

# Some mutualism models

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## Holland & DeAngelis (2010)

General framework for both uni- and bidirectional mutualism, with a reminder on resource exploitation. Two-species model.

$$\begin{aligned}\frac{1}{N_1} \frac{dN_1}{dt} &= r_1 - d_1 N_1 + c_1 f_1(R_2(N_1, N_2)) - q_1 g_1(R_1(N_1, N_2)) \\ \frac{1}{N_2} \frac{dN_2}{dt} &= r_2 - d_2 N_2 + c_2 f_2(R_1(N_1, N_2)) - q_2 g_2(R_2(N_1, N_2))\end{aligned}$$

(and conversely for species 2). In a unidirectional mutualism,  $q_2 g_2(R_1(N_1, N_2)) = 0$

The structure of the model implies that there is a cost for providing its resource for each species, with  $c_i$  and  $q_i$  the relative costs.

When adapted to the Rosensweig-MacArthur model:

$$\begin{aligned}\frac{1}{N_1} \frac{dN_1}{dt} &= r_1 - d_1 N_1 + c_1 \frac{\alpha_{12} N_2}{h_2 + N_2} - q_1 \frac{\beta_1 N_2}{e_1 + N_1} \\ \frac{1}{N_2} \frac{dN_2}{dt} &= r_2 - d_2 N_2 + c_2 \frac{\alpha_{21} N_1}{h_1 + N_1} - q_2 \frac{\beta_2 N_1}{e_2 + N_1}\end{aligned}$$

where  $\alpha_{ij}$  is the saturation level and  $h_j$  is the half-saturation density of species  $j$ ; and there is a similar saturating function for resource supply by species  $i$ , for which  $b_i$  is the saturation level and  $e_i$  the half-saturation constant.

## García-Algarra *et al.* (2014)

Logistic growth. Specifically adapted to animal vs plant species. Here,  $N_i^a$  is the population of species  $i$  of animals, and there is  $n_a$  animal species. Similarly,  $N_j^p$  is the population of species  $j$  of plants.

$$\begin{aligned}\frac{1}{N_i^a} \frac{dN_i^a}{dt} &= r_i + \sum_{k=1}^{n_p} b_{ik} N_k^p - (a_i + c_i \sum_{k=1}^{n_p} b_{ik} N_k^p) N_i^a \\ \frac{1}{N_j^p} \frac{dN_j^p}{dt} &= r_j + \sum_{k=1}^{n_a} b_{jk} N_k^a - (a_j + c_j \sum_{k=1}^{n_a} b_{jk} N_k^a) N_j^p\end{aligned}$$

In this model, the effective growth rate of a species is divided between growth rate potentially reduced/sustained by consumption ( $r_i + \sum_{k=1}^{n_p} b_{ik} N_k^p$ ) and the reduction in growth rate that is alleviated by mutualism ( $a_i + c_i \sum_{k=1}^{n_p} b_{ik} N_k^p$ ). In the absence of mutualism,  $K_i = \frac{r_i}{a_i}$ . With a strong mutualism,  $K_i = \frac{1}{c_i}$ .

$c_i$  is a proportion to establish a maximum population for species  $i$  in the interaction limit  $c_i \sum_{k=1}^{n_p} b_{ik} N_k^p \gg a_i$ .

## Martorell & Freckleton (2014)

Removed the notion of metacommunity in patches to focus on the effect of mutualism in one patch only, but the focus of the model is the colonisation. (BH look-alike if we ignore the facilitation product)

$$N_{j,t+1} = (1 - m_j) N_{j,t} \frac{\lambda_{i,k} \prod_{i \neq j} (1 + N_{i,t})^{\beta_i}}{1 + \sum_i \alpha_i N_{i,t}}$$

where  $m_i$  is the dispersal rate (could as well be a loss term such as a death rate,  $\lambda_{i,k}$  is the population growth during the  $k^{th}$  year in absence of interactions (maximum growth rate measured in isolation?) and  $\beta_i$  and  $\alpha_i$  measure the effect of interactions.

## Moore *et al.* (2018)

Lotka-Volterra with  $\theta$ -density dependence. Only intra-competition and inter-facilitation.

$$\frac{1}{N_1} \frac{dN_1}{dt} = r_1 - a_1 N_1^{\theta_1} + \beta_1 N_2$$

(and conversely for  $N_2$ )

Specific constraints between parameters can stabilize the communities even in the presence of non-saturating positive interactions. With  $\theta > 1$ , i.e., accelerating density-dependence, the community is always stable even with a strong mutualism. With  $\theta < 1$  (decelerating density-dependence), more complex compartments that need to be studied individually (strong mutualism  $\rightarrow$  positive feedback on the given species, explosion of the population).

## Qian & Akçay (2020)

We focus here on the implementation of competition and facilitation. We remove the part on exploitation.

$$\begin{aligned} \frac{1}{N_i} \frac{dN_i}{dt} &= r_1 + \frac{s_i}{K_i} N_1 - \sum_j^{S_{c_i}} a_{ij} N_j + \sum_j^{S_{M_i}} \frac{a_{ij} N_j}{h + N_j} & (\text{Unique Interaction Model}) \\ \frac{1}{N_i} \frac{dN_i}{dt} &= r_1 + \frac{s_i}{K_i} N_1 - \sum_j^{S_{c_i}} a_{ij} N_j + \sum_j^{S_{M_i}} \frac{a_{ij} N_j}{h + \sum_j^{S_{M_i}} N_j} & (\text{Individual Interaction Model}) \end{aligned}$$

where  $S_{c_i}$  is the set of species that have competitive interactions with  $N_i$  while  $S_{M_i}$  is the set of species that have mutualistic interactions with  $N_i$ . One of the main features of this model is the fact that competition is not saturating while mutualism is. I wonder if that has biological grounds.

## References

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