Quantum Optics

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Generation of vacuum squeezing

We consider a degenerate Optical Parametric Oscillator, where the combination of a cavity with a $\chi^{(2)}$ nonlinear crystal and a pump beam (mode $\hat{b}$, frequency $2\omega_0$, treated classically and assumed undepleted) creates a degenerate signal/idler beam (mode $\hat{a}$ at ω_0).
The coupling hamiltonian is :

$$\hat{H}_c = \frac{i\hbar\chi^{(2)}}{2}\left(\hat{a}^2\hat{b}^\dagger - \hat{a}^{\dagger 2}\hat{b}\right) = \frac{i\hbar\, b\,\chi^{(2)}}{2}\left(\hat{a}^2 - \hat{a}^{\dagger 2}\right), \tag{1}$$

where $\chi^{(2)}$ (assumed $\in \mathbb{R}^+$) is characteristic of the nonlinear crystal.
For all fields, a represents the classical mean amplitude, $\hat{a}$ the quantum fluctuations.
Mirror 1 (with reflection coefficient $r_1 = 1 - \gamma_1$ (with $\gamma_1 \ll 1$) at ω_0, high-reflectivity coated at $2\omega_0$) can be used to inject a field a_{in}.
Mirror 2 is used for pump injection at $2\omega_0$ and is high-reflectivity coated at ω_0.
We however model the optical losses of the cavity by a reflection coefficient $r_2 = 1 - \gamma_2$ (with $\gamma_2 \ll 1$).

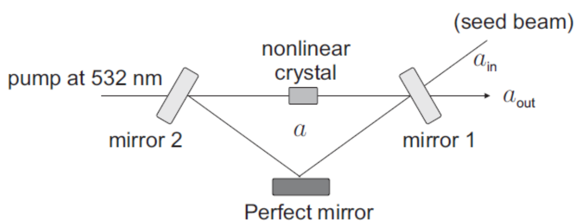

Figure 1 – Optical cavity with a nonlinear medium.

We first assume there is an injected field (*seed beam*) $\hat{a}_{\text{in}}$ at the mirror 1 location.

1. Show that the time evolution equation for the field $\hat{a}$ in a resonant cavity is :

$$\tau\dot{\hat{a}} = -\gamma\,(a + \hat{a}) + g\tau(a + \hat{a}^\dagger) + \sqrt{2\gamma_1}\,(a_{\text{in}} + \hat{a}_{\text{in}}) + \sqrt{2\gamma_2}\,\hat{a}_2, \tag{2}$$

   where g (assumed $\in \mathbb{R}^+$) is an optical gain proportional to the pump field amplitude b and τ is the travel time of light around the ring cavity.

2. Show the mean intracavity field is amplified by the nonlinear gain :

$$\frac{|a(g)|^2}{|a(g=0)|^2} = \frac{1}{(1 - g/\kappa)^2}, \tag{3}$$

where $\kappa = \gamma/\tau = (\gamma_1 + \gamma_2)/\tau$ is the cavity decay rate.

The OPO is operated **below threshold** (with a pump beam such that $g < \kappa$), with no injected field, i.e. $a_{\text{in}} = 0 \Rightarrow a = 0$.

3. Write the time evolution equations for the intracavity field fluctuations $\hat{a}$ and $\hat{a}^\dagger$.
4. Compute the quadratures $\hat{X}[\Omega]$ and $\hat{Y}[\Omega]$ in Fourier space for the intracavity field and for the output field.
5. Show that the output noise spectra are :

$$S_X^{\text{out}}[\Omega] = 1 + \frac{4g}{\kappa}\frac{\gamma_1}{\gamma}\frac{1}{(\Omega/\kappa)^2 + (1 - g/\kappa)^2} \tag{4}$$

$$S_Y^{\text{out}}[\Omega] = 1 - \frac{4g}{\kappa}\frac{\gamma_1}{\gamma}\frac{1}{(\Omega/\kappa)^2 + (1 + g/\kappa)^2}. \tag{5}$$

The ratio $\eta_{\text{esc}} = \gamma_1/\gamma$ is called the *escape efficiency*.

6. Comment the dependence of the noise spectra with the gain g and the frequency Ω.
7. Assuming $\Omega \ll \Omega_{\text{cav}}$, what is the maximum squeezing that can be delivered ? Comment the η_{esc} dependence.
8. How could one get squeezing for a different quadrature ?