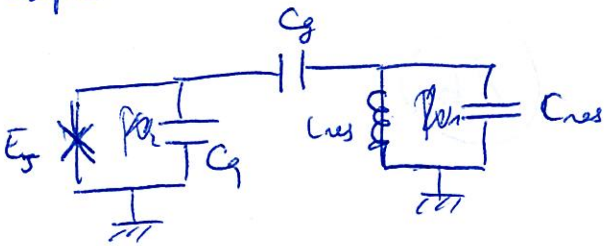
Coupling to the res: JC $\hat{H}$

Capacitor between qbit and res:



$$\Rightarrow H_{int} = \beta\, Q_1 Q_2$$

$Q_1 \Leftrightarrow P_1$

$$\hookrightarrow Q_1 \propto a_1 - a_1^+$$
$$\propto \sigma^- - \sigma^+$$
$$Q_2 \propto a_2 - a_2^+$$
$$\cancel{\propto \sigma}$$

$$\hookrightarrow H_{int} = \hbar g \, (a - a^+)(\sigma^+ - \sigma^-)$$

RWA: Heisenberg rep:

$$a(t) = a(0)\, e^{-i\omega_m t}$$
$$a^+(t) = a^+(0)\, e^{i\omega_m t}$$
$$\sigma^+(t) = \sigma^+(0)\, e^{i\omega_q t}$$
$$\sigma^-(t) = \sigma^-(0)\, e^{-i\omega_q t}$$

$\hookrightarrow$ Terms in $a\sigma^-$ and $a^+\sigma^+$ oscillate at freq $\sim \omega_q + \omega_m \sim 2\omega$.
$\rightarrow$ average to 0 in the dynamics

JC Ham:

$$\boxed{H_{JC} = \frac{\hbar\omega_q}{2}\sigma_z + \hbar\omega_m\left(a^+a + \frac{1}{2}\right) + \hbar g\,(a\sigma^+ + a^+\sigma^-)}$$

Under this approx: exact diagonalisat° possible:

H space:

$|g,2\rangle$ —— - $\Delta 1$ $|e,1\rangle$

$|g,1\rangle$ —— - $\Delta 2$ $|e,0\rangle$

$|g,0\rangle$ ——

$\Delta = \omega_q - \omega_c$ detuning.

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Subspace gated by $|g,n+1\rangle, |e,n\rangle$ stable under the act of $\hat{H}$:

$$H_{\{|g,n+1\rangle, |e,n\rangle\}} = \hbar \begin{pmatrix} (n+1)\omega_m - \frac{\Delta}{2} & g\sqrt{n+1} \\ g\sqrt{n+1} & (n+1)\omega_m + \frac{\Delta}{2} \end{pmatrix} = \begin{pmatrix} -\hbar\omega_q + \hbar\omega_m(n+1+\frac{1}{2}) & \hbar g\sqrt{n+1} \\ \hbar g\sqrt{n+1} & \frac{\hbar\omega_q}{2} + \hbar\omega_m(n+\frac{1}{2}) \end{pmatrix}$$

NB: $|g,0\rangle$ eigstate w/ E $E_{g0} = -\Delta/2$.

Eig states:

$$\begin{cases} |+,n\rangle = \cos\theta_n\, |e,n\rangle + \sin\theta_n\, |g,n+1\rangle \\ |-,n\rangle = -\sin\theta_n\, |e,n\rangle + \cos\theta_n\, |g,n+1\rangle. \end{cases}$$

$$\theta_n = \frac{1}{2}\,\mathrm{Arctan}\left(\frac{2g\sqrt{n+1}}{\Delta}\right)$$

$$H = \hbar \omega_m (n+1) \mathbb{1} + \hbar \begin{pmatrix} -\Delta_h & g\sqrt{n+1} \\ g\sqrt{n+1} & \Delta_h \end{pmatrix}$$

$$= E_0 + \frac{\hbar}{2} \begin{pmatrix} -\Delta & \Omega \\ \Omega & \Delta \end{pmatrix} , \quad \Omega = 2g\sqrt{n+1} \qquad \sin 2\theta_n = \frac{\Omega}{\sqrt{\Omega^2+\Delta^2}}$$

$$= E_0 + \frac{\hbar}{2} \sqrt{\Delta^2+\Omega^2} \underbrace{\begin{pmatrix} -\cos 2\theta_n & \sin 2\theta_n \\ \sin 2\theta_n & \cos 2\theta_n \end{pmatrix}}_{} \qquad \tan 2\theta_n = \frac{\Omega}{\Delta} .$$

$\lambda_\pm = \pm 1$, vect propres : $\begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}$, $\begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}$

$$\hookrightarrow E_\pm = E_0 \pm \frac{\hbar}{2} \underbrace{\sqrt{\Omega^2+\Delta^2}}_{\tilde{\Omega}}$$

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Schrö evol w/ $|\psi(0)\rangle = |e,0\rangle$:

$$\begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} |e,0\rangle \\ |g,1\rangle \end{pmatrix}$$

$$\begin{pmatrix} |e,0\rangle \\ |g,1\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix}$$

$$|e,0\rangle = \cos\theta_1 \, |+\rangle - \sin\theta_1 \, |-\rangle .$$

$$|\psi(t)\rangle = \cos\theta_1 \, e^{-iE_+/\hbar\, t} |+\rangle - \sin\theta_1 \, e^{-iE_-/\hbar\, t} |-\rangle.$$

$$= e^{-iE_0/\hbar\, t} \left(\cos\theta_1 \, e^{-i\frac{\tilde\Omega}{2}t} |+\rangle - \sin\theta_1 \, e^{i\frac{\tilde\Omega}{2}t} |-\rangle \right)$$

$$\cancel{P_1(\Delta t) = |\langle g,1|\psi(t)\rangle|^2}$$

$$\cancel{= \cos\theta_1 \, e^{-i\frac{\tilde\Omega}{2}t}}$$

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$$= e^{-i\frac{E_0}{\hbar}t} \Big[\cos\theta_1 \, e^{-i\frac{\tilde\Omega}{2}t} \left(\cos\theta_1 |e,0\rangle + \sin\theta_1 |g,1\rangle \right)$$

$$- \sin\theta_1 \, e^{i\frac{\tilde\Omega t}{2}} \left(-\sin\theta_1 |e,0\rangle + \cos\theta_1 |g,1\rangle \right) \Big]$$

$$P_1(\Delta,t) = |\langle g,1|\psi(t)\rangle|^2$$

$$= \left| \cos\theta_1 \sin\theta_1 \, e^{-i\frac{\tilde\Omega}{2}t} - \sin\theta_1 \cos\theta_1 \, e^{i\frac{\tilde\Omega t}{2}} \right|^2 = \left| \sin(2\theta_1) \sin\left(\frac{\tilde\Omega}{2}t\right) \right|^2$$

$$= \cancel{|\cos\theta_1 \sin\theta_1|^2} \sin^2 2\theta_1 \sin^2\left(\frac{\tilde\Omega}{2}t\right)$$

$$= \frac{1}{2} \sin^2 2\theta_1 \left(1 - \cos(\tilde\Omega t) \right) \quad \text{où } \sin 2\theta_1 = \frac{\Omega}{\sqrt{\Omega^2+\Delta^2}}$$

$$= \frac{\Omega^2}{\Omega^2+\Delta^2} \; \frac{1-\cos(\tilde\Omega t)}{2}$$

$$\tilde\Omega = \sqrt{\Omega^2+\Delta^2}$$

$$= \Omega \sqrt{1+\left(\frac{\Delta}{\Omega}\right)^2}$$

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Result :

$$P_1(\Delta, t) = \frac{\Omega^2}{\Omega^2 + \Delta^2} \; \frac{1 - \cos\left(\sqrt{\Omega^2 + \Delta^2}\, t\right)}{2}$$