QMeS: *mathematica* package for symbolic derivation of functional equations

Jan M. Pawlowski,[1,2] Coralie S. Schneider,[1] and Nicolas Wink[1]

[1] *Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany*
[2] *ExtreMe Matter Institute EMMI, GSI, Planckstr. 1, 64291 Darmstadt, Germany*

We present the *mathematica* package *QMeS*. It derives symbolic functional equations from a master equation (FRG, mSTI, DSE) by using a superfield formalism. Explicitly it's modules allow to derive DSEs, take functional derivatives, trace over field space and do a momentum routing for 1-loop diagrams while keeping track of prefactors and signs that arise from fermionic commutation relations. The package furthermore contains an installer as well as *mathematica* notebooks with showcase examples.

I. INTRODUCTION

Functional approaches are nowadays a well-established tool to study non-perturbative aspects of quantum field theories. In comparison to perturbative approaches, there are still only a few computer algebraic tools for functional methods published [1**? ?** , 2]. In this work we present a package of *QMeS* (*Quantum Master equations: environment for numerical solutions*), that can be used for the symbolic derivation of functional equations arising from a master equation. Common examples are the Functional Renormalization Group (FRG), Dyson-Schwinger Equation (DSE) or the modified Slavnov-Taylor Identities (mSTI). The package is written in *Mathematica* and can be used to derive the Dyson-Schwinger Equation from a classical action or to extract the symbolic equations for different n-Point functions, i.e. the moments of the master equations. It works in a general field space, allowing for arbitrary theories and can include momentum routing for the diagrammatic/symbolic results. here

First, we define the conventions that are used by deriving the DSE, FRG equation and mSTI, introducing a diagrammatic notation of the master equations and summarizing the derivative rules that are used in *QMeS*. We proceed by describing the package in section III, i.e. how the modules are connected by the interface and the installation process. Then we give an overview of the input, objects and their notation in *QMeS* and output options. Section V contains two examples: Yang-Mills and Yukawa theory ($N_f = 1$ and $N_f = 2$). We summarize the main features in section VI.

II. CONVENTIONS

In this section we describe how one arrives at a Quantum Master Equation (FRG, mSTI or DSE) starting from an action S for a general theory. We use the superfield index notation which will also be explained.

A. Deriving a Quantum Master Equation

For a general theory the euclidean action S can be written as:

$$S[\phi] = \sum_{n=2} S^{(a_1 \dots a_n)} \phi_{a_1} \dots \phi_{a_n}. \tag{1}$$

From this one obtains the generating functional $W[J]$ via:

$$e^{W[J]} = \int D\phi \exp\left(-S[\phi] + J^a \phi_a\right) = Z[J], \tag{2}$$

where Φ is the expectation value:

$$\frac{\delta W[J]}{\delta J^a} = W_a = \langle \phi_a \rangle_J = \Phi_a. \tag{3}$$

The summation over internal indices as well as the integration over space-time is implied by the superfield index notation:

$$J^a \phi_a = \int d^d x J^a(x) \phi_a(x). \tag{4}$$

The effective action Γ is obtained by Legendre transforming the Schwinger functional with respect to the source:

$$\Gamma[\Phi] = \sup_J \left(J^a \Phi_a - W[J]\right). \tag{5}$$

The effective action generates 1PI Greens functions.
Now the sources can be expressed as:

$$\frac{\delta \Gamma[\Phi]}{\delta \Phi_a} = \Gamma^a = \gamma^a{}_b J^b. \tag{6}$$

Here we have introduced a metric γ which deals with fermions. It is diagonal in field space for bosons and scalars. For a fermion anti-fermion pair $(f, \bar{f})_a$ the metric reads:

$$(\gamma^{ab}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{7}$$

With this metric one can raise and lower indices:

$$\Phi_a = \Phi^b \gamma_{ba},$$
$$\Phi^a = \gamma^{ab}\Phi_b. \tag{8}$$

Furthermore:

$$\gamma_b{}^a = \gamma^{ac}\gamma_{bc} = \delta^a_b,$$
$$\gamma^a{}_b = \gamma^{ac}\gamma_{cb} = (-1)^{ab}\delta^a_b, \tag{9}$$

with

$$(-1)^{ab} = \begin{cases} -1 & a \text{ and } b \text{ fermionic}, \\ 1 & \text{otherwise}. \end{cases} \tag{10}$$

Thus one obtains:

$$J^a\Phi_a = \Phi^a J_a = J_a \Phi^b \gamma^a{}_b = \Phi_b J^a \gamma^a{}_b. \tag{11}$$

We furthermore define field and source derivatives as:

$$\frac{\delta}{\delta\Phi_{a_1}} \cdots \frac{\delta}{\delta\Phi_{a_n}} \Gamma[\Phi] = \Gamma^{a_1 \ldots a_n},$$
$$\frac{\delta}{\delta J^{a_1}} \cdots \frac{\delta}{\delta J^{a_n}} W[J] = W_{a_1 \ldots a_n}. \tag{12}$$

Thus it follows that:

$$W_{ac}\Gamma^{cb} = \gamma^b{}_a. \tag{13}$$

1. DSE

Now one can already derive the *Dyson-Schwinger equation* (DSE) for 1PI Greens functions by taking a total derivative of the integral (2) which then vanishes:

$$\begin{aligned} 0 &= \int D\phi \frac{\delta}{\delta\phi_a} \exp\left(-S[\phi] + J^a\phi_a\right) \\ &= \int D\phi \left(-\frac{\delta S}{\delta\phi_a} + (-1)^{aa} J^a\right) \exp\left(-S[\phi] + J^a\phi_a\right) \\ &= \left(-\frac{\delta S}{\delta\phi_a} + (-1)^{aa} J^a\right)_{\phi_b = \frac{\delta}{\delta J^b}} Z[J]. \end{aligned} \tag{14}$$

When pulling the derivative of the action and the source out of the integral one has to replace the field ϕ with a derivative with respect to the source.
Using the relation:

$$e^{-W[J]}\left(\frac{\delta}{\delta J^a}\right) e^{W[J]} = \frac{\delta W[J]}{\delta J^a} + \frac{\delta}{\delta J^a}. \tag{15}$$

one obtains the DSE for connected Greens functions:

$$-\left.\frac{\delta S[\phi]}{\delta\phi_a}\right|_{\phi_b = \frac{\delta W[J]}{\delta J^b} + \frac{\delta}{\delta J^b}} + (-1)^{aa} J^a = 0. \tag{16}$$

By rewriting the derivative with respect to J as and using the definition of the propagator $W_{ab} = G_{ab}$:

$$\begin{aligned} \frac{\delta}{\delta J^a} &= \frac{\delta \Phi_b}{\delta J^a}\frac{\delta}{\delta\Phi_b} \\ &= \frac{\delta}{\delta J^a}\frac{\delta W[J]}{\delta J^b}\frac{\delta}{\delta\Phi_b} \\ &= G_{ab}\frac{\delta}{\delta\Phi_b}, \end{aligned} \tag{17}$$

one can express the DSE in terms of the effective action:

$$\frac{\delta\Gamma[\Phi]}{\delta\Phi_a} = \left.\frac{\delta S[\phi]}{\delta\phi_a}\right|_{\phi_b = \Phi_b + G_{bc}\frac{\delta}{\delta\Phi_c}}. \tag{18}$$

The generalized DSE for quantum symmetries can be derived by inserting a generic function $\Psi[\phi]$ in the derivation of equation (14):

$$\frac{1}{Z[J]}\int D\phi \frac{\delta}{\delta\phi_a}\left(\Psi[\phi]\exp\left(-S[\phi] + J^a\phi_a\right)\right), \tag{19}$$

thus yielding:

$$\langle\Psi[\phi]\rangle \frac{\delta\Gamma[\Phi]}{\delta\Phi_a} = \left\langle \Psi[\phi]\frac{\delta S[\phi]}{\delta\phi_a}\right\rangle - \left\langle\frac{\delta\Psi[\phi]}{\delta\phi_a}\right\rangle. \tag{20}$$

2. FRG

In this section we want to derive *functional renormalisation group* (FRG) equations. It can be viewed as a differential form of the DSE but has the advantage that only 1-loop diagrams are produced. For this one defines an effective average action Γ_k which interpolates between the physical action S and the effective action Γ of a theory. One can already see that a momentum scale k is introduced. The idea then is solve the path integral by integrating out quantum fluctuations momentum shellwise. Thus we modify the Schwinger functional by:

$$Z[J,R] = e^{W[J,R]} = e^{-\Delta S[\phi,R]}e^{W[J]}, \tag{21}$$

with the so-called regulator insertion:

$$\Delta S[\phi, R] = \frac{1}{2}R^{ab}\phi_a\phi_b. \tag{22}$$

The flow of the generating functional can be written as:

$$\begin{aligned} k\partial_k Z[J,R] &= -\left(k\partial_k \Delta S[\phi,R]\right) Z_k[J,R] \\ &= -\frac{1}{2}\left(k\partial_k R^{ab}\right)\frac{\delta^2 Z[J,R]}{\delta J^a \delta J^b}. \end{aligned} \tag{23}$$

Using the relation:

$$\frac{1}{Z[J,R]}\frac{\delta^2 Z[J,R]}{\delta J^a J^b} = W_{ab} + W_a W_b, \tag{24}$$

the flow equation in terms of the Schwinger functional is:

$$k\partial_k W = -\frac{1}{2}\left(k\partial_k R^{ab}\right)\left(W_{ab} + W_a W_b\right) m \tag{25}$$

where W is a function of J and R.
One can define the propagator as:

$$\begin{aligned} G_{ac}\left(\Gamma + \Delta S\right)^{cb} &= \gamma^b{}_a, \\ \leftrightarrow G_{ac}\left(\Gamma^{cb} + R^{bc}\right) &= \gamma^b{}_a. \end{aligned} \tag{26}$$

One can again take the Legendre transform to obtain the effective average action:

$$\Gamma[\Phi, R] = \sup_J \left(J^a \Phi_a - W[J,R] - \Delta S[\Phi, R]\right). \tag{27}$$

Again the relations between the fields and sources in terms of the effective average action are given as:

$$\begin{aligned} \frac{\delta(\Gamma[\Phi,R] + \Delta S[\Phi,R])}{\delta\Phi_a} &= (-1)^{aa} J^a, \\ \frac{\delta W[J,R]}{\delta J^a} &= \Phi_a. \end{aligned} \tag{28}$$

By switching to the RG-time $t = ln(k/\Lambda)$ with $\partial_t = k\partial_k$ one can write:

$$\begin{aligned} \partial_t\Gamma &= -\partial_t W - \partial_t \Delta S \\ &\quad - \partial_t J^a \left(\Phi_a - \frac{\delta W}{\delta J^a}\right) \\ &= \frac{1}{2}\left(\partial_t R^{ab}\right)\left(W_{ab} + W_a W_b\right) - \partial_t \Delta S \\ &= \frac{1}{2}\left(\partial_t R_k^{ab}\right) W_{ab} + \partial_t \Delta S - \partial_t \Delta S \\ &= \frac{1}{2}\dot{R}^{ab} G_{ab}. \end{aligned} \tag{29}$$

where $\Gamma \equiv \Gamma[\Phi]$, $W \equiv W[J,R]$, $\Delta S \equiv \Delta S[\Phi, R]$ and we have used equation (25) as well as:

$$\begin{aligned} \frac{1}{2}\left(\partial_t R^{ab}\right)\frac{\delta W}{\delta J^a}\frac{\delta W}{\delta J^b} &= \frac{1}{2}\left(\partial_t R^{ab}\right)\Phi_a\Phi_b \\ &= \partial_t \Delta S. \end{aligned} \tag{30}$$

Note that the superfield index notation above implies the summation and thus trace over all fields.

3. STI

The classical Yang-Mills action of non-abelian gauge theories is gauge invariant, but neither the ghost nor the gauge fixed action are. Thus:

$$\delta^a_{gauge} e^{-S_A} = \delta^a_{gauge}\left(S_{gf} + S_{gh}\right), \tag{31}$$

where δ^a_{gauge} is the generator of a gauge transformation. Additionally it has the form of an operator in the generalized DSE (20) with $\delta/\delta\phi_a \Psi[\phi] = \delta^a_{gauge}$ which means that:

$$\begin{aligned} &\frac{1}{Z[J]}\int D\phi\, \delta^a_{gauge}\left(\exp(-S_A[\phi] + J^a\phi_a)\right) \\ &= \left\langle J^a(\delta^a_{gauge}\phi_a) - \delta^a_{gauge}(S_{gf} + S_{gh})\right\rangle = 0. \end{aligned} \tag{32}$$

Carrying out the expectation value leads to the Slavnov-Taylor identities (STI) of the theory. These identities guarantee the gauge invariance of observables.
Since δ^a_{gauge} is not a symmetry of the underlying classical theory we would like to find a transformation that is. This is satisfied by the BRST transformation:

$$\delta_{BRST}\phi_a = \delta\lambda\mathfrak{s}\phi_a, \tag{33}$$

where the infinitesimal parameter $\delta\lambda$ as well as the BRST generator $\mathfrak{s}$ are Grassmannian.
The action is invariant under BRST transformations:

$$\mathfrak{s}S_A[\phi] = 0. \tag{34}$$

Again with $\mathfrak{s}$ as an operator the generalized DSE can be written as:

$$\frac{1}{Z[J]}\int D\phi\, \mathfrak{s}\left(\exp(-S_A[\phi] + J^a\phi_a)\right) = 0. \tag{35}$$

Thus the expectation value vanishes:

$$(-1)^{aa}\left\langle J^a \mathfrak{s}\phi_a\right\rangle = 0. \tag{36}$$

Note that the prefactor $(-1)^{aa}$ is due to the grassmannian nature of $\mathfrak{s}$.

$$\mathfrak{s}J^a\phi_a = (-1)^{aa} J^a \mathfrak{s}\phi_a. \tag{37}$$

Since the BRST transformations of fields are usually quadratic in the fields, it seems as if one looses the algebraic nature of the symmetry on quantum level. To resolve this one may introduce additional source terms Q^a for the BRST transformations of the fields:

$$Z[J,Q] = \int D\phi \exp(-S_A[\phi] + J^a\phi_a + Q^a\mathfrak{s}\phi_a). \tag{38}$$

Since $\mathfrak{s}^2 = 0$, this does not change (36).
Then one can write the STI takes again algebraic form as:

$$\langle\mathfrak{s}\phi_a\rangle = \frac{1}{Z[J,Q]}\frac{\delta Z[J,Q]}{\delta Q^a}. \tag{39}$$

By Legendre transforming the Schwinger functional $\ln Z[J,Q]$ one obtains the effective action in the presence of source terms for the BRST transformation:

$$\Gamma[\Phi, Q] = J^a\Phi_a - \ln Z[J,Q]. \tag{40}$$

We can directly see that:

$$\langle \mathfrak{s}\phi_a \rangle = -\frac{\delta \Gamma[\Phi, Q]}{\delta Q^a} = \frac{1}{Z[J,Q]} \frac{\delta Z[J,Q]}{\delta Q^a}. \tag{41}$$

Rewriting the expectation value (36) yields the STI:

$$\frac{\delta \Gamma}{\delta Q^a} \frac{\delta \Gamma}{\delta \Phi_a} = 0. \tag{42}$$

Fulfilling this relation guarantees gauge invariance of observables.

4. mSTI

Due to the presence of the cutoff (22) in the effective average action, gauge and hence BRST symmetry are broken. By ensuring the validity of *modified Slavnov-Taylor identities* (mSTIs) at a non-vanishing momentum scale k one automatically fulfills the STIs at $k = 0$.
Starting from the generating functional with the cutoff term $\Delta S[\phi, R]$ one can derive the mSTIs:

$$Z[J,Q] = \int D\phi \exp\left(-S[\phi] - \Delta S[\phi, R]\right) \cdot \exp\left(J^a \phi_a + Q^a \mathfrak{s}\phi_a\right). \tag{43}$$

Note that either the BRST charge or the field itself is of grassmanian nature.
Thus the expectation value (36) changes to:

$$(-1)^{aa} \langle J^a \mathfrak{s}\phi_a \rangle = \langle \mathfrak{s}\Delta S[\phi, R] \rangle . \tag{44}$$

After Legendre transforming the momentum scale dependent Schwinger functional one obtains the following relation:

$$\langle \mathfrak{s}\phi_a \rangle = -\frac{\delta \Gamma[\Phi, Q, R]}{\delta Q^a} = \frac{\delta \ln Z[J,Q,R]}{\delta Q^a} = \frac{\delta W[J,Q,R]}{\delta Q^a}. \tag{45}$$

Rewriting the sources J^a in terms of the effective average action (28) one is left with:

$$\frac{\delta \Gamma}{\delta Q^a} \frac{\delta\left(\Gamma + \Delta S\right)}{\delta \Phi_a} = \langle \mathfrak{s}\Delta S[\phi, R] \rangle . \tag{46}$$

Moving all terms that contain ΔS to the right, one can further simplify by using relation (45):

$$\begin{aligned} &\langle \mathfrak{s}\Delta S[\phi, R] \rangle - \frac{\delta \Delta S[\Phi, R]}{\delta \Phi_a} \frac{\delta \Gamma[\Phi, Q, R]}{\delta Q^a} \\ &= \langle \mathfrak{s}\Delta S[\phi, R] \rangle + \mathfrak{s}\Delta S[\Phi, R]. \end{aligned} \tag{47}$$

Inserting the cutoff term (22) one arrives at:

$$\begin{aligned} \left\langle R^{ab} (\mathfrak{s}\phi_a) \phi_b \right\rangle + R^{ab} (\mathfrak{s}\Phi_a) \Phi_b &= -R^{ab} \frac{\delta}{\delta J^b} \frac{\delta}{\delta Q^a} W[J,Q,R] \\ &= R^{ab} \frac{\delta}{\delta J^b} \frac{\delta \Gamma[\Phi, Q, R]}{\delta Q^a} \\ &= R^{ab} G_{bc} {\Gamma^c}_{Q[a]}. \end{aligned} \tag{48}$$

Thus the full mSTI reads:

$$\frac{\delta \Gamma}{\delta Q^a} \frac{\delta \Gamma}{\delta \Phi_a} = R^{ab} G_{bc} {\Gamma^c}_{Q[a]}. \tag{49}$$

Fulfilling the mSTI at each momentum scale k ensures gauge invariance of observables at $k = 0$.

III. DESCRIPTION

This section outlines the basic design of *QMeS*, i.e. its modules and how they are connected via an interface. Furthermore we give instructions on how to install the package.

A. Modules and Interface

The code consists of four main modules - *getDSE.m*, *FunctionalDerivatives.m*, *SuperindexDiagrams.m* and *FullDiagrams.m* - which are connected by the interface (*DeriveFunctionalEquation.m*).
The four modules correspond to the four output options described in IV D 1. The workflow is depicted in figure 1.

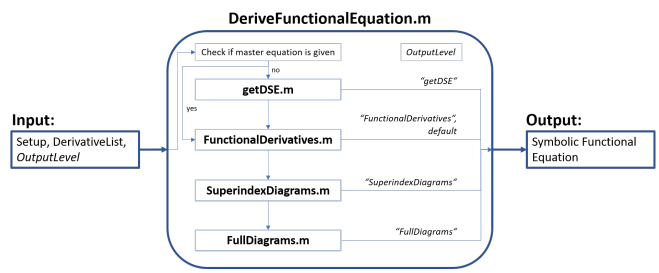

FIG. 1: Depiction of the workflow of *QMeS* with its interface and modules.

The user is required to provide a setup which consists of either a master equation or an association which indicates, that *QMeS* first needs to derive the DSE for a given classical action, a definition of the field space and a truncation, as well as a list of field derivatives. Specifying the preferred form of the output (i.e. `"OutputLevel"`) is optional.
Depending on whether or not a master equation was provided the interface calls the *FunctionalDerivatives* or first the *getDSE.m* module which then generates the Dyson-Schwinger equation of the theory and setup and passes it on to the *FunctionalDerivatives.m* module along with the setup and the derivative list. Within this module the (remaining) field derivative of the master equation are performed and fields are set to zero.
The output and user provided input is again passed on in the interface to the *SuperindexDiagrams.m* module, where the trace in field space is performed, objects are sorted, prefactors are computed and the truncation is applied.

The result together with the initial input is then used by the *FullDiagrams.m* module to replace the superfield indices with physical indices and the objects are replaced by functions of indices.
If the user has specified an output option, the workflow is terminated after the corresponding module providing the user with said output. The default output option is `"FunctionalDerivatives"`.

B. Requirements and Installation

Funcionality of *QMeS* is supported in *mathematica* 12.0 or higher, although it may also work with older versions.
To install the package download the installer via:

```
Import["https://lin0.thphys.uni-heidelberg.de:
4443/qmes/qmes-\derivation/-/tree/Version_0.2
/QMeSInstaller.m"];
```

This does not work yet as one has to sign in to access this git...Furthermore the path needs to be changed when we publish the code.
Other options are to either save the package files in the `"../Mathematica/Applications"` folder or append the path (`yourpath`) where the package is saved to the list of paths where *mathematica* searches for packages via:

```
AppendTo[$Path, "yourpath"];
```

Then the package can be loaded:

```
Needs["DeriveFunctionalEquation`"];
```

IV. INPUT, FUNCTIONS AND OPTIONS

To compute functional derivatives of a master equation one needs to define said equation as well as the theory one is working in. Both must be collected in an association.

```
Setup = <|"MasterEquation" -> masterEquation,
"FieldSpace" -> fields,
"Truncation" -> truncation|>;
```

If one first wants to derive a DSE of a given theory, the setup must be provided as:

```
SetupDSE = <|
"MasterEquation" -> <|"getDSE" -> "True",
"classicalAction" -> classicalAction|>,
"FieldSpace" -> fields,
"Truncation" -> truncation|>;
```

Note that one then needs a definition of the classical action via possible vertices.

A. Master Equations/Objects

Within the QMeS framework a master equation is defined as a list of objects, the first being an overall prefactor. Each object is of a specific `"type"` (e.g. propagator, n-point function, regulator or regulator derivative). Furthermore every object contains a list of `"indices"` which are superfield indices. The indices should be closed.

An example of a master equation is the Wetterich equation:

$$\partial_t \Gamma = \frac{1}{2} \dot{R}^{ab} G_{ab}, \tag{50}$$

```
FRGEq = {"Prefactor" -> {1/2},
<|"type" -> "Regulatordot",
"indices" -> {a, b}|>,
<|"type" -> "Propagator",
"indices" -> {a, b}|>};
```

as well as the modified Slavnov-Taylor identity (mSTI):

$$\frac{\delta\Gamma}{\delta Q^a}\frac{\delta\Gamma}{\delta\Phi_a} = R^{ab} G_{bc} {\Gamma^c}_{Q[a]}. \tag{51}$$

The mSTI can be written as:

```
LHSmSTIEq = {"Prefactor" -> {1},
<|"type" -> "nPoint", "indices" -> {Q[i]},
"nPoint" -> 1, "spec" -> "BRST"|>,
<|"type" -> "nPoint", "indices" -> {i},
"nPoint" -> 1, "spec" -> "none"|>};

mSTIEq = {"Prefactor" -> {1},
<|"type" -> "Regulator", "indices" -> {a, b}|>,
<|"type" -> "Propagator", "indices" -> {b, c}|>,
<|"type" -> "nPoint", "indices" -> {c, Q[a]},
 "nPoint" -> 2, "spec" -> "BRST"|>};
```

It is furthermore possible to derive the DSE of a given theory with the formerly mentioned setup. For further information see section IV D 1.

Prefactors

The first entry in every diagram is the `Prefactor`. It can contain numbers (1,−1,1/2,...) or a metric factor $(-1)^{ab}$. For example the prefactor:

```
"Prefactor" -> {-1/2, {a,b}, {b,b}, {b,c}};
```

translates into:

$$-\frac{1}{2}(-1)^{ab}(-1)^{bb}(-1)^{bc}, \tag{52}$$

where again the superfield index convention (10) is used.

Regluator and Regulator Derivative

```
<|"type" -> "Regulatordot", "indices" -> {a, b}|>;
```

```
<|"type" -> "Regulator", "indices" -> {a, b}|>;
```

A regulator R^{ab} or regulator derivative $\dot{R}^{ab}$ is an object with two superfield indices which correspond to the incoming and outgoing fields with their respective momenta and indices. Note that these are upper indices.

Propagator

```
<|"type" -> "Propagator", "indices" -> {a, b}|>;
```

A propagator G_{ab} is a fully dressed object with two superfield indices corresponding to the fields and their indices. These are lower indices. Note that for FRG and mSTI equations the propagator is k-dependent whereas it is not for DSEs.

n-Point Functions

```
<|"type" -> "nPoint", "indices" -> {a, b, c, d},
"nPoint" -> 4, "spec" -> "none"|>;

<|"type" -> "nPoint", "indices" -> {a, b},
"nPoint" -> 2, "spec" -> "classical"|>;

<|"type" -> "nPoint", "indices" -> {a, b, Q[c]},
"nPoint" -> 3, "spec" -> "BRST"|>;
```

n-point functions are field derivatives of the effective action. The value of `"nPoint"` indicates the number of derivatives, whereas the `"indices"` again represent the superfield indices. The specification `"spec"` implies whether one of the fields is a BRST source of a field (`"BRST"`, ${\Gamma^{ab}}_{Q[c]}$), a 1PI (`"none"`, Γ^{abcd}) or a classical vertex (`"classical"`, S^{ab}). If one of the fields is a BRST source, then its superfield index needs to be written as `"Q[i]"` to indicate that this is a lower index, which belongs to the BRST source of a `field`, `Q[field]`. Again it is worth mentioning, that in case of FRG or mSTI equations the 1PI and BRST vertices are k-dependent objects.

Fields

```
<|"type" -> "Field", "indices" -> {a}|>;
```

Fields Φ_a are objects with one lower index. Note that after taking all functional derivatives, external fields that are left over are set to zero.

B. Theory

The user is required to define a specific theory. This breaks down into two main parts: defining the fields with the respective indices and the truncation.

1. *Fields with indices*

The fields of a theory can either have fermionic or bosonic statistics (i.e. scalars are also bosonic fields). Antifermion/fermion pairs must be combined in a list.

```
fields =
<|"bosonic" -> {A[p, {mu, i}], B[p,{i}]},

"fermionic" -> {{cbar[p, {i}], c[p, {i}]},
{af[p,{i}], f[p,{i}]}},

"BRSTsources" ->
{{Q[A], "fermionic"}, {Q[B], "fermionic"},
{Q[cbar], "bosonic"}, {Q[c], "bosonic"},
{Q[af], "bosonic"}, {Q[f], "bosonic"}}|>;
```

If a theory contains no fields of either bosonic or fermionic statistics, it is then required to assign an empty list.
When computing mSTIs one also needs to define the BRST charges of fields. They are indicated by `Q[field]` followed by the respective statistics of the charge (either `"fermionic"` or `"bosonic"`). For the computation of DSE or FRG equations it is not necessary to define the sources.
The respective indices are provided as arguments of the fields, where the momentum is always the first entry, followed by a list of further indices (e.g. group or Lorentz indices). Note that the names of the indices for different fields does not need to be unique. For better readability it is recommended to define the same kind of indices with the same letter.

2. *Truncation and classical Action*

For the derivation of DSEs it is necessary to define the classical action via vertices. This is done by giving a list of combination of fields that appear as a classical vertex in the action:

```
classicalAction = {{A, A}, {c, cbar}, {A, A, A},
{A, A, A, A}, {A, c, cbar}};
```

Furthermore the truncation of the full theory is defined by specifying the truncation of 1PI and BRST vertices. It is worth mentioning that the user is also required to include the possible propagators in this list:

```
Truncation = {{A, A}, {c, cbar}, {A, A, A},
{A, A, A, A}, {A, c, cbar}, {A, A, c, cbar}};
```

The truncation may be similar or include more vertices than the classical action.
In both definitions the order of fields is irrelevant.

C. Derivative List

Lastly one needs to specify a list of field derivatives. Note that the last entry of the list will be the first derivative.

```
DerivativeList1 = {A, A};
DerivativeList2 = {A[a], A[b]};
DerivativeList3 = {A[-p, {mu, m}], A[p, {nu, n}]};
```

Generally one has three options: The first is to only give fields. This can be combined with the output options "getDSE" and "FunctionalDerivatives". The second is to assing superindices to the fields which additionally works with "SuperindexDiagrams". If we want to obtain full diagrams with momentum routing ("FullDiagrams"), then we need to give the indices and momenta of the fields as input.

D. Outputs

The main function takes two arguments: the setup and the list of field derivatives:

```
DeriveFunctionalEquation[Setup, DerivativeList];
```

The output is always a list of all diagrams that are produced. The specification of the diagrams can be altered with `options`.

1. *Options*

Options are called via:

```
DeriveFunctionalEquation[Setup, DerivativeList,
"OutputLevel" ->  options];
```

There are three options which specify the level of the output via "OutputLevel":

getDSE

The first option is to simply derive the Dyson-Schwinger equation via `getDSE`. The user needs to specify the classical vertices as well as at least one field derivative $\frac{\delta}{\delta\phi}$. From this the RHS of the DSE is computed according to the rules in section A. This means the classical action is written as:

$$S[\phi] = S^{a_1a_2}\phi_{a_1}\phi_{a_2} + S^{a_1a_2a_3}\phi_{a_1}\phi_{a_2}\phi_{a_3} + \cdots + S^{a_1\ldots a_n}\phi_{a_1}\ldots\phi_{a_n}, \tag{53}$$

where only those orders appear that are given in the theory. All prefactors (and signs) should be contained in the definition of $S^{(n)}$. Then the *getDSE* module computes:

$$\left.\frac{\delta S}{\delta\phi_i}\right|_{\phi_i=\Phi_i+G_{ij}\frac{\delta}{\delta\Phi_j}}. \tag{54}$$

Terms that end with a field derivative are immediately dropped. One then gets the general diagrams that contribute to the DSE for a given classical action. If more the field derivative list contains more than one entry, the last one is a field, whereas the others are already expectation values of fields.

```
DerivativeListDSE =
{Phi[a], Phi[b], Phi[c], Phi[d]}
```

The list above thus produces:

$$\Gamma^{abcd} = \frac{\delta^3}{\delta\Phi_a\Phi_b\Phi_c}\left(\frac{\delta S}{\delta\phi_d}\right)_{\phi_i=\Phi_i+G_{ij}\frac{\delta}{\delta\Phi_j}}. \tag{55}$$

Functional Derivatives

If the option is set to "FunctionalDerivatives" then the user obtains a list of diagrams which are generated by taking functional derivatives of the quantum master equation. The trace over fields in the diagrams is however not taken. Therefore one gets symbolic diagrams with fields set to zero. When choosing this option with a given master equation, the user is not required to specify a truncation.

Superindex Diagrams

The third option is called "SuperindexDiagrams". If this is chosen, the trace over fields is taken, the fields in the objects are sorted canonically. This means that upper indices (eg. for regulators, regulator derivatives or vertices, indcluding BRST-vertices) are sorted as (*bosonic*, *antifermionic*, *fermionic*) and lower indices in reverse order. If two indices have the same statistics, they are sorted alphabetically. Finally the prefactors are evaluated and only those diagrams remain which fulfill the truncation.

Full Diagrams

Lastly one may call the main function with the option "FullDiagrams", which means that in addition to the previous option also the momentum routing is done for all 1-loop diagrams (i.e. FRG, mSTI, but not for all DSE diagrams). Superfield indices are replaced by physical indices and objects are transformed into function of indices sucht that one can insert Feynman rules easily. This module only works for 1-loop diagrams of quantum master equations.

The default output level is equal to calling the "FunctionalDerivatives" option.

V. EXAMPLES

In this section we give different examples of deriving symbolic functional equations with *QMeS*.
The first example is deriving functional equations (i.e. FRG, mSTI and DSE) within Yang-Mills theory which serves as a prerequisite for QCD. Studying QCD with functional methods is an ab initio approach to investigate the non-perturbative regime.
Then we derive FRG equations in $N_f = 1$ and $N_f = 2$

Yukawa theory. It illustrates and emphasizes how *QMeS* handles multiple fermions and sorts the vertices accordingly. Furthermore a simple Yukawa model can already be used to describe nuclear forces between fermions which are mediated by pions thus approximating QCD with an effective field theory.

A. Yang-Mills theory

In the following we want to give the crucial steps one needs to take to compute functional equations in Yang-Mills theory with *QMeS*.

Definition of Theory:
The theory we work in is SU(3) Yang-Mills theory, thus one has bosonic gauge fields $A^i_\mu(p)$, fermionic ghosts $c^i(p)$ and antighosts $\bar{c}^i(p)$. Thus we can define the fields in *QMeS* as:

```
fieldsYM =
<|"bosonic" -> {A[p, {mu, i}]},
"fermionic" -> {{cbar[p, {i}], c[p, {i}]}}|>;
```

Note that fermions need to be defined as a pair of the fermion and corresponding antifermion.
Next we specify the truncation. It is important to also define the two-point functions in order to get the possible propagators .

```
TruncationYM = {{A, A}, {c, cbar}, {A, A, A},
{A, A, A, A}, {A, c, cbar}, {A, A, c, cbar},
{c, c, cbar, cbar}};
```

The classical Yang-Mills action is given by:

```
classicalActionYM = {{A, A}, {c, cbar},
{A, A, A}, {A, A, A, A}, {A, c, cbar}};
```

1. Flow of the gluon two-point function

To compute the flow of the gluon two-point function we need to define the Quantum Master, in this case the FRG equation (29). It translates to *QMeS* input as:

```
FRGEq = {"Prefactor" -> {1/2},
<|"type" -> "Regulatordot",
"indices" -> {i, j}|>,
<|"type" -> "Propagator",
"indices" -> {i, j}|>};
```

Now we can define the setup as:

```
SetupYMFRG = <|"MasterEquation" -> FRGEq,
"FieldSpace" -> fieldsYM,
"Truncation" -> TruncationYM|>;
```

The only thing that is missing is a specification of the field derivatives that we want to take.

```
DerivativeListAA =
{A[-p, {mu, a}], A[p, {nu, b}]};
```

Now we can derive symbolic diagrams. In general we have different output options IV D 1.
First we can take a look at the general structure of diagrams that are produced when taking two funcional derivatives with respect to the superfields Φ_a and Φ_b by calling the *QMeS* command `DeriveFunctionalEquation` with the output option `"OutputLevel" -> "FunctionalDerivatives"`. One then obtains:

$$\begin{aligned}\dot{\Gamma}^{ab} =& -\frac{1}{2}(-1)^{ia}(-1)^{ib}(-1)^{nn}\dot{R}^{ij}G_{im}\Gamma^{mabn}G_{nj} \\ &+\frac{1}{2}(-1)^{ia}(-1)^{ib}(-1)^{nn}(-1)^{n'n'} \\ &\dot{R}^{ij}G_{im}\Gamma^{man}G_{nm'}\Gamma^{m'bn'}G_{n'j} \\ &+\frac{1}{2}(-1)^{ia}(-1)^{ib}(-1)^{nn}(-1)^{n'n'}(-1)^{ab} \\ &\dot{R}^{ij}G_{im}\Gamma^{mbn}G_{nm'}\Gamma^{m'an'}G_{n'j}.\end{aligned} \tag{56}$$

One thus obtains a tadpole diagram and two diagrams with two three-point vertices respectively.
Next we want to get the fully traced diagrams by evaluating:

```
FRGDiagramsAA = DeriveFunctionalEquation[
SetupYMFRG, DerivativeListAA,
"OutputLevel" -> "FullDiagrams"];
```

As a result we obtain in superindex notation where now $a \simeq (-p, \mu, a)$ and $b \simeq (p, \nu, b)$ are superindices:

$$\begin{aligned}\dot{\Gamma}^{A_aA_b} =& -\dot{R}^{\bar{c}c}G_{c\bar{c}}\Gamma^{A_aA_b\bar{c}c}G_{c\bar{c}} \\ &-\frac{1}{2}\dot{R}^{AA}G_{AA}\Gamma^{AA_aA_bA}G_{AA} \\ &+\frac{1}{2}\dot{R}^{AA}G_{AA}\Gamma^{AA_aA}G_{AA}\Gamma^{AA_bA}G_{AA} \\ &+\frac{1}{2}\dot{R}^{AA}G_{AA}\Gamma^{AA_bA}G_{AA}\Gamma^{AA_aA}G_{AA} \\ &-\dot{R}^{\bar{c}c}G_{c\bar{c}}\Gamma^{A_a\bar{c}c}G_{c\bar{c}}\Gamma^{A_b\bar{c}c}G_{c\bar{c}} \\ &-\dot{R}^{\bar{c}c}G_{c\bar{c}}\Gamma^{A_b\bar{c}c}G_{c\bar{c}}\Gamma^{A_a\bar{c}c}G_{c\bar{c}}.\end{aligned} \tag{57}$$

The *QMeS* output is a list of different traced diagrams such that one can easily define and insert the Feynman rules for the different objects like propagators, regulators or vertices. It can be found in appendix B 1.

2. mSTI of gluon two-point function

To compute the mSTI of the gluon two-point function we need to alter our definition of fields and include the corresponding BRST sources.

```
fieldsYMmSTI = <|"bosonic" -> {A[p, {mu, i}]},
"fermionic" -> {{cbar[p, {i}], c[p, {i}]}},
"BRSTsources" -> {{Q[A], "fermionic"},
{Q[cbar], "bosonic"}, {Q[c], "bosonic"}}|>;
```

The truncation then also changed. The vertices on the right-hand side of the mSTI are truncated as:

```
TruncationYMRHSmSTI = {{A, A}, {c, cbar},
{A, A, A}, {A, A, A, A}, {A, c, cbar},
{A, c, Q[A]}, {c, c, Q[c]}};
```

and for the left-hand side one chooses:

```
TruncationYMLHSmSTI = {{A, A}, {c, cbar},
{A, A, A}, {A, A, A, A}, {A, c, cbar},
{A, Q[cbar]}, {c, Q[A]}, {A, c, Q[A]},
{c, c, Q[c]}};
```

Lastly we need to define the right- and left-hand side of the mSTI equation (49). In the *QMeS* formalism this is done by:

```
mSTIRHS = {"Prefactor" -> {1},
<|"type" -> "Regulator", "indices" -> {i, j}|>,
<|"type" -> "Propagator", "indices" -> {j, l}|>,
<|"type" -> "nPoint", "indices" -> {l, Q[i]},
"nPoint" -> 2, "spec" -> "BRST"|>};

mSTILHS = {"Prefactor" -> {1},
<|"type" -> "nPoint", "indices" -> {Q[i]},
"nPoint" -> 1, "spec" -> "BRST"|>,
<|"type" -> "nPoint", "indices" -> {i},
"nPoint" -> 1, "spec" -> "none"|>};
```

We define the two setups as:

```
SetupYMmSTIRHS = <|"MasterEquation" -> mSTIRHS,
"FieldSpace" -> fieldsYMmSTI,
"Truncation" -> TruncationYMRHSmSTI|>;

SetupYMmSTILHS = <|"MasterEquation" -> mSTILHS,
"FieldSpace" -> fieldsYMmSTI,
"Truncation" -> TruncationYMmSTILHS|>;
```

To obtain the mSTI of the gluon two-point function one needs to take derivatives with respect to the ghost and gluon field:

```
DerivativeListAAmSTI =
{A[-p, {mu, a}], c[p, {b}]};
```

One obtains the full mSTI by evaluating:

```
AAmSTIDiagramsLHS = DeriveFunctionalEquation[
SetupLHSmSTILHS, DerivativeListmSTI,
"OutputLevel" -> "FullDiagrams"];

AAmSTIDiagramsRHS = DeriveFunctionalEquation[
SetupmSTIRHS, DerivativeListmSTI,
"OutputLevel" -> "FullDiagrams"];
```

With the superindices $a \simeq (-p, \mu, a)$ and $b \simeq (p, b)$ the algebraic equations are then given as:

$$\begin{aligned}
{\Gamma^{c_b}}_{Q[A]}\Gamma^{AA_a} - {\Gamma^{A_a}}_{Q[\bar{c}]}\Gamma^{\bar{c}c_b} = {} & R^{AA}G_{AA}\Gamma^{A\bar{c}c_b}G_{c\bar{c}}{\Gamma^{A_a c}}_{Q[A]} \\
& - R^{AA}G_{AA}\Gamma^{AA_aA}G_{AA}{\Gamma^{Ac_b}}_{Q[A]} \\
& - R^{\bar{c}c}G_{c\bar{c}}\Gamma^{A_a\bar{c}c}G_{c\bar{c}}\Gamma_{Q[c]}{}^{cc_b},
\end{aligned} \tag{58}$$

where for the sake of brevity, indices and momenta are dropped. The output of *QMeS* is given in appendix B 2.

3. DSE of ghost-gluon vertex

In this subsection we derive the DSE for the ghost-gluon vertex. This can be done in two ways:

$$\begin{aligned}
\Gamma^{A\bar{c}c} &= \frac{\delta^2}{\delta A \delta \bar{c}} \left(\frac{\delta S}{\delta c} \right)_{\phi_a \to \Phi_a + G_{ab}\frac{\delta}{\delta \Phi_b}}, && \text{(A)} \\
\Gamma^{A\bar{c}c} = \Gamma^{\bar{c}cA} &= \frac{\delta^2}{\delta \bar{c} \delta c} \left(\frac{\delta S}{\delta A} \right)_{\phi_a \to \Phi_a + G_{ab}\frac{\delta}{\delta \Phi_b}}. && \text{(B)}
\end{aligned} \tag{59}$$

We define the setup as:

```
SetupYMDSE = <|"MasterEquation" -> <|"getDSE" ->
"True", "classicalAction" -> classicalActionYM|>,
"FieldSpace" -> fieldsYM,
"Truncation" -> TruncationYM|>;
```

A: For setup A we define the derivative list:

```
DerivativeListAcbarcDSEA = {A[p1, {mu, a}],
cbar[p2, {b}], c[-p1 - p2, {d}]};
```

We get the full result by using the command:

```
DSEDiagramsAcbarcA = DeriveFunctionalEquation[
SetupYMDSE, DerivativeListAcbarcDSEA,
"OutputLevel" -> "FullDiagrams"];
```

Diagrammatically the result is:

$$\begin{aligned}
\Gamma^{A_a\bar{c}_bc_d} = & - S^{A_a\bar{c}_bc_d} \\
& - S^{A\bar{c}c_d}G_{AA}\Gamma^{AA_a\bar{c}_bc}G_{c\bar{c}} \\
& + S^{A\bar{c}c_d}G_{AA}\Gamma^{AAA_a}G_{AA}\Gamma^{A\bar{c}_bc}G_{c\bar{c}} \\
& - S^{A\bar{c}c_d}G_{AA}\Gamma^{A\bar{c}_bc}G_{c\bar{c}}\Gamma^{A_a\bar{c}c}G_{c\bar{c}},
\end{aligned} \tag{60}$$

where we have used the superindices $a \simeq (p1, \mu, a)$, $b \simeq (p2, b)$ and $d \simeq (-p1 - p2, d)$.
As one can see one only obtains 1-loop diagrams with this setup.
The full equation can be found in appendix B 3.

B: For setup B we define the derivative list:

```
DerivativeListDSEB = {cbar[p2, {b}],
c[-p1 - p2, {d}], A[p1, {mu, a}]};
```

This produces not only 1-loop but also 2-loop diagrams:

$$\begin{aligned}
\dot{\Gamma}^{A_a\bar{c}_b c_d} &= -S^{A_a\bar{c}_b c_d}\\
&- 3S^{A_a AA}G_{AA}\Gamma^{AA\bar{c}_b c_d}G_{AA}\\
&- S^{A_a\bar{c}c}G_{c\bar{c}}\Gamma^{\bar{c}\bar{c}_b c_d c}G_{c\bar{c}}\\
&- 3\left(S^{A_a AA}G_{AA}\Gamma^{A\bar{c}_b c}G_{c\bar{c}}\Gamma^{A\bar{c}c_d}G_{AA}\ +\mathrm{perm}(\bar{c}_b\leftrightarrow c_d)\right)\\
&- S^{A_a\bar{c}c}G_{c\bar{c}}\Gamma^{A\bar{c}c_d}G_{AA}\Gamma^{A\bar{c}_b c}G_{c\bar{c}}\\
&+ 12S^{A_a AAA}G_{AA}G_{AA}\Gamma^{AAA}G_{AA}\Gamma^{AA\bar{c}_b c_d}G_{AA}\\
&- 12\left(S^{A_a AAA}G_{AA}G_{AA}\Gamma^{A\bar{c}_b c}G_{c\bar{c}}\Gamma^{AA\bar{c}c_d}G_{AA}\right.\\
&\quad\left.+\mathrm{perm}(\bar{c}_b\leftrightarrow c_d)\right)\\
&+ 12\left(S^{A_a AAA}G_{AA}G_{AA}\Gamma^{AAA}G_{AA}\Gamma^{A\bar{c}_b c}G_{c\bar{c}}\Gamma^{A\bar{c}c_d}G_{AA}\right.\\
&\quad\left.+\mathrm{perm}(\bar{c}_b\leftrightarrow c_d)\right)\\
&- 12\left(S^{A_a AAA}G_{AA}G_{AA}\Gamma^{A\bar{c}_b c}G_{c\bar{c}}\Gamma^{A\bar{c}c}G_{c\bar{c}}\Gamma^{A\bar{c}c_d}G_{AA}\right.\\
&\quad\left.+\mathrm{perm}(\bar{c}_b\leftrightarrow c_d)\right). \qquad (61)
\end{aligned}$$

The full output can again be found in the appendix B 3.

B. Yukawa theory

In this example we want to compute simple two-point flows in Yukawa theory. To further illustrate how *QMeS* handles multiple fermions we do this in $N_f = 1$ as well as $N_f = 2$. As a master equation we again use the FRG equation:

```
FRGEq = {"Prefactor" -> {1/2},
<|"type" -> "Regulatordot",
"indices" -> {i, j}|>,
<|"type" -> "Propagator",
"indices" -> {i, j}|>};
```

1. $N_f = 1$

For $N_f = 1$ we only have one flavour of fermions and thus only one antifermion/fermion pair in the definition of fields. Furthermore Yukawa theory also contains a scalar field, which thus has bosonic statistics.

```
fieldsNf1 = <|"bosonic" -> {Phi[p, {i}]},
"fermionic" ->
{{Psibar[p, {d, i}], Psi[p, {d, i}]}}|>;
```

The truncation is given as:

```
TruncationFRGNf1 = {{Phi, Phi}, {Psi, Psibar},
{Phi, Psi, Psibar}, {Phi, Phi, Phi, Phi},
{Psi, Psi, Psibar, Psibar}};
```

Thus we can summarize the setup:

```
SetupNf1 = <|"MasterEquation" -> FRGEq,
"FieldSpace" -> fieldsNf1,
"Truncation" -> TruncationFRGNf1|>;
```

Computing $\dot{\Gamma}^{\phi\phi}$:
To compute the flow of the scalar two-point function we define the list of derivatives as:

```
DerivativeListScalarTwopoint =
{Phi[-p, {a}], Phi[p, {b}]};
```

To get the full diagrams one has to run the command:

```
FRGDiagramsPhiPhiNf1 = DeriveFunctionalEquation[
SetupNf1, DerivativeListScalarTwopoint,
"OutputLevel" -> "FullDiagrams"];
```

The result with superindices $a \simeq (-p, a)$ and $b \simeq (p, b)$ is given as:

$$\begin{aligned}
\dot{\Gamma}^{\phi_a\phi_b} =& -\frac{1}{2}R^{\phi\phi}G_{\phi\phi}\Gamma^{\phi\phi\phi_a\phi_b}G_{\phi\phi}\\
&- R^{\bar{\psi}\psi}G_{\psi\bar{\psi}}\Gamma^{\phi_a\bar{\psi}\psi}G_{\psi\bar{\psi}}\Gamma^{\phi_b\bar{\psi}\psi}G_{\psi\bar{\psi}}\\
&- R^{\bar{\psi}\psi}G_{\psi\bar{\psi}}\Gamma^{\phi_b\bar{\psi}\psi}G_{\psi\bar{\psi}}\Gamma^{\phi_a\bar{\psi}\psi}G_{\psi\bar{\psi}}. \qquad (62)
\end{aligned}$$

The full output of *QMeS* is given in appendix B 4.

Computing $\dot{\Gamma}^{\bar{\psi}\psi}$:
The derivative list for the flow of the fermionic two-point is:

```
DerivativeListFermionTwopoint =
{Psibar[-p, {a}], Psi[p, {b}]};
```

The full diagrams can be obtained with:

```
FRGDiagramsPsibarPsiNf1 = DeriveFunctionalEquation[
SetupNf1, DerivativeListFermionTwopoint,
"OutputLevel" -> "FullDiagrams"];
```

The result with superindices is then given as:

$$\begin{aligned}
\dot{\Gamma}^{\bar{\psi}_a\psi_b} =& - R^{\bar{\psi}\psi}G_{\psi\bar{\psi}}\Gamma^{\bar{\psi}\bar{\psi}_a\psi_b\psi}G_{\psi\bar{\psi}}\\
&- \frac{1}{2}R^{\phi\phi}G_{\phi\phi}\Gamma^{\phi\bar{\psi}\psi_b}G_{\psi\bar{\psi}}\Gamma^{\phi\bar{\psi}_a\psi}G_{\phi\phi}\\
&- \frac{1}{2}R^{\phi\phi}G_{\phi\phi}\Gamma^{\phi\bar{\psi}_a\psi}G_{\psi\bar{\psi}}\Gamma^{\phi\bar{\psi}\psi_b}G_{\phi\phi}\\
&- \frac{1}{2}R^{\bar{\psi}\psi}G_{\psi\bar{\psi}}\Gamma^{\phi\bar{\psi}_a\psi}G_{\phi\phi}\Gamma^{\phi\bar{\psi}\psi_b}G_{\psi\bar{\psi}}\\
&- \frac{1}{2}R^{\bar{\psi}\psi}G_{\psi\bar{\psi}}\Gamma^{\phi\bar{\psi}\psi_b}G_{\phi\phi}\Gamma^{\phi\bar{\psi}_a\psi}G_{\psi\bar{\psi}}, \qquad (63)
\end{aligned}$$

where indices were again dropped.
The full output of *QMeS* is given in appendix B 4.

2. $N_f = 2$

Since we now want to include two flavours of fermions, we need to implement two antifermion/fermion pairs. For simplicity we call them $(\bar{\psi}_1, \psi_1)$ and $(\bar{\psi}_2, \psi_2)$.

```
fieldsNf2 =
<|"bosonic" -> {Phi[p, {i}]},
"fermionic" ->
{{Psibar1[p, {d, i}], Psi1[p, {d, i}]},
{Psibar2[p, {d, i}], Psi2[p, {d, i}]}}|>;
```

are the number of indices correct?
The truncation is then given by:

```
TruncationFRGNf2 = {{Phi, Phi}, {Psi1, Psibar1},
{Psi2, Psibar2}, {Phi, Psi1, Psibar1},
{Phi, Psi2, Psibar2}, {Phi, Phi, Phi, Phi},
{Psi1, Psi1, Psibar1, Psibar1},
{Psi2, Psi2, Psibar2, Psibar2},
{Psi1, Psi2, Psibar1, Psibar2}};
```

The setup is then given as:

```
SetupNf2 = <|"MasterEquation" -> FRGEq,
"FieldSpace" -> fieldsNf2,
"Truncation" -> TruncationFRGNf2|>;
```

Computing $\dot{\Gamma}^{\phi\phi}$:
As before we define the two scalar field derivatives as:

```
DerivativeListScalarTwopoint =
{Phi[-p, {a}], Phi[p, {b}]};
```

To get the full diagrams one has to run the command:

```
FRGDiagramsPhiPhiNf2 = DeriveFunctionalEquation[
SetupNf2, DerivativeListScalarTwopoint,
"OutputLevel" -> "FullDiagrams"];
```

The result in superindex notation is given as:

$$
\begin{aligned}
\dot{\Gamma}^{\phi_a\phi_b} = &-\frac{1}{2}R^{\phi\phi}G_{\phi\phi}\Gamma^{\phi\phi\phi_a\phi_b}G_{\phi\phi} \\
&- R^{\bar{\psi}_1\psi_1}G_{\psi_1\bar{\psi}_1}\Gamma^{\phi_a\bar{\psi}_1\psi_1}G_{\psi_1\bar{\psi}_1}\Gamma^{\phi_b\bar{\psi}_1\psi_1}G_{\psi_1\bar{\psi}_1} \\
&- R^{\bar{\psi}_1\psi_1}G_{\psi_1\bar{\psi}_1}\Gamma^{\phi_b\bar{\psi}_1\psi_1}G_{\psi_1\bar{\psi}_1}\Gamma^{\phi_a\bar{\psi}_1\psi_1}G_{\psi_1\bar{\psi}_1} \\
&- R^{\bar{\psi}_2\psi_2}G_{\psi_2\bar{\psi}_2}\Gamma^{\phi_a\bar{\psi}_2\psi_2}G_{\psi_2\bar{\psi}_2}\Gamma^{\phi_b\bar{\psi}_2\psi_2}G_{\psi_2\bar{\psi}_2} \\
&- R^{\bar{\psi}_2\psi_2}G_{\psi_2\bar{\psi}_2}\Gamma^{\phi_b\bar{\psi}_2\psi_2}G_{\psi_2\bar{\psi}_2}\Gamma^{\phi_a\bar{\psi}_2\psi_2}G_{\psi_2\bar{\psi}_2} .
\end{aligned} \tag{64}
$$

The full output of *QMeS* is given in appendix B 5.

Computing $\dot{\Gamma}^{\bar{\psi}_1\psi_1}$:
The derivatives with respect to the first antifermionic and fermionic fields is given as:

```
DerivativeListFermion1Twopoint =
{Psibar1[-p, {a}], Psi1[p, {b}]};
```

The full diagrams can be obtained with:

```
FRGDiagramsPsibar1Psi1Nf2 = DeriveFunctionalEquation[
SetupNf2, DerivativeListFermion1Twopoint,
"OutputLevel" -> "FullDiagrams"];
```

The result in superindex notation is then given as:

$$
\begin{aligned}
\dot{\Gamma}^{\bar{\psi}_{1a}\psi_{1b}} = &- R^{\bar{\psi}_1\psi_1}G_{\psi_1\bar{\psi}_1}\Gamma^{\bar{\psi}_1\bar{\psi}_{1a}\psi_{1b}\psi_1}G_{\psi_1\bar{\psi}_1} \\
&+ R^{\bar{\psi}_2\psi_2}G_{\psi_2\bar{\psi}_2}\Gamma^{\bar{\psi}_{1a}\bar{\psi}_2\psi_{1b}\psi_2}G_{\psi_2\bar{\psi}_2} \\
&- \frac{1}{2}R^{\phi\phi}G_{\phi\phi}\Gamma^{\phi\bar{\psi}_{1a}\psi_1}G_{\psi_1\bar{\psi}_1}\Gamma^{\phi\bar{\psi}_1\psi_{1b}}G_{\phi\phi} \\
&- \frac{1}{2}R^{\phi\phi}G_{\phi\phi}\Gamma^{\phi\bar{\psi}_1\psi_{1b}}G_{\psi_1\bar{\psi}_1}\Gamma^{\phi\bar{\psi}_{1a}\psi_1}G_{\phi\phi} \\
&- \frac{1}{2}R^{\bar{\psi}_1\psi_1}G_{\psi_1\bar{\psi}_1}\Gamma^{\phi\bar{\psi}_{1a}\psi_1}G_{\phi\phi}\Gamma^{\phi\bar{\psi}_1\psi_{1b}}G_{\psi_1\bar{\psi}_1} \\
&- \frac{1}{2}R^{\bar{\psi}_1\psi_1}G_{\psi_1\bar{\psi}_1}\Gamma^{\phi\bar{\psi}_1\psi_{1b}}G_{\phi\phi}\Gamma^{\phi\bar{\psi}_{1a}\psi_1}G_{\psi_1\bar{\psi}_1} .
\end{aligned} \tag{65}
$$

Here one can see how the canonical sorting of fields in vertices is followed by an alphabetical one.
The full output of *QMeS* is given in appendix B 5.

VI. CONCLUSION

We presented the *mathematica* package *QMeS*. It allows to derive symbolic functional equations from a master equation (FRG, mSTI, DSE). This includes taking functional derivatives, tracing in field space and a momentum routing for 1-loop diagrams. One of the most notable features is that during this process *QMeS* is able to deal with fermionic signs effectively and consistently. We furthermore gave examples, namely Yang-Mills and Yukawa theory which demonstrate the usage of *QMeS*.

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Appendix A: Summary of derivative rules

The relevant derivative and sign rules which are used in *QMeS* can be summarized as:

$R^{ab} = (-1)^{ab} R^{ba}$, $\quad G_{ab} = (-1)^{ab} G_{ba}$, $\quad \Gamma^{ab} = (-1)^{ab} \Gamma^{ba}$,

$\frac{\delta}{\delta \Phi_a} \frac{\delta}{\delta \Phi_b} O = (-1)^{ab} \frac{\delta}{\delta \Phi_b} \frac{\delta}{\delta \Phi_a} O$,

$\frac{\delta}{\delta \phi_a} R^{bc} = 0$, $\quad \frac{\delta}{\delta \phi_a} \phi_b = \delta_{ab}$, $\quad \frac{\delta}{\delta \phi_a} S^{bcd} = 0$,

$\frac{\delta}{\delta \Phi_a} R^{bc} = 0$, , $\quad \frac{\delta}{\delta \Phi_a} \Phi_b = \delta_{ab}$, $\quad \frac{\delta}{\delta \Phi_a} S^{bcd} = 0$,

$\frac{\delta}{\delta \Phi_a} \Gamma^{b\ldots n} = \Gamma^{ab\ldots n}$, $\quad \frac{\delta}{\delta \Phi_a} G_{bc} = (-1)(-1)^{ab}(-1)^{ee} G_{bd} \Gamma^{dae} G_{ec}$.

Appendix B: Results of the examples

1. YM: Flow of gluon two-point function

The full diagrams of the flow of the gluon two-point function with all indices are:

```
FRGDiagramsAA =

{{1/2 GAA[{-p - q, mu$6522, i$6522, p + q, mu$6517, i$6517}]
GAA[{-q, mu$6511, i$6511, q, mu$6504, i$6504}]
GAA[{q, mu$6528, i$6528, -q, mu$6507, i$6507}]
RdotAA[{q, mu$6504, i$6504, -q, mu$6507, i$6507}]
\[CapitalGamma]AAA[{-p - q, mu$6522, i$6522, q, mu$6528, i$6528, p, nu, b}]
\[CapitalGamma]AAA[{-q, mu$6511, i$6511, p + q, mu$6517, i$6517, -p, mu, a}]},

{-(1/2) Gccbar[{q, i$6569, -q, i$6574}]
Gccbar[{q, i$6591, -q, i$6566}]
Gccbar[{p + q, i$6580, -p - q, i$6585}]
Rdotcbarc[{-q, i$6566, q, i$6569}]
\[CapitalGamma]Acbarc[{-p, mu, a, -q, i$6574, p + q, i$6580}]
\[CapitalGamma]Acbarc[{p, nu, b, -p - q, i$6585,nq, i$6591}]},

{-(1/2) Gccbar[{-p - q, i$6642, p + q, i$6637}]
Gccbar[{-q, i$6627, q, i$6648}]
Gccbar[{-q, i$6631, q, i$6624}]
Rdotcbarc[{q, i$6624, -q, i$6627}]
\[CapitalGamma]Acbarc[{-p, mu, a, p + q, i$6637, -q, i$6631}]
\[CapitalGamma]Acbarc[{p, nu, b, q, i$6648, -p - q, i$6642}]},

{-(1/2) GAA[{-q, mu$6695, i$6695, q, mu$6685, i$6685}]
GAA[{q, mu$6689, i$6689, -q, mu$6682, i$6682}]
RdotAA[{-q, mu$6682, i$6682, q, mu$6685, i$6685}]
\[CapitalGamma]AAAA[{q, mu$6689, i$6689, -q, mu$6695, i$6695, -p, mu, a, p, nu, b}]},

{-(1/2) Gccbar[{-q, i$6725, q, i$6730}]
Gccbar[{-q, i$6737, q, i$6722}]
Rdotcbarc[{q, i$6722, -q, i$6725}]
\[CapitalGamma]AAcbarc[{-p, mu, a, p, nu, b, q, i$6730, -q, i$6737}]},

{-(1/2) Gccbar[{q, i$6765, -q, i$6776}]
Gccbar[{q, i$6769, -q, i$6762}]
```

```
Rdotcbarc[{-q, i$6762, q, i$6765}]
\[CapitalGamma]AAcbarc[{-p, mu, a, p, nu, b, -q, i$6776, q, i$6769}]},

{1/2 GAA[{-p - q, mu$6815, i$6815, p + q, mu$6821, i$6821}]
GAA[{-q, mu$6826, i$6826, q, mu$6805, i$6805}]
GAA[{q, mu$6809, i$6809, -q, mu$6802, i$6802}]
RdotAA[{-q, mu$6802, i$6802, q, mu$6805, i$6805}]
\[CapitalGamma]AAA[{q, mu$6809, i$6809, -p - q, mu$6815, i$6815, p, nu, b}]
\[CapitalGamma]AAA[{p + q, mu$6821, i$6821, -q, mu$6826, i$6826, -p, mu, a}]},

{-(1/2) Gccbar[{-p - q, i$6874, p + q, i$6879}]
Gccbar[{-q, i$6863, q, i$6868}]
Gccbar[{-q, i$6885, q, i$6860}]
Rdotcbarc[{q, i$6860, -q, i$6863}]
\[CapitalGamma]Acbarc[{-p, mu, a, p + q, i$6879, -q, i$6885}]
 \[CapitalGamma]Acbarc[{p, nu, b, q, i$6868, -p - q, i$6874}]},

{-(1/2) Gccbar[{q, i$6921, -q, i$6942}]
Gccbar[{q, i$6925, -q, i$6918}]
Gccbar[{p + q, i$6936, -p - q, i$6931}]
Rdotcbarc[{-q, i$6918, q, i$6921}]
\[CapitalGamma]Acbarc[{-p, mu, a, -q, i$6942, p + q, i$6936}]
\[CapitalGamma]Acbarc[{p, nu, b, -p - q, i$6931, q, i$6925}]}}
```

2. YM: mSTI of gluon two-point function

The left-hand side of the mSTI is given as:

```
AAmSTIDiagramsLHS =

{{\[CapitalGamma]AA[{p, mu$3486, i$3486, -p, mu, a}]
\[CapitalGamma]cQA[{p, b, -p, mu$3486, i$3486}]},

{-\[CapitalGamma]AQcbar[{-p, mu, a, p, i$3507}]
\[CapitalGamma]cbarc[{-p, i$3507, p, b}]}}
```

and the right-hand side:

```
AAmSTIDiagramsRHS =

{{GAA[{q, mu$7871, i$7871, -q, mu$7867, i$7867}]
Gccbar[{p + q, i$7882, -p - q, i$7877}]
RAA[{-q, mu$7864, i$7864, -q, mu$7867, i$7867}]
\[CapitalGamma]Acbarc[{q, mu$7871, i$7871, -p - q, i$7877, p, b}]
\[CapitalGamma]AcQA[{-p, mu, a, p + q, i$7882, -q, mu$7864, i$7864}]},

{-GAA[{p - q, mu$7926, i$7926, -p + q, mu$7932, i$7932}]
GAA[{q, mu$7920, i$7920, -q, mu$7916, i$7916}]
RAA[{-q, mu$7913, i$7913, -q, mu$7916, i$7916}]
\[CapitalGamma]AAA[{q, mu$7920, i$7920, p - q, mu$7926, i$7926, -p, mu, a}]
\[CapitalGamma]AcQA[{-p + q, mu$7932, i$7932, p, b, -q, mu$7913, i$7913}]},

{-Gccbar[{q, i$7969, -q, i$7962}]
Gccbar[{-p + q, i$7980, p - q, i$7975}]
Rcbarc[{-q, i$7962, -q, i$7965}]
\[CapitalGamma]Acbarc[{-p, mu, a, p - q, i$7975, q, i$7969}]
\[CapitalGamma]Qccc[{-q, i$7965, -p + q, i$7980, p, b}]}}
```

3. YM: DSE of ghost-gluon vertex

The full result of setup (A) is:

```
DSEDiagramsAcbarcA =

{{-SAcbarc[{p1, mu, a, p2, n, -p1 - p2, b}]},

{GAA[{p1 - q, mu$12329, i$12329, -p1 + q, mu$12338, i$12338}]
GAA[{q, mu$12348, i$12348, -q, mu$12343, i$12343}]
Gccbar[{-p2 - q, i$12355, p2 + q, i$12332}]
SAcbarc[{p1 - q, mu$12329, i$12329, p2 + q, i$12332, -p1 - p2, d}] \[CapitalGamma]AAA[{-p1 + q, mu$12338, i
\[CapitalGamma]Acbarc[{q, mu$12348, i$12348, p2, b, -p2 - q, i$12355}]},

{-GAA[{p1 + p2 - q, mu$12388, i$12388, -p1 - p2 + q, mu$12397, i$12397}] Gccbar[{-q, i$12404, q, i$12391}]
SAcbarc[{p1 + p2 - q, mu$12388, i$12388, q, i$12391, -p1 - p2, d}]
\[CapitalGamma]AAcbarc[{-p1 - p2 + q, mu$12397, i$12397, p1, mu, a, p2, b, -q, i$12404}]},

{-GAA[{p1 + p2 - q, mu$12429, i$12429, -p1 - p2 + q, mu$12438, i$12438}] Gccbar[{p1 - q, i$12444, -p1 + q,
Gccbar[{-q, i$12455, q, i$12432}]
SAcbarc[{p1 + p2 - q, mu$12429, i$12429, q, i$12432, -p1 - p2, d}]
\[CapitalGamma]Acbarc[{p1, mu, a, -p1 + q, i$12449, -q, i$12455}]
\[CapitalGamma]Acbarc[{-p1 - p2 + q, mu$12438, i$12438, p2, b, p1 - q, i$12444}]}}
```

The full result of setup (B) is:

```
DSEDiagramsAcbarcB =
```

the output is super long (96 diags). do we really want/need that?

4. Yukawa $N_f = 1$: Flow of two-point functions

The flow of the scalar two-poing function is:

```
FRGDiagramsPhiPhiNf1 =

{{-(1/2) GPsiPsibar[{q, i$37675, -q, i$37680}]
GPsiPsibar[{q, i$37697, -q, i$37672}]
GPsiPsibar[{p + q, i$37686, -p - q, i$37691}]
RdotPsibarPsi[{-q, i$37672, q, i$37675}]
\[CapitalGamma]PhiPsibarPsi[{-p, a, -q, i$37680, p + q, i$37686}]
\[CapitalGamma]PhiPsibarPsi[{p, b, -p - q, i$37691, q, i$37697}]},

{-(1/2) GPsiPsibar[{-p - q, i$37748, p + q, i$37743}]
GPsiPsibar[{-q, i$37733, q, i$37754}]
GPsiPsibar[{-q, i$37737, q, i$37730}]
RdotPsibarPsi[{q, i$37730, -q, i$37733}]
\[CapitalGamma]PhiPsibarPsi[{-p, a, p + q, i$37743, -q, i$37737}]
\[CapitalGamma]PhiPsibarPsi[{p, b, q, i$37754, -p - q, i$37748}]},

{-(1/2) GPhiPhi[{-q, i$37801, q, i$37791}]
GPhiPhi[{q, i$37795, -q, i$37788}]
RdotPhiPhi[{-q, i$37788, q, i$37791}]
\[CapitalGamma]PhiPhiPhiPhi[{q, i$37795, -q, i$37801, -p, a, p, b}]},

{-(1/2) GPsiPsibar[{-p - q, i$37842, p + q, i$37847}]
GPsiPsibar[{-q, i$37831, q, i$37836}]
GPsiPsibar[{-q, i$37853, q, i$37828}]
RdotPsibarPsi[{q, i$37828, -q, i$37831}]
\[CapitalGamma]PhiPsibarPsi[{-p, a, p + q, i$37847, -q, i$37853}]
\[CapitalGamma]PhiPsibarPsi[{p, b, q, i$37836, -p - q, i$37842}]},

{-(1/2) GPsiPsibar[{q, i$37889, -q, i$37910}]
GPsiPsibar[{q, i$37893, -q, i$37886}]
GPsiPsibar[{p + q, i$37904, -p - q, i$37899}]
RdotPsibarPsi[{-q, i$37886, q, i$37889}]
\[CapitalGamma]PhiPsibarPsi[{-p, a, -q, i$37910, p + q, i$37904}]
\[CapitalGamma]PhiPsibarPsi[{p, b, -p - q, i$37899, q, i$37893}]}}
```

The flow of the fermionic two-point function with all indices is:

```
FRGDiagramsPsibarPsiNf1 =

{{-(1/2) GPhiPhi[{-q, i$38630, q, i$38623}]
GPhiPhi[{q, i$38646, -q, i$38626}]
GPsiPsibar[{p + q, i$38637, -p - q, i$38642}]
RdotPhiPhi[{q, i$38623, -q, i$38626}]
\[CapitalGamma]PhiPsibarPsi[{-q, i$38630, -p, a, p + q, i$38637}]
\[CapitalGamma]PhiPsibarPsi[{q, i$38646, -p - q, i$38642, p, b}]},

{-(1/2) GPhiPhi[{-p - q, i$38699, p + q, i$38693}]
GPsiPsibar[{-q, i$38684, q, i$38705}]
GPsiPsibar[{-q, i$38688, q, i$38681}]
RdotPsibarPsi[{q, i$38681, -q, i$38684}]
\[CapitalGamma]PhiPsibarPsi[{-p - q, i$38699, q, i$38705, p, b}]
\[CapitalGamma]PhiPsibarPsi[{p + q, i$38693, -p, a, -q, i$38688}]},

{-(1/2) GPsiPsibar[{-q, i$38742, q, i$38747}]
GPsiPsibar[{-q, i$38754, q, i$38739}]
RdotPsibarPsi[{q, i$38739, -q, i$38742}]
\[CapitalGamma]PsibarPsibarPsiPsi[{q, i$38747, -p, a, p, b, -q, i$38754}]},

{-(1/2) GPsiPsibar[{q, i$38782, -q, i$38792}]
GPsiPsibar[{q, i$38786, -q, i$38779}]
RdotPsibarPsi[{-q, i$38779, q, i$38782}]
\[CapitalGamma]PsibarPsibarPsiPsi[{-p, a, -q, i$38792, q, i$38786, p, b}]},

{-(1/2) GPhiPhi[{-q, i$38842, q, i$38822}]
GPhiPhi[{q, i$38826, -q, i$38819}]
GPsiPsibar[{p + q, i$38837, -p - q, i$38832}]
RdotPhiPhi[{-q, i$38819, q, i$38822}]
\[CapitalGamma]PhiPsibarPsi[{-q, i$38842, -p, a, p + q, i$38837}]
\[CapitalGamma]PhiPsibarPsi[{q, i$38826, -p - q, i$38832, p, b}]},

{-(1/2) GPhiPhi[{-p - q, i$38889, p + q, i$38896}]
GPsiPsibar[{-q, i$38880, q, i$38885}]
GPsiPsibar[{-q, i$38902, q, i$38877}]
RdotPsibarPsi[{q, i$38877, -q, i$38880}]
\[CapitalGamma]PhiPsibarPsi[{-p - q, i$38889, q, i$38885, p, b}]
\[CapitalGamma]PhiPsibarPsi[{p + q, i$38896, -p, a, -q, i$38902}]}}
```

5. Yukawa $N_f = 2$: Flow of two-point functions

The flow of the scalar two-point in $N_f = 2$ Yukawa theory is:

```
FRGDiagramsPhiPhiNf2 =

{{-(1/2) GPsi1Psibar1[{q, i$43500, -q, i$43505}]
GPsi1Psibar1[{q, i$43522, -q, i$43497}]
GPsi1Psibar1[{p + q, i$43511, -p - q, i$43516}]
RdotPsibar1Psi1[{-q, i$43497, q, i$43500}]
\[CapitalGamma]PhiPsibar1Psi1[{-p, a, -q, i$43505, p + q, i$43511}]
\[CapitalGamma]PhiPsibar1Psi1[{p, b, -p - q, i$43516, q, i$43522}]},

{-(1/2) GPsi2Psibar2[{q, i$43558, -q, i$43563}]
GPsi2Psibar2[{q, i$43580, -q, i$43555}]
GPsi2Psibar2[{p + q, i$43569, -p - q, i$43574}]
RdotPsibar2Psi2[{-q, i$43555, q, i$43558}]
\[CapitalGamma]PhiPsibar2Psi2[{-p, a, -q, i$43563, p + q, i$43569}]
\[CapitalGamma]PhiPsibar2Psi2[{p, b, -p - q, i$43574, q, i$43580}]},

{-(1/2) GPsi1Psibar1[{-p - q, i$43631, p + q, i$43626}]
GPsi1Psibar1[{-q, i$43616, q, i$43637}]
GPsi1Psibar1[{-q, i$43620, q, i$43613}]
RdotPsibar1Psi1[{q, i$43613, -q, i$43616}]
\[CapitalGamma]PhiPsibar1Psi1[{-p, a, p + q, i$43626, -q, i$43620}]
\[CapitalGamma]PhiPsibar1Psi1[{p, b, q, i$43637, -p - q, i$43631}]},

{-(1/2) GPsi2Psibar2[{-p - q, i$43689, p + q, i$43684}]
GPsi2Psibar2[{-q, i$43674, q, i$43695}]
GPsi2Psibar2[{-q, i$43678, q, i$43671}]
RdotPsibar2Psi2[{q, i$43671, -q, i$43674}]
\[CapitalGamma]PhiPsibar2Psi2[{-p, a, p + q, i$43684, -q, i$43678}]
\[CapitalGamma]PhiPsibar2Psi2[{p, b, q, i$43695, -p - q, i$43689}]},

{-(1/2) GPhiPhi[{-q, i$43742, q, i$43732}]
GPhiPhi[{q, i$43736, -q, i$43729}]
RdotPhiPhi[{-q, i$43729, q, i$43732}]
\[CapitalGamma]PhiPhiPhiPhi[{q, i$43736, -q, i$43742, -p, a, p, b}]},

{-(1/2) GPsi1Psibar1[{-p - q, i$43783, p + q, i$43788}]
GPsi1Psibar1[{-q, i$43772, q, i$43777}]
GPsi1Psibar1[{-q, i$43794, q, i$43769}]
RdotPsibar1Psi1[{q, i$43769, -q, i$43772}]
\[CapitalGamma]PhiPsibar1Psi1[{-p, a, p + q, i$43788, -q, i$43794}]
\[CapitalGamma]PhiPsibar1Psi1[{p, b, q, i$43777, -p - q, i$43783}]},

{-(1/2) GPsi2Psibar2[{-p - q, i$43841, p + q, i$43846}]
GPsi2Psibar2[{-q, i$43830, q, i$43835}]
GPsi2Psibar2[{-q, i$43852, q, i$43827}]
RdotPsibar2Psi2[{q, i$43827, -q, i$43830}]
\[CapitalGamma]PhiPsibar2Psi2[{-p, a, p + q, i$43846, -q, i$43852}]
\[CapitalGamma]PhiPsibar2Psi2[{p, b, q, i$43835, -p - q, i$43841}]},

{-(1/2) GPsi1Psibar1[{q, i$43888, -q, i$43909}]
GPsi1Psibar1[{q, i$43892, -q, i$43885}]
GPsi1Psibar1[{p + q, i$43903, -p - q, i$43898}]
RdotPsibar1Psi1[{-q, i$43885, q, i$43888}]
\[CapitalGamma]PhiPsibar1Psi1[{-p, a, -q, i$43909, p + q, i$43903}]
```

```
\[CapitalGamma]PhiPsibar1Psi1[{p, b, -p - q, i$43898, q, i$43892}]},

{-(1/2) GPsi2Psibar2[{q, i$43946, -q, i$43967}]
GPsi2Psibar2[{q, i$43950, -q, i$43943}]
GPsi2Psibar2[{p + q, i$43961, -p - q, i$43956}]
RdotPsibar2Psi2[{-q, i$43943, q, i$43946}]
\[CapitalGamma]PhiPsibar2Psi2[{-p, a, -q, i$43967, p + q, i$43961}]
\[CapitalGamma]PhiPsibar2Psi2[{p, b, -p - q, i$43956, q, i$43950}]}}
```

The flow of the fermionic two-point function in $N_f = 2$ Yukawa theory is:

```
FRGDiagramsPsibarPsiNf2 =

{{-(1/2) GPhiPhi[{-q, i$46269, q, i$46262}]
GPhiPhi[{q, i$46285, -q, i$46265}]
GPsi1Psibar1[{p + q, i$46276, -p - q, i$46281}]
RdotPhiPhi[{q, i$46262, -q, i$46265}]
\[CapitalGamma]PhiPsibar1Psi1[{-q, i$46269, -p, a, p + q, i$46276}]
\[CapitalGamma]PhiPsibar1Psi1[{q, i$46285, -p - q, i$46281, p, b}]},

{-(1/2) GPhiPhi[{-p - q, i$46338, p + q, i$46332}]
GPsi1Psibar1[{-q, i$46323, q, i$46344}]
GPsi1Psibar1[{-q, i$46327, q, i$46320}]
RdotPsibar1Psi1[{q, i$46320, -q, i$46323}]
\[CapitalGamma]PhiPsibar1Psi1[{-p - q, i$46338, q, i$46344, p, b}]
\[CapitalGamma]PhiPsibar1Psi1[{p + q, i$46332, -p, a, -q, i$46327}]},

{-(1/2) GPsi1Psibar1[{-q, i$46381, q, i$46386}]
GPsi1Psibar1[{-q, i$46393, q, i$46378}]
RdotPsibar1Psi1[{q, i$46378, -q, i$46381}]
\[CapitalGamma]Psibar1Psibar1Psi1Psi1[{q, i$46386, -p, a, p, b, -q, i$46393}]},

{1/2 GPsi2Psibar2[{-q, i$46421, q, i$46426}]
GPsi2Psibar2[{-q, i$46433, q, i$46418}]
RdotPsibar2Psi2[{q, i$46418, -q, i$46421}]
\[CapitalGamma]Psibar1Psibar2Psi1Psi2[{-p, a, q, i$46426, p, b, -q, i$46433}]},

{-(1/2) GPsi1Psibar1[{q, i$46461, -q, i$46471}]
GPsi1Psibar1[{q, i$46465, -q, i$46458}]
RdotPsibar1Psi1[{-q, i$46458, q, i$46461}]
\[CapitalGamma]Psibar1Psibar1Psi1Psi1[{-p, a, -q, i$46471, q, i$46465, p, b}]},

{1/2 GPsi2Psibar2[{q, i$46501, -q, i$46511}]
GPsi2Psibar2[{q, i$46505, -q, i$46498}]
RdotPsibar2Psi2[{-q, i$46498, q, i$46501}]
\[CapitalGamma]Psibar1Psibar2Psi1Psi2[{-p, a, -q, i$46511, p, b, q, i$46505}]},

{-(1/2) GPhiPhi[{-q, i$46561, q, i$46541}]
GPhiPhi[{q, i$46545, -q, i$46538}]
GPsi1Psibar1[{p + q, i$46556, -p - q, i$46551}]
RdotPhiPhi[{-q, i$46538, q, i$46541}]
\[CapitalGamma]PhiPsibar1Psi1[{-q, i$46561, -p, a, p + q, i$46556}]
\[CapitalGamma]PhiPsibar1Psi1[{q, i$46545, -p - q, i$46551, p, b}]},

{-(1/2) GPhiPhi[{-p - q, i$46608, p + q, i$46615}]
GPsi1Psibar1[{-q, i$46599, q, i$46604}]
GPsi1Psibar1[{-q, i$46621, q, i$46596}]
RdotPsibar1Psi1[{q, i$46596, -q, i$46599}]
\[CapitalGamma]PhiPsibar1Psi1[{-p - q, i$46608, q, i$46604, p, b}]
\[CapitalGamma]PhiPsibar1Psi1[{p + q, i$46615, -p, a, -q, i$46621}]}}
```

[1] M. Q. Huber and J. Braun, Comput.Phys.Commun. **183**, 1290 (2012), arXiv:1102.5307 [hep-th].
[2] J. Vermaseren, (2000), arXiv:math-ph/0010025 [math-ph].