Interaction and growth

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Starting Point

Growth rates

%Here, describe the variability of species growth rate in the literature. Insist on the fact that we need to have growth rate values in isolation (so lab conditions) and that may vary from one study to the other. Most of the time, these are not taken at different temperature, so we cannot take that into account to reproduce a seasonal environment.

%We can start with the values from Scranton and Vasseur but they are much lower than the ones found in the literature, in lab conditions. They also rely on optimal temperatures for phytoplankton growth, and we rarely have access to that. Finally, they do not lead to However, they enable us to show blooms due to seasonal environments.

Interaction matrix

Interaction matrices are inspired by previous works on phytoplankton abundance time-series (Barraquand *et al.*, 2018; Picoche & Barraquand, 2019). The model used in these two papers is a multivariate autoregressive (MAR) model, that is a discrete-time linear model of the dynamics in which $X_{t+1} = BX_t$ where X_t is the vector of log-abundances of the phytoplankton species and B is the interaction matrices. Based on Picoche & Barraquand (2019), $b_{ij,i\neq j} \sim \mathcal{N}(\mu,\sigma)$ with μ and σ around 0.0 and 0.01. Centric diatoms, pennate diatoms and dinoflagellates can only interact with each other, i.e. the community matrix is modular. Self-regulation is computed as a function of vulnerability, with $b_{ii} \sim -0.49 b_{i.} - 0.37$. Assuming that nutrients are rarer in the ocean than in the coast [ref], $\alpha_{ij,c} << \alpha_{ij,o}$, with a fixed coefficient k such that $b_{ij,c} = k_{c2o} b_{ij,o}$

Certain *et al.* (2018)[1] showed that MAR and Beverton-Holt interaction coefficients, respectively b_{ij} and α_{ij}, could map once abundances at equilibrium N_i^* are defined.

$$\begin{cases} b_{ii} - 1 = & \frac{-\alpha_{ii} N_i^*}{1+\sum_l \alpha_{ij} N_l^*} \\ b_{ij,i\neq j} = & \frac{-\alpha_{ij} N_j^*}{1+\sum_l \alpha_{il} N_l^*} \end{cases}$$

Let's define $f_A(i) = \sum_l \alpha_{ij} N_l^*$.

$$b_{ij}(1 + f_A(i)) = -\alpha_{ij} N_j^*$$

We then sum on columns (on j).

$$\sum_j [b_{ij}(1 + f_A(i))] = -f_A(i)$$

$$\Leftrightarrow -f_A(i)(1 + \sum_j b_{ij}) = \sum_j b_{ij}$$

$$\Leftrightarrow f_A(i) = -\frac{\sum_j b_{ij}}{(1 + \sum_j b_{ij})}$$

$$\Leftrightarrow \alpha_{ij} = -\frac{1}{N_j^*} b_{ij} (1 - \frac{\sum_j b_{ij}}{1 + \sum_j b_{ij}})$$

[1] Corrected in the Appendices of Picoche & Barraquand (2019)

$$\Leftrightarrow \alpha_{ij} = -\frac{1}{N_j^*}\frac{b_{ij}}{1+\sum_j b_{ij}}$$

%However, strong variability between sites and calibration on biweekly data (so, we should actually use $\mathbf{B}^{\frac{1}{15}}$ but this leads to complex numbers and right now, I'm not sure what to do with that).

Quadratic programming

%Due to the strong uncertainty for all parameters, we need to calibrate more precisely. For that, we use quadratic programming [ref], applied to interaction matrices and growth rates by Maynard et al. 2019[2].

In the following, I am basically copying the package documentation to make the code understandable for later. Would maybe be better with a Rmarkdown file. This algorithm aims at finding $\mathbf{x}$ that minimizes $||\mathbf{Cx}-\mathbf{d}||^2$ under the constraints $\mathbf{Ex}=\mathbf{f}$ and $\mathbf{Gx}>=\mathbf{h}$.

Here, $\mathbf{C}=\mathbf{I}$, $\mathbf{d}=[\text{vec}(\mathbf{A}^T)^\frown\mathbf{r}']$ with $\mathbf{r}'=-(\mathrm{e}^{\mathbf{r}}-\mathbf{1})$, $\mathbf{E}$ so that we verify the equality $\mathbf{AN}^*+\mathbf{r}'=0$, and $\mathbf{G},\mathbf{h}$ so that $\mathbf{r}>0$ and $\forall i, a_{ii}>0$.

References

Barraquand, F., Picoche, C., Maurer, D., Carassou, L. & Auby, I. (2018). Coastal phytoplankton community dynamics and coexistence driven by intragroup density-dependence, light and hydrodynamics. *Oikos*, 127, 1834–1852.

Certain, G., Barraquand, F. & G, A. (2018). How do MAR(1) models cope with hidden nonlinearities in ecological dynamics? *Methods in Ecology and Evolution*, 9, 1975–1995.

Picoche, C. & Barraquand, F. (2019). Strong self-regulation and widespread facilitative interactions between genera of phytoplankton. preprint, bioRxiv.

[2]Actually, Maynard et al. 2019 uses a Least Square Inverse Problem solver, with a package (limSolve::lsei) that also offers quadratic programming