

解: 令 $L(\omega_{c1}) = 0 \Rightarrow \omega_{c1} = 1.18$

$$\varphi(\omega_{c1}) = -90^\circ - \arctan \omega_{c1} - \arctan 0.25\omega_{c1} = -156.156^\circ$$

$$\gamma = 180^\circ + \varphi(\omega_{c1}) = 23.84^\circ$$

$$\varphi_m = 40^\circ - \gamma + 7^\circ = 23.156^\circ$$

设超前相位为 $G_c(s) = \frac{1+\alpha Ts}{1+Ts}$ $0 < \alpha < 1$

由于 $\sin \varphi_m = \frac{\alpha-1}{\alpha+1} \Rightarrow \alpha = \frac{1+\sin \varphi_m}{1-\sin \varphi_m} = 2.296$

则 $L(\omega_c) = 20 \lg 2.5 - 20 \lg \omega \sqrt{\omega^2+1} \sqrt{(0.25\omega)^2+1} = -10 \lg \alpha = -3.61$

$$20 \lg \omega \sqrt{\omega^2+1} \sqrt{(0.25\omega)^2+1} = 11.5688$$

$$\omega \sqrt{\omega^2+1} \sqrt{(0.25\omega)^2+1} = 3.788$$

$$\Rightarrow \omega_{c2} = 1.7352$$

当 $\omega_m = \omega_{c2}$ 时

$$\omega_m = \frac{1}{\sqrt{\alpha} T} \Rightarrow T = \frac{1}{\sqrt{\alpha} \omega_m} = 0.3803$$

故 $G_c(s) = \frac{1+0.8732s}{1+0.3803s} \cdot \frac{1}{2.296}$

故 $G(s) = \frac{1+0.8732s}{1+0.3803s} \cdot \frac{1}{2.296} * \frac{2.5}{s(1+s)(0.25s+1)}$

$$= \frac{0.9508s+1.0889}{0.09515s^4+0.7254s^3+1.6303s^2+s+1}$$

Bode Diagram

$G_m = 16 \text{ dB}$ (at 3.11 rad/s), $P_m = 53.2 \text{ deg}$ (at 0.941 rad/s)

