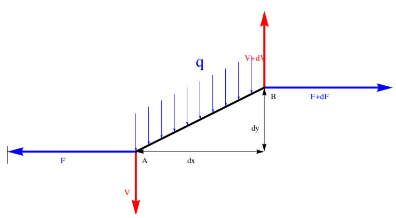
Lines for OpenGlider

29. April 2014

1 Sag:

1.1 dgl:



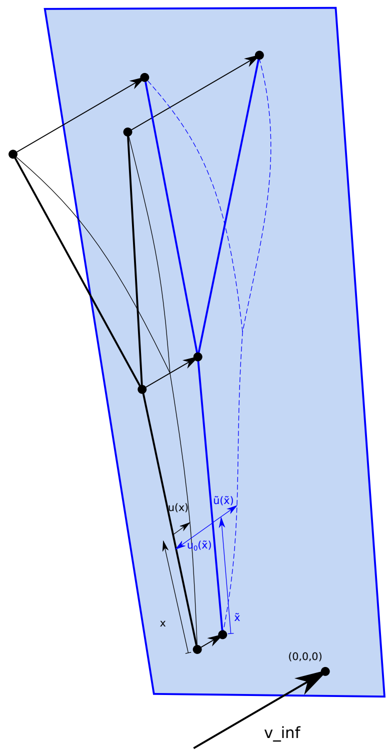
$$\sum F_x = 0 = V + dV - V - q \cdot dx \Rightarrow dV = q \cdot dx \Rightarrow \frac{dV}{dx} = V' = -q$$

$$\sum F_y = 0 = F + dF - F \Rightarrow dF = 0 \Rightarrow F' = 0$$

$$\frac{dy}{dx} = y'(x) = \frac{V(x)}{F(x)} \Rightarrow y''(x) = \frac{V'(x)}{F(x)} - \frac{V(x) \cdot F'(x)}{F(x)^2} = \frac{V'(x)}{F} = -\frac{q}{F}$$

$$y''(x) = -\frac{q}{F}$$

1.2 projection



$$
\begin{aligned}
u(x) = y(x) \quad u''(x) &= -\frac{q}{F} \\
u(x) &= \tilde{u}(x) + u_0(x) \\
u'(x) &= \tilde{u}'(x) + u_0' \\
u''(x) = \tilde{u}''(x) &= -\frac{q}{F} \\
\\
\tilde{u}''(x) &= -\frac{q}{F} \\
\tilde{u}'(x) &= -\frac{q}{F} \cdot x + C_1 \\
\tilde{u}(x) &= -\frac{q}{F} \cdot \frac{x^2}{2} + C_1 \cdot x + C_2
\end{aligned}
$$

1.3 bc for $u_0(x)$:

The boundary contitions for the linear sag function are sattisfied by the chooce of u_0.

1.4 bc for $\tilde{u}(x)$

$$\begin{aligned}\tilde{u}'(x=0) &= C_1\\ \tilde{u}'(x=l) &= \frac{q}{\tilde{F}} + C_1\\ \tilde{u}(x=0) &= C_2\\ \tilde{u}(x=l) &= -\frac{q}{\tilde{F}} \cdot \frac{l}{2}^2 + C_1 \cdot l + C_2\end{aligned}$$

1.4.1 lower Node:

- i is the number of the current line
- j is the number of the correspondending lower line
- node_type $= 0$

$$C_{i2} = 0 \tag{1}$$

- node_type $= 1$

$$C_{i2} - C_{j1} \cdot l_j - C_{j2} = -\frac{q_{jl}}{\tilde{F}_{jl}} \cdot \frac{l_{jl}}{2} \tag{2}$$

1.4.2 upper Node

- node_type $= 1$

$$\begin{aligned}C_{l1} \cdot l_{il} + C_{l2} - C_{u2} &= \frac{q_{il}}{\tilde{F}_{il}} \cdot \frac{l_{il}^2}{2}\\ -\frac{q_l}{\tilde{F}_l} \cdot l_l + C_{l1} &= \sum_{k=1}^{K} C_{u_k1} \cdot |f_{u_k}|\\ C_{l1} - \sum_{k=1}^{K} C_{u_k1} \cdot |f_{u_k}| &= \frac{q_l}{\tilde{F}_l} \cdot l_l \qquad (3)\\ f_{u_k} &= \vec{\tilde{F}}_{u_k} . \vec{v_{il}}\\ |f_{u_k}| &= \frac{f_{u_k}}{\sum f_{u_j}}\end{aligned}$$

- node_type = 2

$$
\begin{aligned}
-\frac{q_{il}}{\tilde{F}_{il}} \cdot \frac{l_{il}}{2} + C_{i1} \cdot l_i + C_{i2} = 0 \\
C_{i1} \cdot l_i + C_{i2} = \frac{q_{il}}{\tilde{F}_{il}} \cdot \frac{l_{il}}{2}
\end{aligned} \tag{4}
$$

1.5 System Entries

1. upper nodes:

   (a) if node_type = 1:
   $A[2i, 2i] = -1 \quad A[2i, 2j_k] = f_{jk}$
   $rhs[2i] = \frac{q_i \cdot l_i^2}{\tilde{F}_i \cdot 2}$

   (b) if node_type = 2:
   $A[2i, 2i] = l_i \quad A[2i, 2i+1] = 1$
   $rhs = -\frac{q_i \cdot l_i^2}{\tilde{F}_i \cdot 2}$

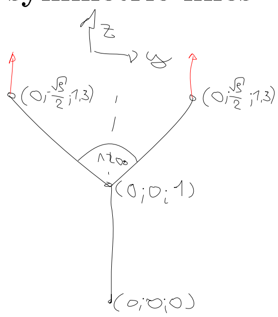
2. lower nodes:

   (a) if node_type = 0: $A[2i+1, 2i+1] = 1$

   (b) if node_type =1: $A[2i+1, 2j] = -l_j \quad A[2i+1, 2j+1] = -1 \quad A[2i+1, 2i+1] = 1$
   $rhs[2i+1] = \frac{q_j \cdot l_j^2}{\tilde{F}_j \cdot 2}$

2 Example:

2.1 symmetric lines



2.1.1 inputfile:

```
TEST_INPUT_FILE_1

NODES
#   n_nr    TYP     x       y       z           fx          fy          fz
    0       0       0.      0.      0.          None        None        None
    1       1       None    None    None        None        None        None
    2       2       0.      0.866   1.5         0           0           1
    3       2       0.      -0.866  1.5         0           0           1

LINES
#   l_nr    LOWER   UPPER   LENGTH  TYP
    0       0       1       1       liros
    1       1       2       None    liros
    2       1       3       None    liros

LINEPAR
#   TYP     CW      B       STRETCH
    liros   1.      0.1     0.1

CALCPAR
#   GEOSTEPS        SAGSTEPS        ITER        SPEED       GLIDE
    2               10              10          10          4
```

2.1.2 matrix:

```
[[ 1.          0.          -0.5         0.          -0.5         0.         ]
 [ 0.          1.           0.          0.           0.          0.         ]
 [ 0.          0.           0.99259766  1.           0.          0.         ]
 [-0.9701425  -1.           0.          1.           0.          0.         ]
 [ 0.          0.           0.          0.           0.99259766  1.         ]
 [-0.9701425  -1.           0.          0.           0.          1.         ]]
```

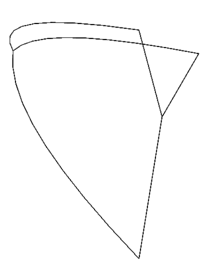
2.1.3 rhs:

```
[ 2.5         0.          1.24074708 -1.21267813  1.24074708 -1.21267813]
```

2.1.4 solution

```
[ 2.5143009   0.          0.0143009   1.22655204  0.0143009   1.22655204]
```

2.1.5 visual output:



3 some functions

3.1 proj force

$$(f_l \cdot \vec{l}).\vec{v} = f$$
$$f_l = \frac{f}{\vec{l} \cdot \vec{v}}$$

3.2 proj vec to surface:

$$(\vec{p} + \vec{n} \cdot \lambda) = \vec{x}$$
$$\vec{n}.\vec{x} = \vec{n}.\vec{p}_0$$
$$\vec{n}.(\vec{p} + \vec{n} \cdot \lambda) = \vec{n}.\vec{p}_0$$
$$p_0 = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \implies \vec{n}.(\vec{p} + \vec{n} \cdot \lambda) = 0$$
$$\vec{n}.\vec{p} = -(\vec{n}.\vec{n}) \cdot \lambda$$
$$\lambda = \frac{-\vec{n}.\vec{p}}{\vec{n}.\vec{n}}$$