezing in this system?

3 Homework: damping of the oscillations

When looking at figure 3 in the article `https://doi.org/10.1103/PhysRevLett.83.4037`, we see that the oscillation of the momentum variance with τ_2 is damped. Equation (8) is plotted in dash-dotted line, whereas a finer model fitting the data is plotted in solid line. The authors explain that this damping is due to the spread in the oscillation frequency seen by each atom depending on its transverse position in the trap, with a spread in ω of 10%.

1. Using the language of your choice, reproduce the dash-dotted line and the solid line in a plot. We recall that the variance of the average of independant random variables is given by the average of the variances of these random variables.