

I thank the Authors for thoroughly addressing all my previous points. I just wanted to briefly revisit the problem of the random matrix results. By all means, if the Authors still think that this is worthwhile including, then they should go ahead and do so. In that case though, I have one extra technical point to make.

Right now, the claim is that the discrete-time analogue of the matrix A is $B - I$ (where A is the continuous-time interaction matrix, B is the discrete time one, and I is the identity matrix). My understanding is slightly different. Let us start from the linearized system

$$\frac{dx(t)}{dt} = Ax(t),$$

where x is the state vector, t is time, and A is the interaction matrix. Solving this equation using the matrix exponential $\exp(A)$ (obtained by substituting A into the Taylor series of the exponential function), we get

$$x(t) = e^{At}x(0).$$

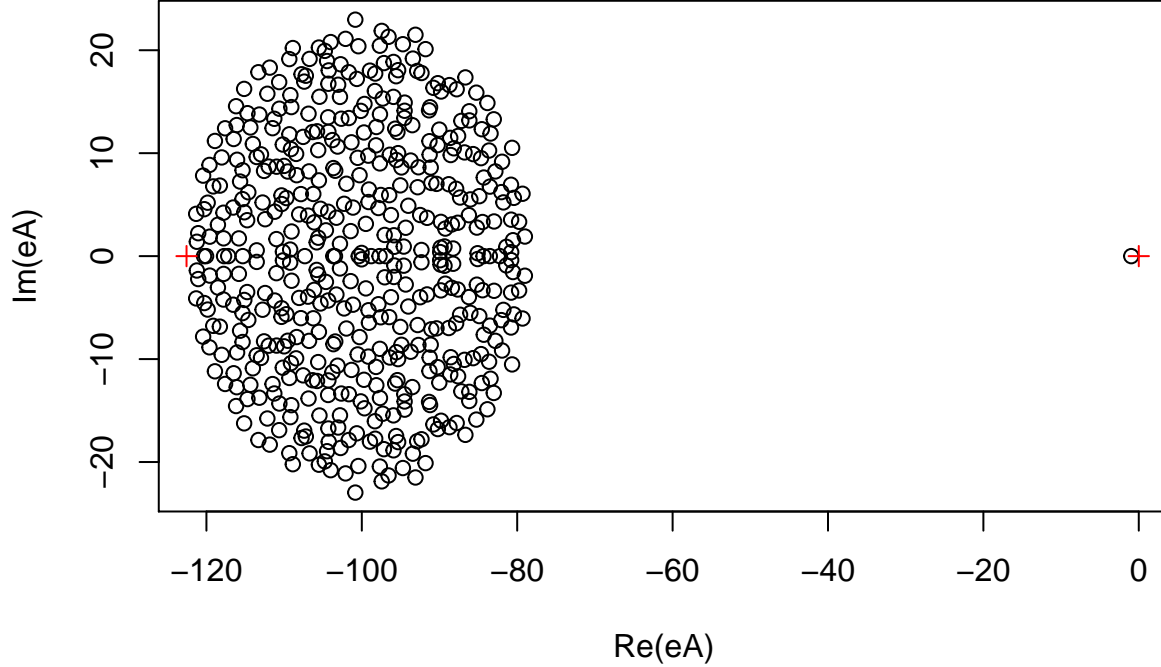
For one time step then,

$$x(t+1) = e^{A(t+1-t)}x(t) = e^A x(t),$$

so the discrete-time analogue of A is actually $B = \exp(A)$. From this, $A = \log(B)$, where we take the matrix logarithm of B . Now, if B is almost equal to the identity matrix, this indeed reduces to $A \approx B - I$ (by Taylor expanding the log function to first order near I). But this only holds in that special case.

I am mentioning this because relating certain metrics to $\max(|\lambda|)$ (like SV , as in Figure 2d) will only connect to existing literature well if $A \approx B - I$ holds. The reason is that SV has been related to continuous-time stability, not discrete-time, and the Authors' system is discrete time. However, as long as $A \approx B - I$, it is indeed true that any effect making B more stable will also make A more stable. The difference between discrete- and continuous-time formulations therefore disappears. But $A \approx B - I$ is not true generally. For instance, consider the example below, with a matrix whose offdiagonal entries are skewed towards mutualisms:

```
set.seed(1) ## set random seed for reproducibility; feel free to change this
S <- 500 ## number of species
mu <- 0.2 ## mean of offdiagonal coefficients
sigma <- 1 ## standard deviation of offdiagonal coefficients
d <- -100 ## diagonal of matrix
A <- matrix(rnorm(S*S, mu, sigma), S, S) ## generate random matrix
diag(A) <- d ## set its diagonal entries to d
eA <- eigen(A)$values ## obtain eigenvalues
plot(eA) ## plot the spectrum
## add analytical predictions of the dominant (largest modulus) and leading
## (largest real part) eigenvalues to the plot, based on random matrix theory
## (e.g., Barabas et al 2016 Am Nat, Appendix B4.2)
points(c(d - mu - sqrt(S * sigma^2) + 0i, S * mu + d + 0i), col="red", pch=3)
```



In this example, the leading eigenvalue (the one with maximum real part) is small, while the dominant eigenvalue (the one with maximum modulus) is large. Here, if we e.g. change μ from 0.2 to 0, this will have a large effect on the leading eigenvalue, but a negligible effect on the dominant one.

Long story short: in general, the effects that will result in stability in discrete time (small dominant eigenvalue) are different from those resulting in stability in continuous time (small leading eigenvalue). The only time they coincide is when B is almost the identity, because then $A \approx B - I$. Now, the Authors' matrices happen to be diagonally dominated, so there is good reason to think that $A \approx B - I$ is a fine approximation. However, it would be important to perform a quick calculation to make sure - e.g., by comparing $B - I$ and $\log(B)$.

In light of this, I also feel that the last section of the Authors' supplement ("Connection to Lotka-Volterra dynamics") is not really necessary - and it was somewhat hand-waivy to begin with. Instead, it could simply be argued that $B - I$ is indeed almost equal to $\log(B)$ (in case it is true).

But I emphasize that these are just minutia whose purpose is to make the manuscript even better than it currently is. Otherwise, I am very much satisfied with the paper and strongly recommend its publication.

Sincerely,

Gyuri Barabas