Steps

$$1-\begin{cases} N_{t+h,i,c/o} & = & \frac{e^{r_i}N_{t,i,c/o}}{1+\sum_{j/a_{ij}\in\mathbb{C}}\frac{a_C N_{t,j,c/o}}{H_{ij}+N_{t,jc/o}}+\sum_{j/a_{ij}\in\mathbb{F}}\frac{a_F N_{t,j,c/o}}{H_{ij}+N_{t,j,c/o}}} - lN_{t,i,c/o} \\ N_{t+h,i,b} & = & N_{t,i,b}(1-m-\zeta) \end{cases}$$

$$2-\begin{cases} N_{t+1,i,c} & = & N_{t+h,i,c}(1-s_i-e)+\gamma N_{t+h,i,b}+eN_{t+h,i,o} \\ N_{t+1,i,o} & = & N_{t+h,i,o}(1-s_i-e)+eN_{t+h,i,c} \\ N_{t+1,i,b} & = & N_{t+h,i,b}(1-\gamma)+s_iN_{t+h,i,c} \end{cases}$$

Growth rate

$$\begin{aligned} e^{r_i(T)} &= f(T)E(T) \\ \text{where } E(T) &= d\times 0.81e^{0.0631T_{c^\circ}} \\ \text{and } f_i(T) &= \begin{cases} e^{-|T_K-T_{K,i}^{opt*}|^3/b_i}, & T_K\leq T_{K,i}^{opt*} \\ e^{-5|T_K-T_{K,i}^{opt*}|^3/b_i}, & T>T_{K,i}^{opt*} \end{cases} \end{aligned}$$

with d is the daylength (growth rate proposed by Bissinger et al. 2008 is for a whole day but phytoplankton does not growth during the night: in field conditions, growth rate is therefore reduced), $T_{C^\circ}$is the temperature in Celsius degree, b_i is a measure of the species-specific niche width and $T_{K,i}^{opt*}$ is the modified (see below) optimal temperature of each species in Kelvin.

The latter two parameters are estimated on field-data phenology. For each genus and each year, the beginning of the bloom of a given genus is defined by the date at which its abundance exceeds its median abundance over the year. The duration of the bloom is the number of days between the beginning and the date where abundance fall below the median value. Generalists are characterized by one long bloom in the year or several blooms when the abundances oscillate around their median. Specialists tend to appear only once or twice in the year for shorter amounts of time. A genus is therefore defined as a generalist if its annual cumulated blooms $L_{i,y}$ last more than the average duration (137 days) for at least 15 years over the 20 years of the time series.

With these values in mind, we can define two range of values for the niche width (5-10 for specialists, 15-30 for the generalists) and then order the species in these ranges as a function of the mean sum of their bloom length, i.e. $\overline{L_i}>\overline{L_j}\Rightarrow b_i>b_j$ where $\overline{L}$ is the mean over 20 years of the annual cumulated lengths of the bloom.

The thermal optimum T_i^{opt} has first been defined as the mean temperature at the beginning bloom throughout the whole time series. This led to very low value and to blooms preferentially in winter and autumn. Therefore, $T_i^{opt*}=T_i^{opt}+5$.

Interactions

Maximum competitive and facilitative interaction strengths a_C and a_F, and half saturation coefficients H_{ih} are computed with the following steps.

- Compute α_{ij} from the derivation of classical Beverton-Holt and its relationship with MAR interaction coefficients b_{ij}
- Compute a_C from α_{ij} as $\sum|\alpha_{ij}|N_{j,max}$ and $a_F=\frac{1}{0.7}\left(\frac{1-O_y}{O_y}-0.3a_C\right)$; where 0.7 is the fraction of positive interaction coefficients in Auger and O_y is an overyielding factor[1]
- Compute $H_{ij}=\frac{a_\cdot}{\alpha_{ij}}$

Interactions are the same in the ocean and in the coast.

No quadratic programming can be applied to this model as it requires the equilibrium constraint to be linear with respect to the interaction matrix.

[1]

– see more explanations in exploratory/saturating_interactions.pdf