TD Q gs of mech. resonator

Main theme: macroscopic object in the q. regime?

↳ cooling to gs.

↳ control (q. states g$^{\text{ation}}$).

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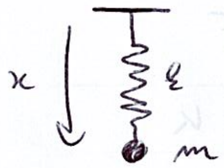
~~Orders of magnitude~~

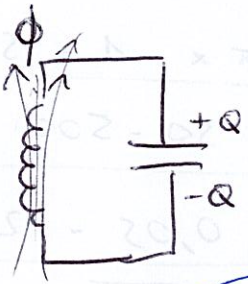
2 types: mechanical or ~~state~~ electrical:



$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\cancel{H = \frac{k x^2}{2} + \frac{p^2}{\ }}$$

$$H = \frac{p^2}{2m} + \frac{1}{2} k x^2$$



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

from $U = L\frac{di}{dt}$

$i = C\frac{dU}{dt}$

$$\Phi(t) = \int_{-\infty}^{t} V(t')\,dt'$$

$$Q(t) = \int_{-\infty}^{t} i(t')\,dt'$$

$$\cancel{H} = \frac{1}{2}Li^2 + \frac{1}{2}CU^2$$

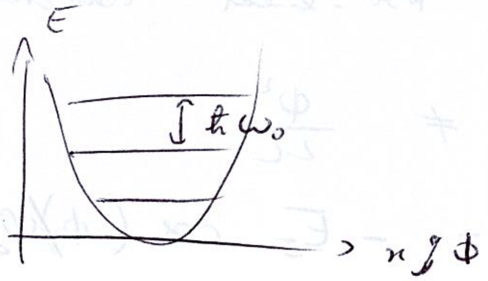
$$H = \frac{\Phi^2}{2L} + \frac{Q^2}{2C}$$

Analogy:

x	Φ
p	Q
k	L^{-1}
m	C

Pq $x \leftrightarrow \Phi$ et pas $x \leftrightarrow Q$! Non lin. sur Φ, donc $\Leftrightarrow$ potentiel sur x !

$$[\hat{x}, \hat{p}] = i\hbar \iff [\hat{\Phi}, \hat{Q}] = i\hbar$$

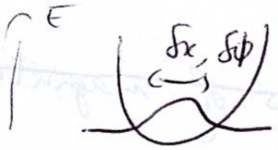


Orders of magnitude:

Q regime when $k_B T \ll \hbar\omega_0$ (macro pop in gs)

$\hookrightarrow$ $\boxed{T_Q = \frac{\hbar\omega_0}{k_B}}$.

And $\underbrace{Q \gg 1}_{\text{quality factor}}$ $\Rightarrow$ well-def $\mathcal{E}$-levels.



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	Mech		Elec
ω_0	$\sim 2\pi \times 1 - 50\,\text{MHz}$	ω_0	$\sim 2\pi \times 2 - 20\,\text{GHz}$
m	$\sim 50 - 500\,\text{pg}$	C	$\sim 0{,}2 - 2\,\text{pF}$
T_Q	$\sim 0{,}05 - 2\,\text{mK}$	T_Q	$\sim 0{,}1 - 1\,\text{K}$
δx	$\sim \sqrt{\frac{\hbar}{2m\omega_0}} \sim 0{,}5 - 5\,\text{fm}$		$\delta V \sim 1\,\mu V$ / $\delta i \sim 10\,nA$.
	very small.		meas!

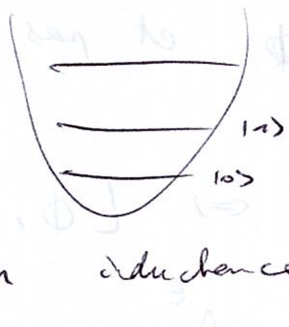
Dilut° cryo: $\sim 10\,\text{mK}$

$\rightarrow$ further cooling for mech.

NOT HERE! $\rightarrow$ Specificity of this art:

6 GHz Mech res! very high.

Need anharmonicity:



Prepare $|1\rangle$?

Josephson junct°: Non-linear inductance:

$$\cancel{E(\Phi) =} \quad E_\Phi \neq \frac{\Phi^2}{2L}$$

$$E_\Phi = -E_J \cos(\Phi/\varphi_0)$$

Lin inductor : $I(t) = \frac{1}{L}\Phi(t)$.

Conjt. φ :

JJ $\quad I(t) = I_0 \sin\left(2\pi\,\Phi(t)/\Phi_0\right) \cancel{= I_0 \sin(\varphi)}$; $\quad V = \frac{\hbar}{2e}\frac{d\varphi}{dt} = \cancel{\varphi_0 \frac{d\varphi}{dt}}\ \frac{d\Phi}{dt}$

$\Phi_0 = \frac{h}{2e}$ ⚠ flux quantum, $\quad \varphi_0 = \Phi_0/2\pi$ reduced flux quantum.

period imposed by discreteness of Cooper pairs of charge 2e. tunnelling through the barrier.

$$\frac{d\varphi}{dt} = \frac{d}{dt}\left(\frac{2\pi\,\Phi(t)}{\Phi_0}\right) = \frac{2\pi}{\Phi_0}\frac{d\Phi}{dt} = \frac{1}{\varphi_0}\frac{d\Phi}{dt}$$

$$\cancel{\varphi = \frac{2\pi\,\Phi(t)}{\Phi_0}}.$$

I_0 = critical current.

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Energy stored in the JJ:

$$\int_{-\infty}^{t} I(t')V(t')\,dt' \,.= \cancel{I_0\int_{-\infty}^{t} \sin\left(2\pi\,\Phi(t')/\Phi_0\right) V(t')\,dt'}$$

$$= \cancel{I_0\varphi_0 \int_{-\infty}^{t} \sin(\varphi(t))}$$

$$= I_0 \int_{-\infty}^{t} \sin\left(\frac{2\pi\,\Phi(t')}{\Phi_0}\right)\frac{d\Phi(t')}{dt'}dt'$$

$$= I_0 \int_{\Phi(-\infty)}^{\Phi(t)} \sin\left(\frac{2\pi\,\Phi}{\Phi_0}\right) d\Phi$$

$$= -I_0 \underbrace{\frac{\Phi_0}{2\pi}}_{\varphi_0} \cos\left(\frac{2\pi\,\Phi(t)}{\Phi_0}\right) + \text{cte}$$

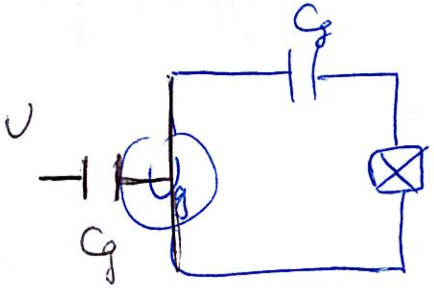
$$\boxed{E_J = + I_0\varphi_0}$$

And charge var: $\Phi \to \varphi = \frac{2\pi\Phi}{\Phi_0}$.

$Q \to N = \frac{Q}{2e}$

$[\varphi, N] = i$

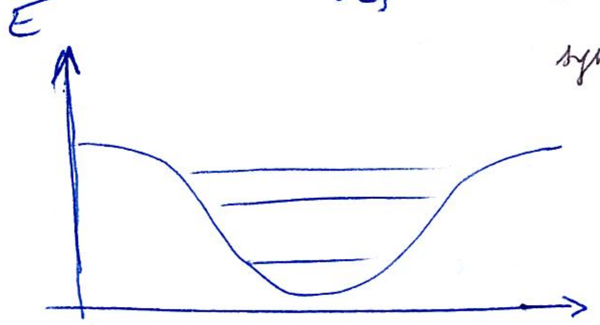
1st type of qubit: Cooper-pair box:



$N_g = \frac{C_g U}{2e}$

$$H = 4E_c (N - N_g)^2 - E_J \cos\varphi.$$

$E_c = \frac{e^2}{2C}$, $E_J = \frac{\Phi_0^2}{2L_J}$



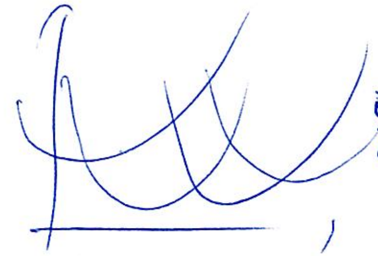
Only excite the syst w/ freq st.

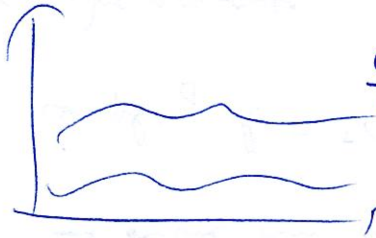
$|\omega - \omega_{01}| \ll |\omega - \omega_{12}|$

see only 2 levels:

$H \simeq \hbar\omega_{01}\sigma_z$.

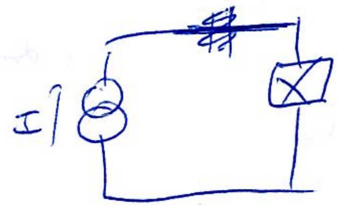
eig-energies vs N_g:





our case: Current-biased JJ:

~~charge~~



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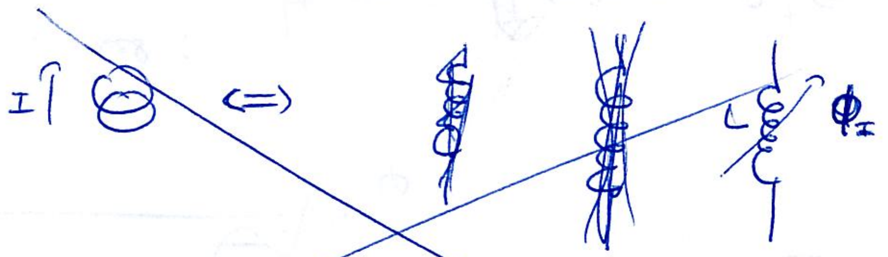

~~$L \to \infty$, $\Phi_I \to \infty$ ac $I = \frac{\Phi_I}{L}$!~~

done: Now

$$H = E_{CJ} N^2 - I\varphi_0\delta - I_0\varphi_0\cos\varphi$$

$$\int \frac{\phi}{L}\frac{d\phi}{dt}\,dt = \frac{1}{L}\int \phi\, d\phi = \frac{1}{L}\frac{\phi^2}{2} = \frac{1}{2} I\phi$$

Time-varying $\hat{H}$: drive

Drive I w/ RF:

$$I(t) = I_d \cos(\omega t)$$

$$\hat{H} = \underbrace{4E_c n^2 - E_J \cos\varphi}_{\text{naked qubit}} - \underbrace{I_d \varphi_0 \cos(\omega t)\,\varphi}_{\text{drive}}$$

$$\varphi \equiv x \quad \propto \quad a_{qb} + a_{qb}^+$$

$$\propto \sigma^+ + \sigma^- \quad \text{in the } \{|0\rangle, |1\rangle\} \text{ RF}$$

$$\propto \sigma_x \qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\boxed{\hat{H}_{\text{drive}} = \frac{\hbar\omega_{01}}{2}\sigma_z + \hbar\Omega_R^{(t)}\sigma_x \cos(\omega t)}$$

~~Coupling to the resonator~~ ~~: χC~~

The controllable $\hat{H}$:

How to deal w/ t-varying $\hat{H}$? RWA:

Unitary transfo $U = e^{i\frac{\omega t}{2}\sigma_z}$:

$$i\hbar\,\partial_t|\psi'\rangle = H'|\psi'\rangle \qquad , U|\psi\rangle = |\psi'\rangle$$

$$i\hbar(\partial_t U)|\psi\rangle + i\hbar\, U\partial_t|\psi\rangle = H' U|\psi\rangle.$$

$$(i\hbar\dot{U} + UH)|\psi\rangle = H' U|\psi\rangle$$

$$\hookrightarrow \quad i\hbar\dot{U} + UH = H'U$$

$$\boxed{i\hbar\dot{U}U^+ + UHU^+ = H'}$$

$$H' = \frac{\hbar\omega_q}{2}\sigma_z + \hbar\Omega_R\, e^{i\frac{\omega t}{2}\sigma_z}\sigma_x e^{-i\frac{\omega t}{2}\sigma_z}\cos\omega t$$

$$-\frac{\hbar}{2}\omega\sigma_z$$

$$= \frac{\hbar\delta}{2}\sigma_z + \hbar\Omega_R\, e^{i\frac{\omega t}{2}\sigma_z}\sigma_x e^{-i\frac{\omega t}{2}\sigma_z}\cos(\omega t)$$

$$e^{i\frac{\omega t}{2}\sigma_z} = \begin{pmatrix} e^{i\omega t/2} & 0 \\ 0 & e^{-i\omega t/2} \end{pmatrix}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

so: $e^{i\frac{\omega t}{2}\sigma_z} \sigma_x e^{-i\frac{\omega t}{2}\sigma_z} \cos(\omega t) = \begin{pmatrix} e^{i\frac{\omega t}{2}} & 0 \\ 0 & e^{-i\frac{\omega t}{2}} \end{pmatrix}$

$$= \frac{1}{2}\begin{pmatrix} 0 & e^{i\omega t} \\ e^{-i\omega t} & 0 \end{pmatrix} \left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$= \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & e^{2i\omega t} \\ e^{-2i\omega t} & 0 \end{pmatrix}$$

$$H' = \frac{\hbar\delta}{2}\sigma_z + \frac{\hbar\Omega_R}{2}\sigma_x + \underbrace{\frac{\hbar\Omega_R}{2}\begin{pmatrix} 0 & e^{2i\omega t} \\ e^{-2i\omega t} & 0 \end{pmatrix}}_{\text{fast-osc terms}}$$

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RWA.

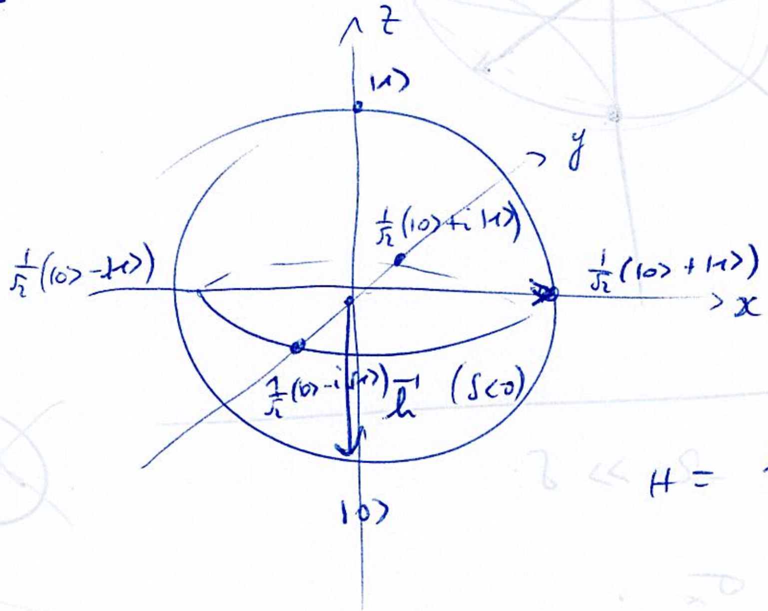
⚠ Valid if $\omega \gg \delta, \Omega_R$!

MW pulse → rot around σ_x on Bloch sphere.

$$H' = \frac{\hbar\delta}{2}\sigma_z + \frac{\hbar\Omega_R}{2}\sigma_x .$$

Interp Bloch sphere:

$\Omega_R = 0$ (No drive):

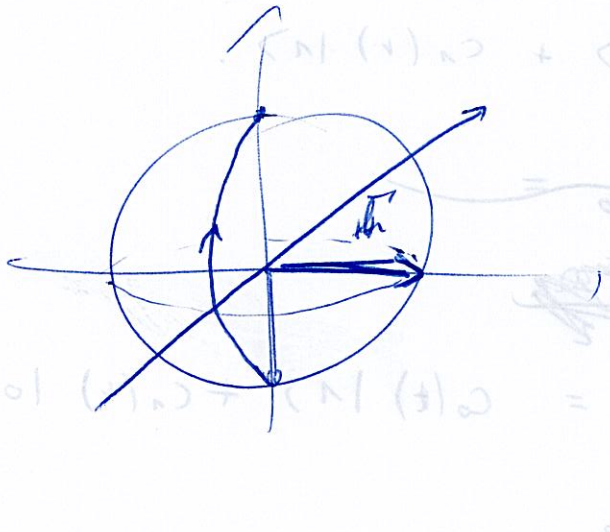


$$H = \frac{\hbar\delta}{2}\hat{\sigma}_z .$$

$$\boxed{12}$$

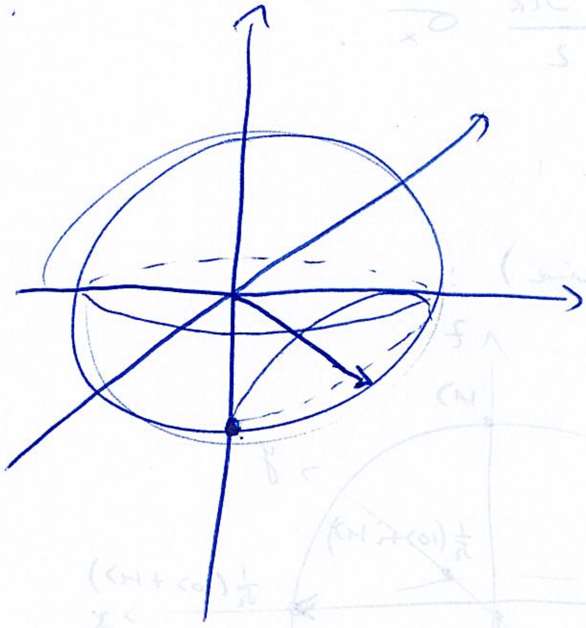
$$H = \alpha\,\vec{n}\cdot\vec{\hat{\sigma}} \Rightarrow \text{Rotate around } \vec{n}.$$

$\Omega_R \gg \delta$:



$|0\rangle \to |1\rangle$ transit possible!

$\Omega_R \sim \delta$:



By calculat : $\Omega_R \gg \delta$:

$\boxed{13}$

$$H' \simeq \frac{\hbar \Omega_R}{2} \sigma_x .$$

$$|\psi(0)\rangle = |0\rangle .$$

$$i\hbar \partial_t |\psi\rangle = \hat{H} |\psi\rangle$$

$$i \partial_t |\psi\rangle = \frac{\Omega_R}{2} \sigma_x |\psi\rangle$$

$$|\psi(t)\rangle = c_0(t)|0\rangle + c_1(t)|1\rangle .$$

$$\sigma_x |\psi(t)\rangle = c_0(t)|1\rangle + c_1(t)|0\rangle .$$

$$\Leftrightarrow \quad i\partial_t c_0 = \frac{\Omega_R}{2} c_1 \qquad (1)$$

$$i\partial_t c_1 = \frac{\Omega_R}{2} c_0 \qquad (2)$$

∂_t (2) :

$$i\,\partial_t^2 C_1 = \frac{\Omega_R}{2}\,\partial_t C_0$$

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$$= -i\,\frac{\Omega_R}{2}\,C_1$$

$$\partial_t^2 C_1 + \frac{\Omega_R}{2}\,C_1 = 0$$

$$\hookrightarrow\quad C_1(t) = A\cos\left(\frac{\Omega_R}{2}t\right) + B\sin\left(\frac{\Omega_R}{2}t\right)$$

$$C_1(0) = 0 = A$$

$\cancel{C_1(0) =}$

$$\partial_t C_1(0) = -i\,\frac{\Omega_R}{2}\,C_0(0) = -i\,\frac{\Omega_R}{2}$$

$$\text{et } \partial_t C_1(0) = B\,\frac{\Omega_R}{2}\ \cancel{\cos\left(\frac{\Omega_R}{2}t\right)}$$

$$\hookrightarrow\quad B = -i$$

$$C_1(t) = -i\sin\left(\frac{\Omega_R}{2}t\right)$$

$$P_1(t) = |C_1^*(t)|^2 = \sin^2\left(\frac{\Omega_R}{2}t\right)$$

$$= \frac{1-\cos(\Omega_R t)}{2}.$$

$$\boxed{P_1\left(\frac{\pi}{\Omega_R}\right) = 1} \qquad \tau_\pi = \frac{\pi}{\Omega_R}\ !$$

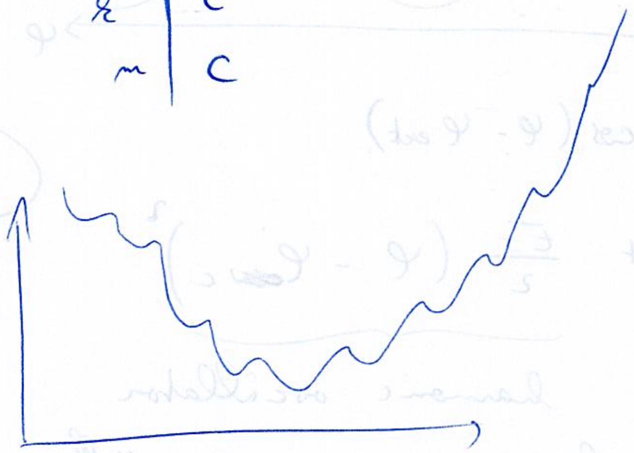
Heavy fluxonium

Recall the analogy : help for the calc :

1

x	φ
p	Q
k	L^{-1}
m	C

1)



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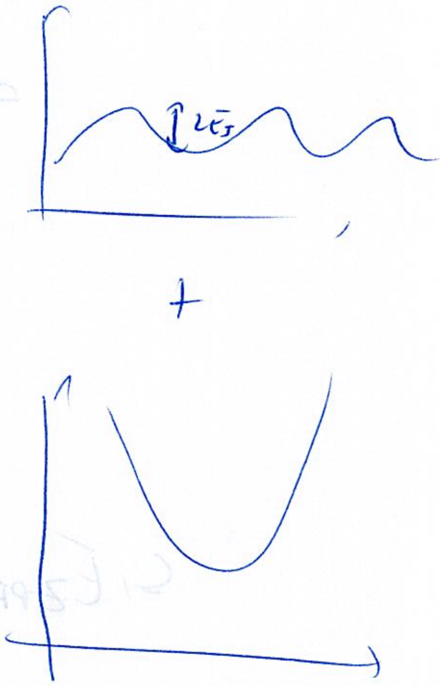


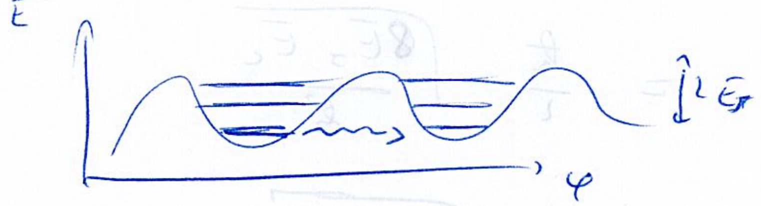
Sur qq. osc au fond du puit :

$$E_L \left(\Phi - \Phi_{ext}\right)^2 \sim E_L$$

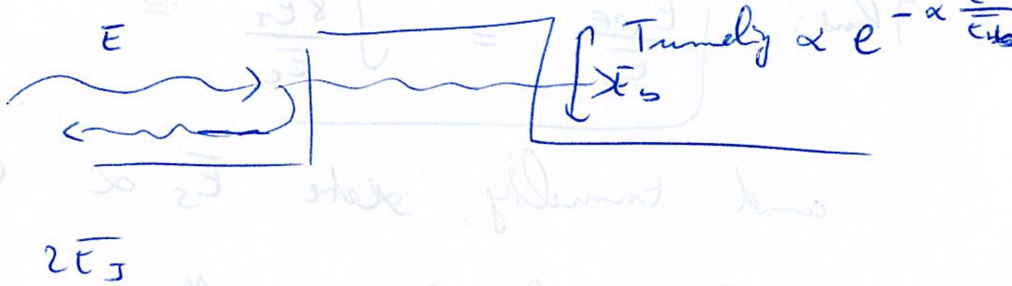
Prof des puits : $\sim 2E_J$

$\boxed{\text{Wells well def if : } E_J \gg E_L.}$

Low tunnelling rate between wells :



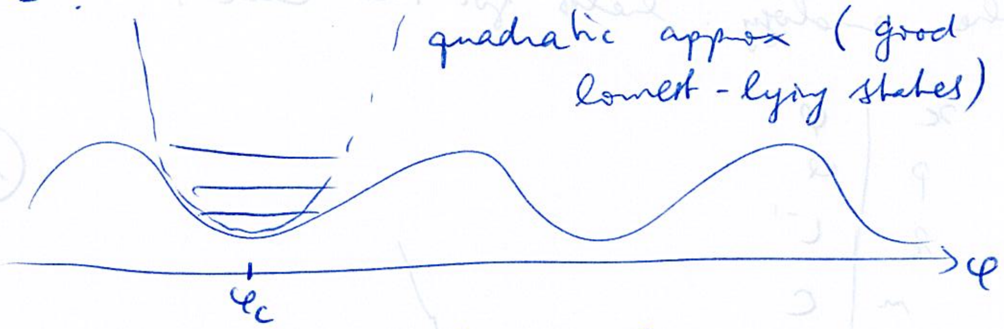
Tunneling rate :



Ici : $E_b = 2E_J$

$E = E_{ZPF} = ?$

quadratic approx (good for lowest-lying states)



$U(\varphi) \simeq E_J \cos(\varphi - \varphi_{ext})$

$$\simeq \text{cte} + \underbrace{\frac{E_J}{2}(\varphi - \varphi_c)^2}_{\text{harmonic oscillator}}$$

of mass C and stiffness L_J^{-1}

w/ $E_J = \frac{\Phi_0^2}{2 L_J}$

$$\text{So } E_{ZPF} = \frac{\hbar \omega_{HO}}{2} = \frac{\hbar}{2} \frac{1}{\sqrt{L_J C}}$$

$$= \frac{\hbar}{2} \frac{1}{\sqrt{\frac{\Phi_0^2}{2E_J} \frac{e^2}{2E_C}}} \quad \text{w/ } \Phi_0 = \frac{\hbar}{2e}$$

$$= \frac{\hbar}{2} \sqrt{\frac{8 E_J E_C}{\hbar^2}}$$

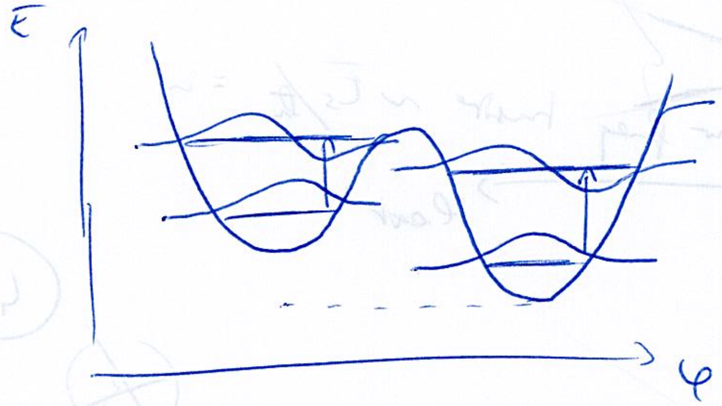
$$= \frac{1}{2} \sqrt{8 E_J E_C}$$

Thus: $\boxed{\frac{E_{ZPF}}{E_b} = \sqrt{\frac{8 E_J}{E_C}}}$

and tunneling rate $E_S \propto e^{-\alpha \sqrt{\frac{8E_J}{E_C}}}$

Cond^n for low tunneling: $E_J \gtrsim 10 E_C$

2) $\underline{\varphi_{ext} \geq \pi}$:



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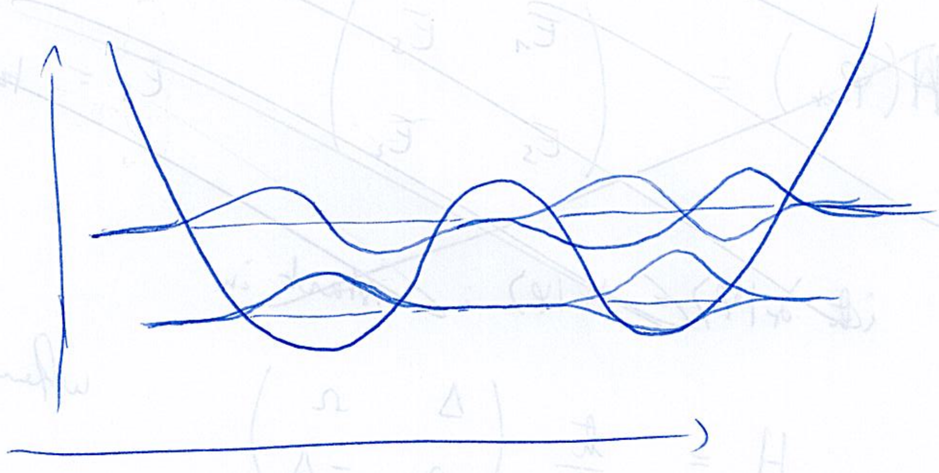
Intra-well transitions (= plasmonic transitions) : ~ indep of ext flux (only dep on E_J).

Interwell transit° : ~ linear w/ flux

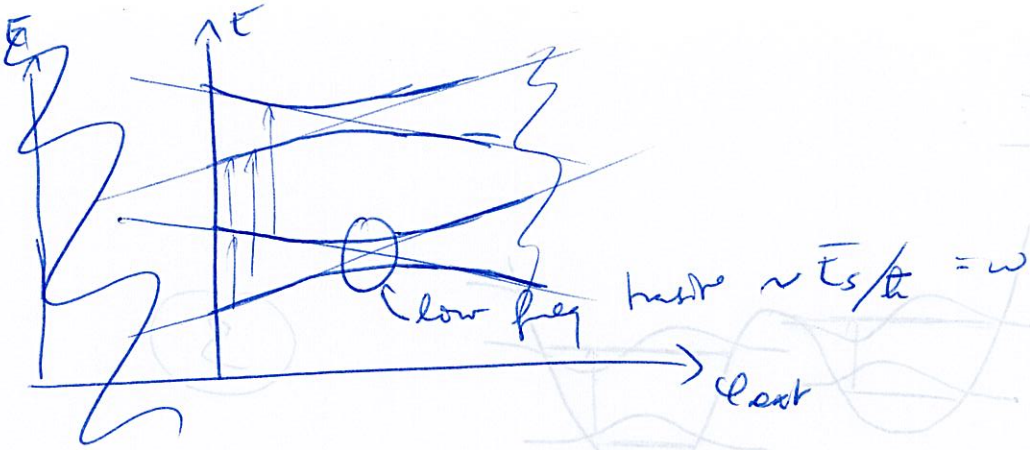
3) $\underline{\varphi_{ext} = \pi}$;

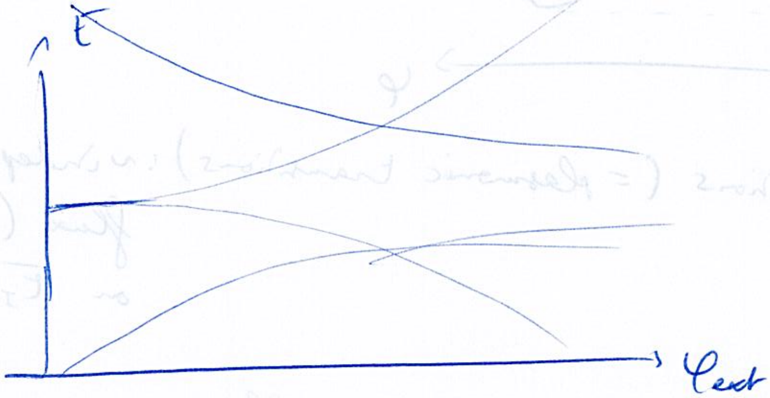


~~Degen~~ Quasi-degeneracy !

Degeneracy lifted by small tunneling rate !

4)





~~3~~

4

Why is the gap $= E_s/\hbar$?

↳ Anticrossing of 2 levels w/ small coupling :

$$\cancel{H(\varphi_{ext}) = \begin{pmatrix} E_1 & E_s \\ E_s & E_2 \end{pmatrix}}$$ ~~E_s = tunneling rate.~~

~~$i\hbar\,\partial_t|\psi\rangle = H|\psi\rangle$: start in~~

$$H = \frac{\hbar}{2}\begin{pmatrix} \Delta & \Omega \\ \Omega & -\Delta \end{pmatrix}$$

when $\Omega \to 0$: eig states

$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ w/ $E = \frac{\hbar\Delta}{2}$

$|-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ w/ $E = -\frac{\hbar\Delta}{2}$

↳ eig vals : $\pm \frac{\hbar}{2}\sqrt{\Omega^2 + \Delta^2}$

$\Delta \gg \Omega$: $\pm \frac{\hbar\Delta}{2}$: initial eig states same as w/out coupling.

$\Delta \ll \Omega$ (close to degen) :

$\pm \frac{\hbar\Omega}{2}$: gap given by coupling

w/ when $\Delta \to 0$: eig states $|+\rangle + |-\rangle$ et $(|-\rangle - |+\rangle)$! $E = \frac{\hbar\Omega}{2}$ and $-\frac{\hbar\Omega}{2}$!