Quantum Optics – M1

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TD2: Superconducting qubits: from Cooper-pair box to heavy fluxonium

1 Description of a superconducting qubit: the Cooper-pair box

1.1 Current-biased Josephson qubit

Consider an electrical LC oscillator. We denote Φ the flux through the inductor and Q the charge on the capacitor's plates.

1. Writing the total energy of the system, show that the Hamiltonian of the oscillator can be cast in the form of a harmonic oscillator.
2. Promoting Φ and Q to quantum operators, recall the eigenenergies of this system. What temperature regime do we need to reach in order to perform quantum manipulations of this system. In view of performing quantum computations, justify the need of an anharmonicity in the oscillator.
3. We now replace the linear inductor by a Josephson junction: a superconducting element described by the two constitutive equations given below (1). Why is the junction called a "non-linear induction"? Show that the Hamiltonian of the oscillator now presents a strong anharmonicity.

$$\begin{cases} I(t) = I_0 \sin\left(2\pi \frac{\Phi(t)}{\Phi_0}\right), \quad \Phi_0 = \frac{h}{2e} \\ V(t) = \dfrac{\mathrm{d}\Phi}{\mathrm{d}t} \end{cases} \tag{1}$$

4. Show that biasing the Josephson junction with a constant current I results in a tilted washboard potential in the Hamiltonian. Explain how this can be used to read out the state of the qubit defined by the two lowest levels of the central well of the washboard.

1.2 Coherent control of the qubit

We restrict to the study of the two lowest energy states of the central well of our qubit, from now on respectively called $|0\rangle$ and $|1\rangle$, forming the qubit Hilbert space. The energy difference between these two states is denoted $\hbar\omega_q$.

5. Let's consider that a bias current can be sent to the resonator through an inductive flux line. How does the Hamiltonian change if we send a microwave signal at frequency ω through the flux line? Write its restriction to the qubit Hilbert space and show it is of the form:

$$\frac{\hbar\omega_q}{2}\,\sigma_z + \hbar\Omega_R \cos(\omega t)\,\sigma_x \tag{2}$$

In order to deal with this time-dependant Hamiltonian, we perform the following unitary transformation to go to a rotating frame:

$$U(t) = e^{i\frac{\omega t}{2}\sigma_z} \tag{3}$$

6. Give the expression of the new Hamiltonian in the rotating frame, and justify that some terms may be neglected in the regime $\omega \gg \delta, \Omega_R$ where $\delta = \omega - \omega_q$.

7. Show that starting from $|0\rangle$ and applying a strong microwave pulse ($\Omega_R \gg \delta$), we can bring the qubit to the state $|1\rangle$. Give the duration of the pulse in terms of the Rabi frequency Ω_R.

2 Heavy fluxonium

2.1 Qualitative description of the qubit spectrum

We will now consider a new kind of qubit called fluxonium: it is made of a Cooper-pair box shunted by a strong linear inductance. We admit that the hamiltonian describing this qubit is given by:

$$H = -E_J \cos(\varphi - \varphi_{\text{ext}}) + 4E_C(n - n_g(t))^2 + \frac{E_L}{2}\varphi^2 \tag{4}$$

were $E_J = \Phi_0^2/(2L_J)$ is the Josephson energy, $E_C = e^2/(2C)$ is the capacitance energy, and $E_L = \Phi_0^2/(2L)$ is the inductance energy. φ_{ext} and $n_g(t)$ can be respectively controlled externally *via* a flux line and a coplanar waveguide coupled to the capacitance pad. We want to operate our qubit in the heavy fluxonium regime, meaning that the potential seen by the position-like variable φ consists of multiple wells with distinct minima, and the two lowest energy eigenstates of each well are well localized within each well, with a small tunnelling probability between each well. We will study this qubit close to the *flux frustration point* $\varphi_{\text{ext}} = \pi$.

1. Give two conditions on E_J, E_L and E_C that have to be satisfied in order to be in the heavy fluxonium regime. Argue that the tunneling rate is exponentially suppressed with the dimensionless parameter $\sqrt{8E_J/E_C}$.

2. Draw qualitatively the four lowest eigenstates of the system in the φ variable representation in the presence of an external flux φ_{ext} very close to π. Show that there exist two kinds of transitions in this system, one with frequencies relatively independant on the external flux, and the other with frequencies scaling linearly with the flux.

3. Show that the eigenstates of the system undergo a transition at the flux frustration point $\varphi_{\text{ext}} = \pi$, from being well-localized in their respective wells to being delocalized in the two lowest wells.

4. Draw qualitatively the energies of these eigenstates with respect to external flux φ_{ext} and show that at the flux frustration point, the qubit frequency can be reduced by several orders of magnitude compared to that of a Cooper-pair box qubit with equivalent design.

2.2 Homework: Quantitative description of the qubit spectrum

5. Using the Python library QuTip, solve for the eigenstates of the qubit for the parameters found in the following article `https://arxiv.org/abs/2307.14329`, and reproduce the spectrum found on the figure 1.