Chapter 5

Tutorial on Interventions and Explanatory Variables

5.1 Interventions

Intervention variables are dummy (or indicator) variables which are used to take account of outlying observations and structural breaks. These data irregularities are usually thought of as arising from a specific event, for example a strike in the case of an outlier or a change in policy in the case of a structural break.

An *outlier* can be thought of as an unusually large value of the *irregular* disturbance at a particular time. It can be captured by an *impulse* intervention variable which takes the value one at the time of the outlier and zero elsewhere.

A *structural break* in which the level of the series shifts up or down is modelled by a *step* intervention variable which is zero before the event and one after. Alternatively it can be modelled in exactly the same way by adding an outlying intervention to the *level* equation. In other words the break is identified with an unusually large value of the level disturbance.

A *structural break in the slope* can be modelled by a *staircase* intervention which is a trend variable taking the values, $1, 2, 3, \ldots$, starting in the period after the break. It can be thought of as arising from a large value of the *slope* disturbance.

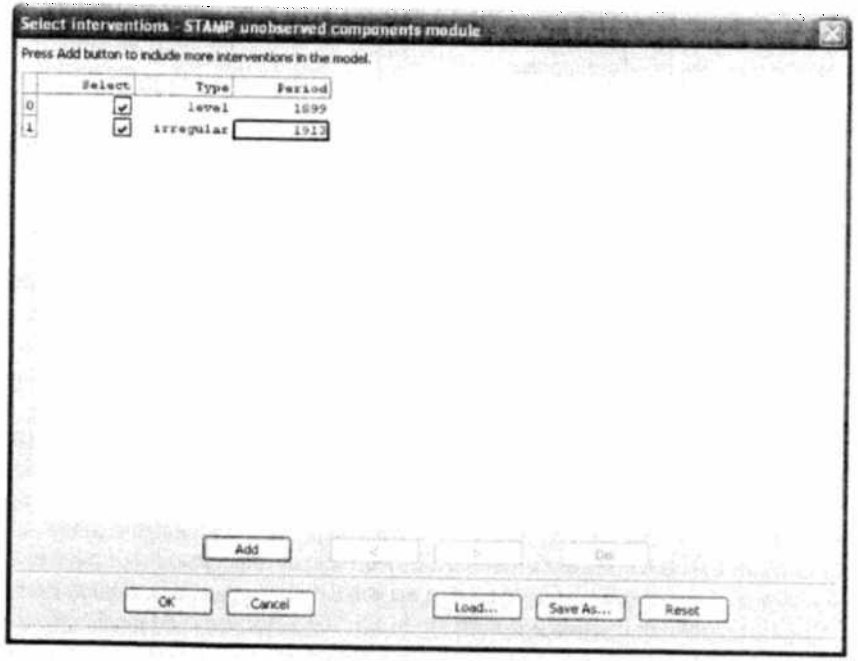
The easiest way to include intervention variables in the model is by by choosing the option 'Select interventions automatically' using the appropriate radio-button in section 'Options' of the Select a model dialog. The program selects the appropriate outliers and breaks in the time series using an intellegent procedure and described below. Alternatively, the interventions can be inputted manually by accessing the Intervention dialog. This is done from the Select components dialog by pressing the radio-button 'Select interventions manually' in section 'Options'. Level (step) and slope (staircase) interventions formed in this way have their effects included in the trend. The selected interventions can be saved for future use. Alternatively, intervention variables can be created as explanatory variables by entering them directly in the database or by using the Calculator. Interventions formed in this way can be saved in the database for future use.

The next subsection will use the database NILE.IN7.

5.1.1 Modelling

The series contained in NILE is of the volume of the flow of the river, in cubic metres ($\times 10^8$) in the years from 1871 to 1970. We will model this as a fixed level plus a cycle plus an irregular with a structural break in 1899, corresponding to the building of a dam at Aswan and an outlier in 1913. The route by which we might have arrived at such a model is explored in the next section. Our methodology is very easy to implement — it is instructive to compare it with the ARIMA approach described in Balke (1993).

Having loaded the data, go to the Formulate dialog in the Model menu (`Alt+y`), mark 'Nile', press << and, finally, press OK to move to Select components. Enter the specification suggested above in the standard way by selecting 'Level, Fixed', deselecting 'Slope' and selecting 'Medium cycle' in the 'Cycle(s)' section. To access the Intervention dialog, mark 'Select interventions manually' in the 'Options' section of the Select components dialog and press OK:



This dialog has one entry as a default. Entries can be added by clicking on the 'Add' button. Put in the break by selecting the entry in the 'Select' column, select 'Level' in the drop-box of the second column 'Type' and input the year 1899 in the

'Period' column. The level break for 1899 has become a part of the trend component. The outlier in 1913 can be inputted in the same way but selecting the 'Irregular' in the 'Type' column and 1913 in the 'Period' column. (It will be also useful to note that you can clear, from the present model, the interventions by deselecting the intervention in the 'Select' column.) Once the interventions have been added, click on OK and estimate the model.

A part of the default output is given by:

```
Regression effects in final state at time 1970
                     Coefficient        RMSE     t-value      Prob
Level break 1899. 1  -243.28229     30.02331    -8.10311 [0.00000]
Outlier 1913. 1      -377.86556    119.51527    -3.16165 [0.00209]
```

As can be seen, both interventions are statistically significant when judged against t-distributions. The two-sided probability values are shown in square brackets. Although a t-distribution is, strictly speaking, only valid if the (relative) variances are known, it provides a good guide in practice.

Now move to the **Test**/Component graphics dialog and keep the default sets of graphs. An inspection of 'Trend plus Regression effects' and 'Fixed intervention effects' will show a clear shift in the level after 1899, as the step intervention has been absorbed into the trend component.

Finally enter the **Test**/Forecasting dialog and select 'Signal' and 'Trend plus Regression effects'. Note that the forecast of the signal continues at the lower level, incorporating the effect of the intervention.

5.1.2 Detection using auxiliary residuals

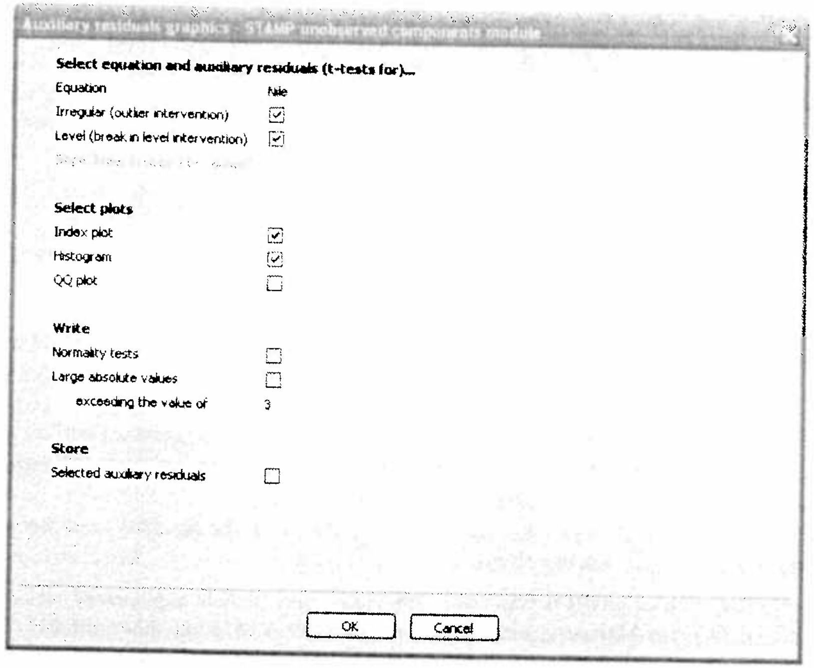
Outliers and structural breaks can often be detected simply by looking at a graph of the series. When a strong seasonal pattern is present, it may be better to seasonally adjust the data first. If you look at the seasonally adjusted ofuCOALl series, for example, possible breaks and outliers can be seen which are less apparent from an inspection of the original series.

Although looking at a graph is always helpful, it is often not possible to detect outliers and structural breaks or to distinguish between them. This is where the auxiliary residuals, due to Harvey and Koopman (1992), are useful. *The auxiliary residuals are smoothed estimates of the irregular and level disturbances* and although they are neither serially uncorrelated nor uncorrelated with each other, they play a valuable role in that they go some way towards separating out pieces of information which are mixed up together in the innovation residuals. It is possible to construct estimates of other disturbances, such as the one driving the slope, but experience suggests that these are less useful.

We illustrate the use of the auxiliary residuals by looking at how we might have gone about specifying the model for the Nile flow estimated in the previous subsection.

If you still have the NILE data loaded, go to **Model**/Select components and specify a model with no slope, but allow the level to remain stochastic. Do not specify a cycle and remember to take out the interventions by accessing the Interventions dialog and deselecting them. The model is therefore a simple random walk plus noise. This might well be a plausible initial specification.

Having estimated the model, examine the trend in **Test**/Components graphics. It can be seen how the level adapts to the change in 1899 and picks up the cycle. The Auxiliary residuals graphics dialog, which can be found in the **Test** menu, allows the inspection of irregular, level and slope residuals:



They provide an even clearer pointer to the structural break in the Nile series. Figure 5.1 shows a pronounced dip in the levels residual. The outlier in 1913 shows up clearly in the irregular. The normality statistics are not significant, but this only serves to stress the importance of looking at graphical output rather than relying solely on test statistics.

Once interventions are put into a model (re-activate them in the Select interventions dialog) with a fixed level, a cyclical pattern can be seen in the residual autocorrelation function, though the first-order autocorrelation, $r(1)$, is not significant.

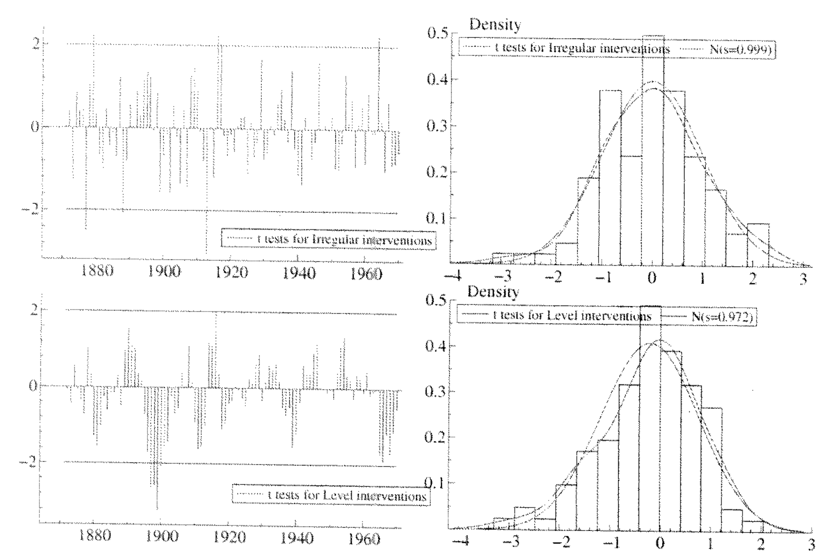


Figure 5.1 Auxiliary residuals for random walk + noise model of the Nile.

5.1.3 Automatic outlier and break detection

The inspection of auxiliary residuals is useful for the analysis and modelling of time series but can be time-consuming when the primary interest of the analysis is not focussed on outliers and breaks. The option 'Select interventions automatically' in the **Model**/Select components dialog allows the program to detect significant outliers and breaks and to include them in the model automatically without the interference of the user.

The strategy of finding outliers and breaks is based on the auxiliary resiudals and consists of the following two steps:

(1) The selected model is estimated (this model may include explanatory variables and imposed interventions, see discussion below). The auxiliary residuals are computed and the time periods of residuals that have absolute values exceeding 2.3 (for irregular), 2.5 (for level residual) or 3.0 (for slope residual) are recorded. Adjacent time periods for level and slope residuals are removed within a distance of upto 3 (for level) or 4 (for slope) periods from the largest residual. This addresses the issue that level and slope residuals are serially correlated over time, see Harvey and Koopman (1992).

(2) The model with the inclusion of interventions detected in Step 1 is re-estimated. The estimated interventions with t-values that do not exceed the value 3 in absolute values are de-selected as intervention variables from the model. The output reported in STAMP is based on this model.

After Step 2, the model with some intervention variables excluded is not re-estimated. It should be re-estimated since the model specification has changed. However, since the removed interventions are not estimated as significant, the estimates of the remaining variables (including disturbance variances of the components and regression coefficients) may not be affected by much when these non-significant effects are removed from the model specification. It is nevertheless advisable, after re-inspecting the resulting model (using the various diagnostic statistics available in STAMP) and after re-considering the automatically selected set of interventions, to re-estimate the model. The automatic procedure will be illustrated using the NILE data.

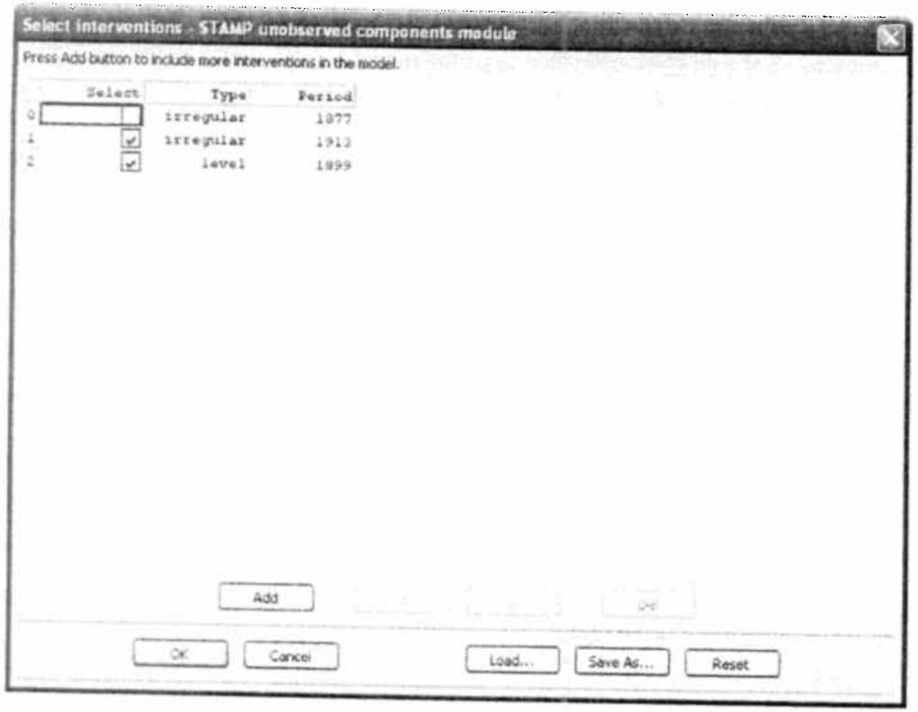
Go to the **Model**/Select components dialog and specify a model with 'stochastic level', no 'slope', do not include a cycle, and activate 'Select interventions automatically' by pressing the appropriate radio-button. By pressing OK and accepting the settings in the Estimate dialog, the program carries out the two-step procedure as decribed above. In the results window, we find the output

```
Variances of disturbances:
                  Value     (q-ratio)
Level          0.000000  (   0.0000)
Irregular       14124.7  (    1.000)

State vector analysis at period 1970
            Value       Prob
Level   1097.75000 [0.00000]

Regression effects in final state at time 1970
                    Coefficient         RMSE      t-value      Prob
Level break 1899. 1  -242.22887     26.52156     -9.13328 [0.00000]
Outlier 1913. 1      -399.52113    119.68141     -3.33821 [0.00120]
```

The automatic procedure confirms the findings in the previous section. To check whether other outliers and breaks have also been considered in Step 1 of the procedure, proceed to the Select components dialog and accept the default settings which include the activated option of 'Select interventions manually'. This last option is activated as a default when intervention variables are part of the current model. In the Select interventions dialog, the level break of 1899 and the outlier of 1913 are activated while the outlier effect for 1877 is de-activated.



The outlier for 1877 was detected as a potential intervention variable in Step 1 of the procedure while it was dismissed as an outlier in Step 2 (its estimated t-value did not exceed the value of 3). The model may now be re-estimated with the two activated intervention variables included in the model. The estimation output has changed slightly since the variances of the components are re-estimated without the outlier intervention variable for 1877.

```
Variances of disturbances:
                    Value     (q-ratio)
Level            0.000000  (    0.0000)
Irregular         14845.9  (     1.000)

State vector analysis at period 1970
              Value        Prob
Level    1097.75000 [0.00000]

Regression effects in final state at time 1970
                      Coefficient        RMSE      t-value        Prob
Level break 1899. 1    -242.22887    27.19026     -8.90866 [0.00000]
Outlier 1913. 1        -399.52113   122.69901     -3.25611 [0.00156]
```

The estimated irregular variance has increased to the value of 14845.9 since the intervention variable for the outlier of 1877 has been excluded and has become part of the irregular component.

Finally, the automatic outlier and break detection procedure may be repeated with imposing pre-selected intervention variables in the model. Return to the Select components dialog and change the default setting from 'Select interventions manually' to 'Select interventions automatically'. By pressing OK, the Select interventions dialog appears and intervention variables can be added, deleted, selected or deselected. The selected interventions will be imposed in the model during the automatic outlier and break detection procedure described. In other words, they are regarded as a set of explanatory variables that remain in the model during the two-step procedure. In case of the NILE data, after repeated applications of the automatic outlier and break detection procedure, no new outliers and breaks will be detected.

In case of noisy data-sets, a long list of selected and non-selected intervention variables may appear in the Select interventions dialog. Although it is useful that the program keeps a record of all considered interventions in the past, it can also be useful to have an option that deletes all (selected and non-selected) intervention variables. This option is simply the 'Select interventions none' in the Select components dialog. When the model is estimated with this option activated, all intervention variables will be deleted from memory.

5.1.4 Specification of more complex interventions

The interventions considered so far have been of a relatively simple form. We now examine a somewhat more complicated case in which:

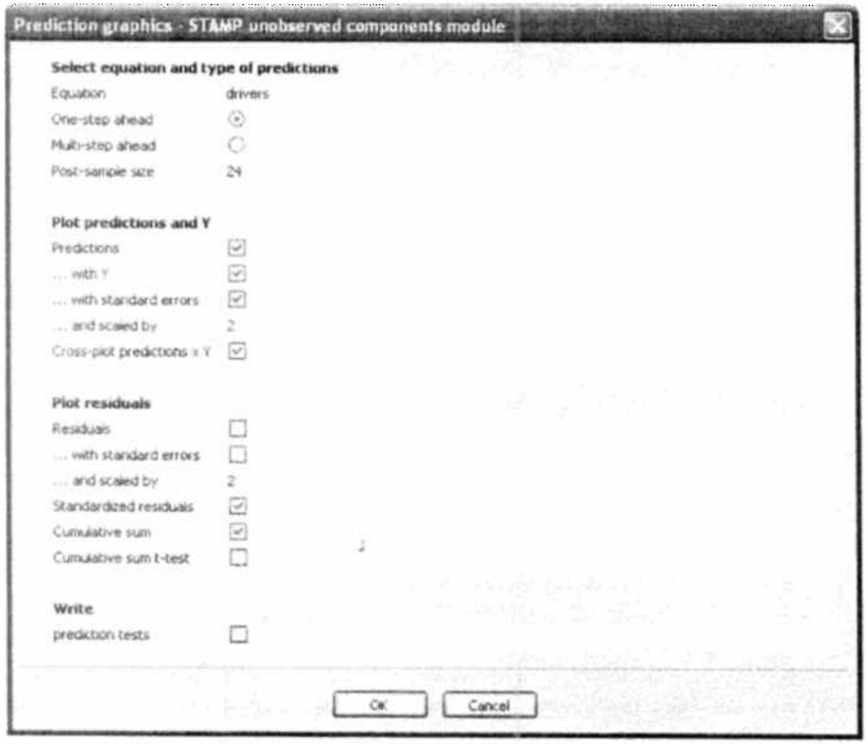
(1) the form of the intervention variable is such that it must be created as an explanatory variable; and
(2) the effect of the intervention in the periods immediately after the occurrence of the intervention is not known with certainty and so the form adopted must be subjected to *post-intervention diagnostics*.

The study by Harvey and Durbin (1986) on the effect of the seat belt law in Great Britain provides a good illustration of the above issues. Load the SEATBELT data. The intervention variable, 'law', is already included. It is basically a step intervention starting from February 1983, but it takes the value 0.18 in January because of increased seat belt usage in anticipation of the coming of the law on the last day of the month. It is a useful exercise to recreate this intervention. Go to **Model**/Calculator of OxMetrics, `Alt+c`. Press 'Dummy', and type 83 and 2, respectively, in the two edit boxes after 'Sample start'. Subsequently, press OK and, in the Calculator dialog, press '='. Give the variable a name, say 'law1', and press OK. Then leave the Calculator. In the database of SEATBELT, go to the field corresponding to row '83-1' and column 'law1' and, by double-clicking it or by selecting it and pressing ↵, you can edit the field and type

0.18. You now have an intervention exactly like 'law', but since you don't need it, go back to the calculator and delete it!

Now formulate a model for 'Drivers' and include 'law' as an explanatory variable. Use a BSM - the default in Components - or do not select 'Slope' and select 'Seasonal, Fixed' and then Estimate the model. The coefficient of the intervention appears in More written output, provided the 'State and Regression output' box is marked. The coefficient (with no slope and fixed seasonals) is -0.26, indicating a fall of 23%, that is $1 - \exp(-.26) = .23$, in car drivers killed and seriously injured after the introduction of the law.

To check the specification of the intervention variable, go to the **Test**/Prediction graphics dialog, keep the default settings and press OK.



The plots, shown in Figure 5.2, show the predictive performance of the model after the intervention. It appears to be good, and the *post-intervention predictive test*, given here as the *Failure Chi2 test*, is not significant.

```
Prediction analysis for 24 post-sample predictions
(with 1 missing values).
             error  stand.err  residual    cusum   sqrsum
83(1)         .NaN    0.07546    0.0000   0.0000   0.0000
```



```
83(2)      0.2607     0.4041    0.6452   0.6452   0.4163
83(3)      0.1212    0.09177     1.321    1.966    2.161
...
84(10)    0.07777    0.07526     1.033    6.568    18.91
84(11)    0.04815    0.07526    0.6399    7.208    19.32
84(12)  -0.007602    0.07526   -0.1010    7.107    19.33

Post-sample predictive tests.
Failure Chi2( 23) test is  19.3299  [0.6819]
Cusum t( 23)      test is   1.4820  [0.1519]

Post-sample prediction statistics.
Sum of 23 absolute prediction errors is 1.58736
Sum of 23 squared  prediction errors is 0.181874
Sum of 23 absolute prediction resids is 17.8311
Sum of 23 squared  prediction resids is 19.3299
```

Although the 'post-sample size' is set to 24 in the Prediction graphics dialog, only 23 residuals can be used for the test statistics. The residual of 83(1), with value .NaN, is lost since the regression effect 'law' takes effect from period 83(1) onwards and this is the first period in the post-sample of length 24. The observation at 83(1) is used to identify the regression effect. The uncertainty of the residuals in the period just after 83(1) is apparent from the higher standard errors and from the confidence interval in the prediction graph of Figure 5.2.

To carry out a post-intervention test over a shorter period, return to the **Test**/Prediction graphics dialog and proceed as follows. Change value 'Post-sample size' from 24 to 6 and mark 'prediction tests' in the 'Write' section. The Chow failure test statistic is still insignificant.

```
Post-sample predictive tests.
Failure Chi2( 6) test is   3.4923  [0.7450]
Cusum t( 6)      test is   1.1186  [0.3061]
```

5.2 Explanatory variables

Including explanatory variables in a structural time series model results in a mixture of time series and regression. Indeed classical regression emerges as a special case in which there are no stochastic components, apart from a single random disturbance term. STAMP will estimate classical regression models quite happily, but its real power comes in combining regression with unobserved components.

The combination of unobserved stochastic components with explanatory variables opens up a wide range of possibilities for dynamic modelling.

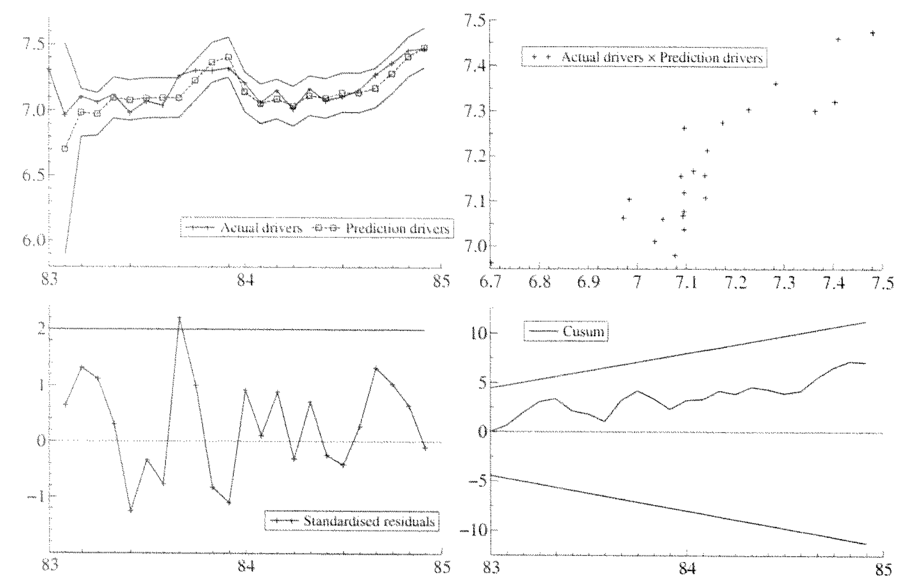


Figure 5.2 Predictive testing for the Drivers series.

5.2.1 Stochastic trend component

The SPIRIT data consists of annual observations, from 1870 to 1938, on the logarithms of three variables, the *per capita* consumption of spirits in the UK, *per capita* income, and the relative price of spirits. The data set is a famous one, having been used as a test bed for the Durbin–Watson statistic in 1951.

The series may be graphed in OxMetrics. Income and price are explanatory variables and the standard econometric approach is to estimate a regression model with a linear or quadratic time trend and an AR(1) disturbance; see Fuller (1996, p. 522). The structural time series modelling approach is simply to use a stochastic trend with the explanatory variables. The aim of the stochastic trend is to pick up changes in tastes and habits which cannot be measured explicitly.

Enter the Formulate dialog. Select the three variables by clicking and press << or by double-clicking. The first variable to be selected is 'Spirits' and it will be marked with 'Y' in the 'Selection' list box. The other two variables 'Income' and 'Price' will not be marked in the 'Selction' list box. This is as it should be. Press OK to enter the Select components dialog which is set for a stochastic level and slope with an irregular. There is no seasonal — obviously since the observations are annual. Since the model includes explanatory variables, the option 'Set regression coefficients' is marked by default. It allows you to enter the Regression coefficients dialog and to change the specification of the various regression coefficients. For example, coefficients can be specified as time-varying and modelled as a random walk process. For this illustration

we concentrate on fixed regression coefficients and therefore accept the default settings in this dialog and press OK. This brings up the Estimation dialog. Change the period of 'Estimation ends at' from 1938 to 1930. Press OK and once estimation is complete, the standard output reads:

```
    The databased used is SPIRIT.IN7
    The selection sample is: 1870 - 1930 (T = 61, N = 1)
    The dependent variable Y is: spirits
    The model is:  Y = Trend + Irregular + Explanatory vars

Log-Likelihood is 217.767 (-2 LogL = -435.535).
Prediction error variance is 0.000486587
Summary statistics
                     spirits
 T                    61.000
 p                    2.0000
 std.error          0.022059
 Normality            7.9501
 H(19)                2.2760
 DW                   2.0779
 r(1)              -0.048381
 q                    8.0000
 r(q)               -0.17059
 Q(q,q-p)             4.1047
 Rd^2                0.72830

 Variances of disturbances:
                     Value     (q-ratio)
Level          9.04323e-005 (   0.6070)
Slope          3.55317e-005 (   0.2385)
Irregular       0.000148994 (    1.000)

State vector analysis at period 1930
                Value      Prob
Level        2.23692 [0.00000]
Slope       -0.01538 [0.12573]

Regression effects in final state at time 1930
                Coefficient       RMSE      t-value       Prob
price              -0.94969    0.07094    -13.38671  [0.00000]
income              0.69526    0.13199      5.26766  [0.00000]
```

All appears to be well, although the normality statistic (which measures the departure of third and fourth moment from their values expected under normality) is somewhat on the high side. Here H is a heteroskedasticity statistic, $r(1)$ and $r(8)$ are the serial correlation coefficients at the 1st and 8th lag. DW is the Durbin–Watson statistic, while $Q(8,6)$ is the Box–Ljung statistic using 8 lags. $Q(8,6)$ should have a χ^2_6 distribution under the null distribution. Finally R^2_d is a measure of goodness-of-fit, computed on the differences of the spirit data set. All of these statistics will be discussed in more detail in Chapter 10.

We now turn to the **Test** menu. The first option is 'Variances' in the More written output dialog:

```
Variances of disturbances:
                      Value     (q-ratio)
Level          9.04323e-005  (   0.6070)
Slope          3.55317e-005  (   0.2385)
Irregular       0.000148994  (    1.000)

Standard deviations of disturbances:
                      Value     (q-ratio)
Level            0.00950959  (   0.7791)
Slope            0.00596084  (   0.4883)
Irregular         0.0122063  (    1.000)
```

The output shows the estimates of both the variances and the standard deviations of the disturbances driving the level and slope and the irregular disturbance. The first two are non-zero indicating a stochastic trend . However, the most useful information on the nature of this trend appears when we examine a plot of it later in Components.

Part of the standard output are the estimates of the coefficients of the explanatory variables. *These can be interpreted in exactly the same way as regression coefficients.* For example, the coefficient of price is approximately $-.95$, indicating that a one per cent increase in price leads, other things being equal, to a fall in spirits consumption of $-.95$. The 't-value' is -13.39. If the (relative) variances were known, this statistic would have a t-distribution. However, because the parameters are estimated all we can say is that it is normally distributed in large samples. The figure in square brackets gives the probability value for a two-sided test based on a standard normal distribution.

The basic results can thus be expressed in much the same way as in a classical regression:

$$y_t = \widetilde{\mu}_{t|T} - \underset{(.071)}{.950}\ x_{1t} + \underset{(.132)}{.695}\ x_{2t} + \widetilde{\varepsilon}_{t|T}, \quad t = 1, \ldots, T,$$

$$\mathrm{R}^2_{\mathrm{d}} = 0.728, \quad \hat{\sigma} = 0.022, \quad \mathrm{Q}(8,6) = 4.10, \quad \mathrm{N} = 7.95.$$

where y_t is spirits, x_{1t} is price, x_{2t} is income and $\widetilde{\mu}_{t|T}$ and $\widetilde{\varepsilon}_{t|T}$ are the smoothed estimates of the trend and irregular components.

The above expression tells us nothing about the trend. To see precisely what the

trend is doing, we move to **Test**/Components graphics. Mark 'Trend' and 'Trend plus Regression effects' in the 'Plots with Y' section. Press OK to get the graphs as displayed in Figure 5.3. The former shows the trend with the actual series. The gradual downward movement in the trend indicates a change in tastes away from spirits. Unlike a univariate series the trend does not pass through the observations. However, adding in the effect of the explanatory variables is shown in the second graph which reflects the equation as written above. (If we go back to the Components dialog and change 'Smoothed' to 'Filtered', this second graph shows one-step-ahead predictions. Although this is a standard output in econometric packages, we find it less useful than the plot of the prediction errors which is given in Residuals.)

As was noted earlier, the standard econometric approach to the SPIRIT data is to fit a deterministic trend and an AR(1) disturbance. To do this, enter the ***Model****/Formulate dialog first and include the Y and X variables. In the Select components dialog, select 'Level, Fixed' and 'Slope, Fixed' (this is a fixed trend), select 'AR(1)' in the 'Cycle(s)' section and proceed as before. Compare the fit, coefficients and diagnostics with those obtained with the stochastic trend model.*

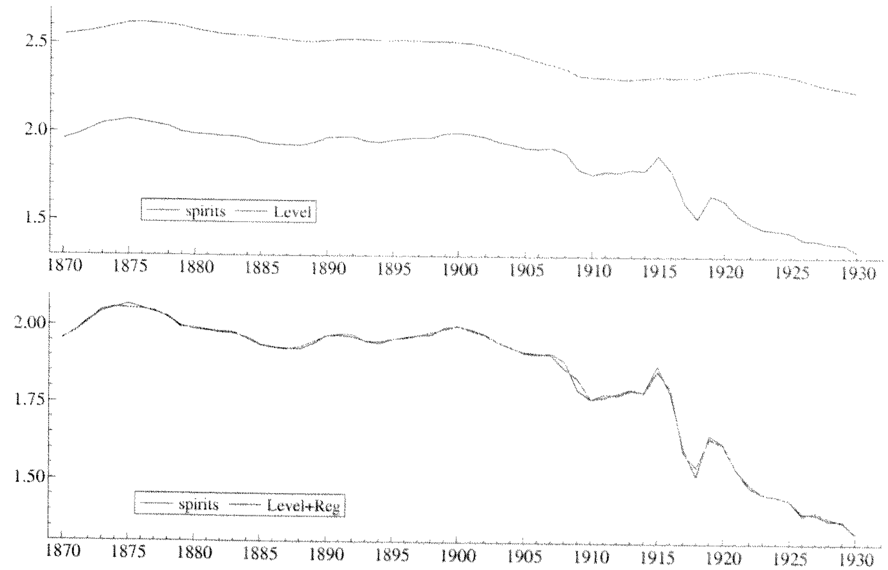


Figure 5.3 Trend and Trend with Explanatory variables for Spirit series.

5.2.2 Outliers and structural breaks

Intervention variables may appear in models together with explanatory variables. Similarly the auxiliary residuals may be used to detect the outliers and structural breaks which give rise to their inclusion. To illustrate, continue with the SPIRIT example. An examination of the output from Auxiliary residuals, indicates some outliers and a shift in the level in 1909; see Harvey and Koopman (1992) for further details.

In case STAMP is used with the option 'Select interventions automatically' activated in the Select components dialog, the following regression output is reported (based on the estimation sample 1870 – 1930):

```
Regression effects in final state at time 1930

                      Coefficient        RMSE      t-value       Prob
Outlier 1918. 1          -0.06011     0.01054     -5.70292  [0.00000]
Level break 1909. 1      -0.09518     0.01484     -6.41588  [0.00000]
price                    -0.82434     0.05563    -14.81940  [0.00000]
income                    0.59481     0.10382      5.72920  [0.00000]
```

In case the full estimation sample 1870 – 1938 is used, we obtain:

```
Regression effects in final state at time 1938
                      Coefficient        RMSE      t-value       Prob
Outlier 1915. 1           0.04525     0.00829      5.45911  [0.00000]
Outlier 1918. 1          -0.06177     0.00815     -7.58320  [0.00000]
Level break 1909. 1      -0.09499     0.01175     -8.08297  [0.00000]
price                    -0.73800     0.04703    -15.69331  [0.00000]
income                    0.67555     0.08072      8.36942  [0.00000]
```

These are also the intervention variables considered in Harvey and Koopman (1992) where it is discussed that the outliers are clearly associated with World War 1 and the level break is associated with the introduction of a law on spirits by prime minister Lloyd George.

5.2.3 Lags and differences

The SPIRIT example involved only current values of the explanatory variables. We can easily test whether lagged price should enter into the model. When the Formulate dialog is active, select 'price' and select 'lag' in the drop box of the 'Lags' section and type 1 in the box below. Press << and both 'price' and price lagged one period — 'price_1' — now appear as explanatory variables. Estimate the model with the same Components specification as before and carry out a 't-test' on lagged price.

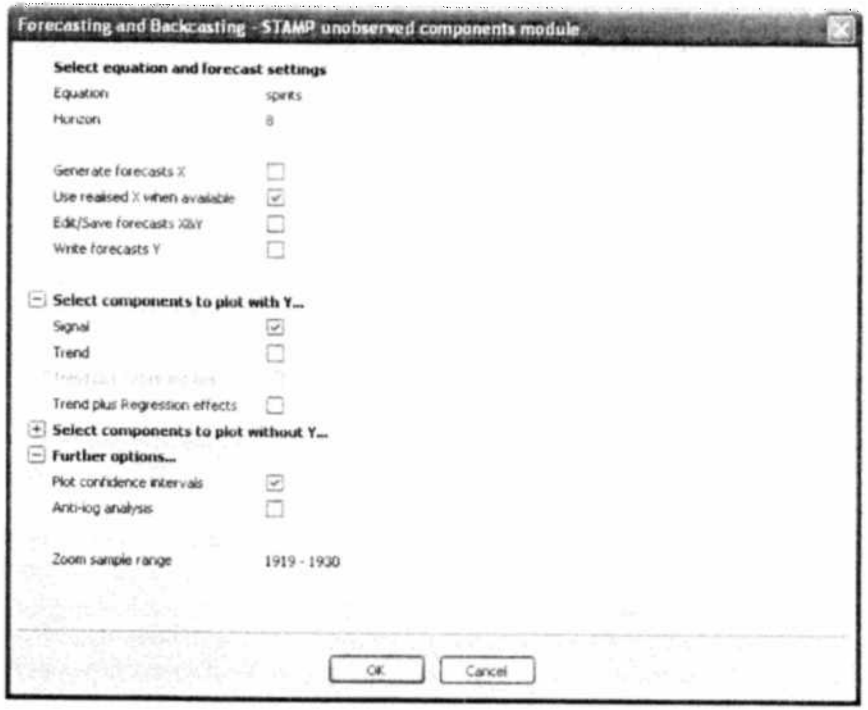
The above example created lags within the Formulate dialog. However, lags can also be created in the Calculator of OxMetrics. This is useful if we wish to retain the lagged variable in the data set for future use.

It is often desirable to reformulate a lag structure in terms of differenced observations. This is for both theoretical reasons and practical reasons because differenced observations typically lead to greater stability in estimation since they suffer less from multicollinearity. The differencing must be carried out before entering the Formulate dialog. The Calculator of OxMetrics may be used for this purpose. The Calculator also

offers other types of time series transformations.

5.3 Forecasting

To make forecasts of future values of the dependent variable, we need corresponding values of the explanatory variables. The Forecasting dialog is given below.



This dialog offers a flexible method of inputting future values for explanatory variables and changing them so that the path of the forecasts can be examined under different scenarios. We will continue with the SPIRIT example, and see how forecasts can be made from after the end of the estimation period in 1931 to 1938. Then within the **Test**/Forecasting dialog the 'Horizon' values remains at its default value of 8. Furthermore, select the options 'Signal' and 'Trend plus Regression effects'. The default zoom period of 1919 – 1930 can be changed but we keep the default. Pressing OK gives the default forecasts in which future values of the explanatory variables (1931 – 1938) are simply set to their values inside the database; see the upper graph of Figure 5.4. Some more interesting possibilities are outlined below.

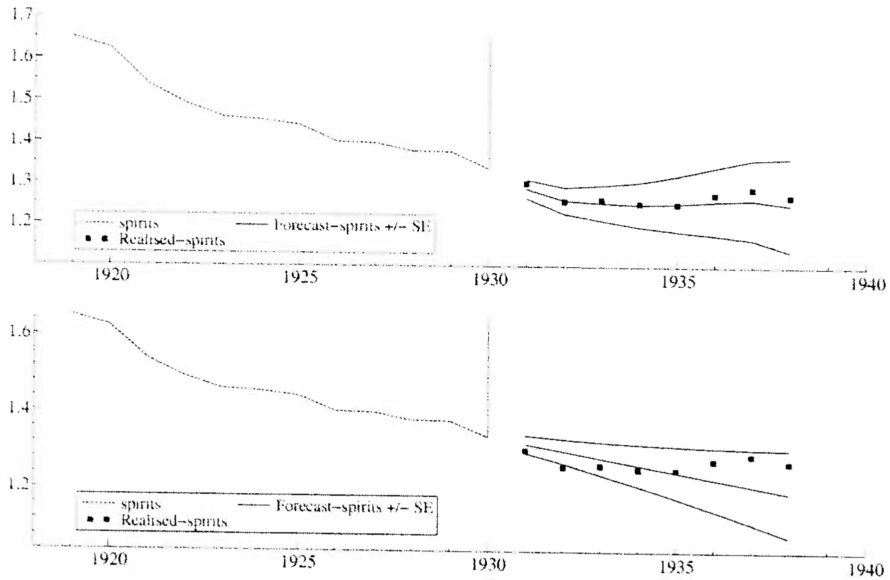
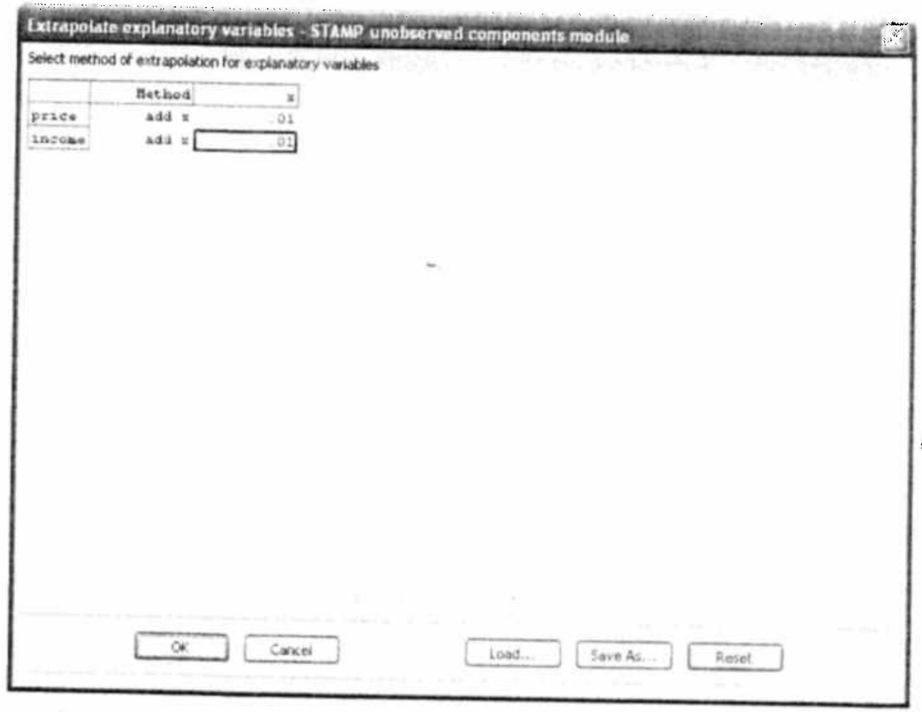


Figure 5.4 Forecasts with Explanatory variables for Spirit series: in upper graph, future Xs are replaced by their realised values in the database; in lower graph, future Xs are extrapolated from their values in period 1930 with an increment of 0.01 for each period.

5.3.1 Incremental change

In the Forecasting dialog, the 'Forecast settings' section has the options 'Generate forecasts X' and 'Edit/Save forecasts X&Y' to provide means of manipulating the values of the explanatory variables each period beyond the database sample. When the option 'Generate forecasts X' is activated, the following dialog appears that allows you to increase explanatory variables by a constant value each period in the forecasting horizon.



If the data are in logarithms, the increment is (one hundredth of) the growth rate per period. Thus, if in the present example, we feel that income and price should grow at 1% per year, '0.01' should be added each time period. By changing the values in the 'x' column to 0.01, both explanatory variables are increased by 0.01 in the forecasting period starting with their corresponding values in 1930. By pressing OK, the graphs of the series together with their forecasts appear. This dialog also allows one to generate future values for explanatory variables by other methods. The column 'Method' has drop boxes to choose methods of generating values for the forecasting period including percentage increases from the last observed values, means of the last x values of the explanatory variables and fixed trends based on the last x values. After generating future values for explanatory variables using this dialog, pressing OK leads to graphs of the series together with their forecasts. The graphs of the forecasts of the explanatory variables allow the user to judge whether the assumed extrapolations of the explanatory variables are plausible; see Figure 5.4.

5.3.2 Manual input

Future values of the explanatory variables can also be entered manually and this can be achieved as follows. Go to the Forecasting dialog and activate the option 'Edit/Save forecasts X&Y' in the 'Forecast settings' section. After selecting other options in the

Forecasting dialog and pressing OK, the Matrix Editor dialog appears that allows the user to change the forecast values of the explanatory variables in standard ways. The Matrix Editor has standard spreadsheet functionalities but it also allows saving the matrix of values on disk and loading values from files saved earlier on disk. In our case, you can edit, save or load the eight forecasts for price and for income. When finishing editing, the button OK can be pressed and the corresponding forecasts of spirits are presented in OxMetrics.

5.3.3 Using models to forecast the explanatory variables

Future values of the explanatory variables may be constructed by fitting models to them. Having fitted time series models, such as local linear trends, to each "explanatory" variable, forecasts can be made using the Forecasting dialog and then saved when the option 'Write forecasts Y' is activated. Suppose in the SPIRIT example that the forecasts of price and income are outputted in the Results window using the option 'Write forecasts Y' in the Forecasting dialog. Once the model has been estimated for spirits, go to Forecasting, select option 'Edit/Save forecasts X&Y' and you can reload the model-based forecasts of the explanatory variables using standard copy/paste facilities.

5.3.4 Interventions

If intervention variables are constructed using the Intervention dialog they will be automatically extended into the forecast period. Interventions constructed in any other way, such as by using the Calculator, must be treated in the same way as any other explanatory variable.

5.4 Statistical features of the models

Some insight into statistical aspects of the models can be obtained by examining a simple model with a single explanatory variable, a random walk trend (level), and irregular and level interventions. The model may be written

$$\begin{aligned} y_t &= \mu_t + \delta x_t + \lambda w_t + \epsilon_t, \quad \epsilon_t \sim \mathsf{NID}(0,\sigma^2_\epsilon), \quad t = 1,\ldots,T, \\ \mu_t &= \mu_{t-1} + \theta z_t + \eta_t, \quad \eta_t \sim \mathsf{NID}(0,\sigma^2_\eta), \end{aligned}$$

where w_t takes the value one at the time an event occurs and is zero otherwise. The specification of z_t is similar.

As noted in the introduction to the chapter, an alternative way of capturing the change in level induced by z_t is to put a step variable in the measurement equation, that is add another w_t variable which is zero before the event in question and unity thereafter. The attraction of the specification written above is that the step change is built into the trend component; see the Nile example in §5.1.

Now consider the model without the interventions and observe the following:

- If σ^2_η is zero the level is a constant and the model reduces to a classical regression.
- If σ^2_ϵ is zero, but σ^2_η is not, the model is a standard regression in first differences, that is

$$\Delta y_t = \delta \Delta x_t + \eta_t, \quad t = 2,\ldots,T.$$

- If both variances are non-zero, the disturbance in the first difference equation is a first-order MA. In levels the stochastic part of the model is ARIMA(0,1,1). Hence the 'reduced form' of the stochastic trend regression model is what is sometimes called a 'transfer function' model; see Harvey (1989, Ch. 7) or Harvey (1993, Ch. 5) for further discussion.

5.5 Exercises

(1) Estimate the Nile model with interventions as specified in §5.1.1. Carry out a post-intervention test on the 1899 level shift based on the observations up to and including 1905.

(2) The new Aswan dam on the Nile was constructed in 1955. Use the NILE data to see if there is any impact on the flow of the river.

(3) Use the auxiliary residuals to detect outliers in the series of US exports to Latin America, LAXQ. Fit an appropriate model — you might perhaps specify a smooth trend and include a cycle.

(4) Re-estimate 'ofuCOALI' in ENERGY with interventions for the miners' strike in 84, Q3 and Q4. How are the forecasts affected?

(5) Test if 'law' is significant in SEATBELT for 'Front' and 'Rear' passengers.

(6) An example of a dynamic model is the employment-output equation fitted to quarterly, seasonally adjusted, UK data by Harvey, Henry, Peters and Wren-Lewis (1986). This is contained in the file EMPL. The dependent variable is 'Empl'. Include a stochastic trend and two lags on the dependent variable and the explanatory variable, which is manufacturing output, 'Output'. Test the significance of the lags and hence simplify the model.

(7) In the book by Fuller (1996, p. 522), a quadratic time trend is fitted to the SPIRIT data. Such a variable may be constructed as indicated by forming an index using Algebra, and then squaring it. Fit a quadratic trend, with and without an AR(1) disturbance, and compare the fit with the stochastic trend model fitted in §5.2.1.

Chapter 6

Tutorial on Multivariate Models

6.1 SUTSE models

Seemingly unrelated time series equations (SUTSE) have a similar form to univariate models, except that $\mathbf{y}_t$ is now an $N \times 1$ vector of observations, which depends on unobserved components which are also vectors. The link across the different series is through the correlations of the disturbances driving the components. In a common factor model, some, or all, of the variance matrices will be of reduced rank. Common factors are of considerable importance, and their interpretation and how they may be imposed is discussed in detail in §6.4.

The fitting of multivariate models proceeds much as in the univariate case. The Formulate dialog in the Model menu (`Alt+y`) allows the marking of multiple variables as dependent (Y). This is done by selecting the appropriate variables in the Database listbox: use the mouse and double-click on a variable or highlight a variable and use the << keys to transfer the variables to the Selection listbox. Once the variables have been transferred to the Selection listbox, the first variable is marked as Y. It suggests that the default model is univariate and the other selected variables are regarded as explanatory variables. By right-clicking the mouse on a variable in the Selection listbox, the status of this variable can be changed to Y variable. This can also be achieved by highlighting the appropriate variables (for multiple highlighting, `Ctrl` and left-clicking the variables) in the Selection listbox, choosing Y variable in the droplist below the Selection listbox and pressing the button 'Set'. By pressing OK, the Select components dialog is entered. The default model is the multivariate basic structural time series model with full variance matrices for the disturbance vectors driving the unobserved component vectors. These default settings can be confirmed by pressing OK and the model is estimated in the usual way.

The output for each equation is listed in turn in the Results window. The output that appears in univariate modelling is presented for each variable in the dependent vector of time series. Furthermore, for each unobserved component the 'variance/correlation matrix' is presented as default output in the Results window: the diagonal elements are the variances, the lower triangular elements are the covariances and the upper triangular

elements are the corresponding correlations. Plots of joint components can easily be produced within OxMetrics. The default component graphs can be found in the Model graphics window (in the Document list of OxMetrics, the left-hand side of the main program).

As an example, load the UKCYP data and build a multivariate BSM for consumption and income; having marked both 'c' and 'y' as dependent Y and then formulate the components model by following the defaults. The hyperparameter results are as follows. The elements of the estimated variance/correlation matrices for the irregular and level noise are:

corrélés

$$\widehat{\Sigma}_\varepsilon = \begin{pmatrix} 6.11 \times 10^{-6} & \mathit{-0.835} \\ -1.15 \times 10^{-5} & 3.11 \times 10^{-5} \end{pmatrix}, \widehat{\Sigma}_\eta = \begin{pmatrix} 4.98 \times 10^{-5} & \mathit{0.933} \\ 8.55 \times 10^{-5} & 1.69 \times 10^{-4} \end{pmatrix}.$$

The elements in italics are the estimated correlations. The correlation is much higher for the disturbances in the levels than for the irregular. The corresponding matrices for the slope and seasonal are:

$$\widehat{\Sigma}_\zeta = \begin{pmatrix} 2.84 \times 10^{-6} & \mathit{1} \\ 2.05 \times 10^{-6} & 1.48 \times 10^{-6} \end{pmatrix}, \widehat{\Sigma}_\omega = \begin{pmatrix} 1.28 \times 10^{-6} & \mathit{-0.213} \\ -4.37 \times 10^{-7} & 3.28 \times 10^{-6} \end{pmatrix}.$$

The slope disturbances are perfectly correlated while the correlation between the seasonals is negative and close to zero. ?!

In the special case of a multivariate local level model

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t, & \boldsymbol{\epsilon}_t &\sim \mathsf{NID}(\mathbf{0}, \boldsymbol{\Sigma}_\epsilon), \\ \boldsymbol{\mu}_t &= \boldsymbol{\mu}_{t-1} + \boldsymbol{\eta}_t, & \boldsymbol{\eta}_t &\sim \mathsf{NID}(\mathbf{0}, \boldsymbol{\Sigma}_\eta), \end{aligned} \qquad (6.1)$$

where $\boldsymbol{\Sigma}_\epsilon$ and $\boldsymbol{\Sigma}_\eta$ are the $N \times N$ variance matrices, and $\boldsymbol{\eta}_t$ and $\boldsymbol{\epsilon}_t$ are multivariate normal disturbances which are mutually uncorrelated in all time periods. For the more general multivariate BSM, the other disturbances similarly become vectors which have $N \times N$ variance matrices. In the case of trigonometric seasonals there are two sets of $N \times 1$ vectors for each seasonal frequency such that

$$E(\boldsymbol{\omega}_{it}\boldsymbol{\omega}_{it}') = \mathbf{E}(\boldsymbol{\omega}_{it}^*\boldsymbol{\omega}_{it}^{*\prime}) = \boldsymbol{\Sigma}_\omega, \quad \mathbf{E}(\boldsymbol{\omega}_{it}\boldsymbol{\omega}_{it}^{*\prime}) = \mathbf{0}, \quad \mathbf{i} = \mathbf{1}, \mathbf{2}, \ldots, [\mathsf{s}/\mathbf{2}],$$

and all disturbances at different frequencies are independent.

In STAMP 8 we allow for a range of variance matrices structures that can be selected for each unobserved component. The following variance matrices can be selected:

- *full*: a full variance matrix of rank N is considered as in the UKCYP example above;
- *scalar*: variance matrix Σ is specified as the unity matrix scaled by a non-negative value, that is $\Sigma = \sigma^2 I_N$ where $\sigma^2 \geq 0$ is a scalar variance;
- *diagonal*: a diagonal variance matrix is considered with N different diagonal elements;

- *ones*: variance matrix Σ is specified as a matrix of ones scaled by a non-negative value, that is $\Sigma = \sigma^2 \iota\iota'$ where $\sigma^2 \geq 0$ is a scalar variance and ι is a vector of ones (note: Σ has rank one in case $\sigma^2 > 0$);
- *cdiag* : variance matrix Σ is specified as a $\Sigma = aa' + D$ where a is an $N \times 1$ non-zero vector and D is a diagonal matrix.

6.2 Cycles

Cycles may be introduced into multivariate models. As with other components, the disturbances can be correlated across the series. Since the cycle in each series is driven by two disturbances, there are two sets of disturbances and these are assumed to have the same variance matrix, that is

$$E(\kappa_t \kappa_t') = \mathbf{E}(\kappa_t^* \kappa_t^{*'}) = \Sigma_\kappa, \quad \mathbf{E}(\kappa_t \kappa_t^{*'}) = \mathbf{0}, \quad \mathrm{t} = 1, \ldots, \mathbf{T},$$

where Σ_κ is an $N \times N$ variance matrix. The homogeneity restriction can be applied to models with cycles if desired.

As in the univariate case, STAMP allows up to three cycles in each series, but it imposes the restriction that, for a given cycle, *the damping factor and the frequency, ρand λ_c, are the same for all series*. This means that the cycles in different series have the same properties, that is they have the same autocorrelation function and spectrum. We call them *similar cycles*. The strength of a cycle in a particular series depends on the variance of its disturbance.

The file MINKMUSK contains two series showing the numbers of furs of minks and muskrats traded annually by the Hudson Bay Company in Canada from 1848 to 1909. A model may be fitted with similar cycles in order to establish the joint stylised facts associated with the two series. Proceed by selecting the logarithms of the two series (Lmink and Lmuskrat) and select 'Fixed' for Level component and select 'Cycle medium (default 10 years)' (keep the other options at their default settings) in the Select components dialog. The last three observations are best omitted as they are somewhat atypical. Thus the end of the sample in the Estimate dialog should be changed to 1906.

Some relevant output for the cycle is given by

```
 Variances of disturbances in Eq Lmink:
                       Value     (q-ratio)
Level               0.000000   (   0.0000)
Slope           5.11905e-005   ( 0.001427)
Cycle             0.00137809   (  0.03842)
Irregular          0.0358708   (    1.000)
 Variances of disturbances in Eq Lmuskrat:
                       Value     (q-ratio)
Level               0.000000   (   0.0000)
Slope            0.000251142   ( 0.007001)
Cycle               0.113689   (    3.169)
Irregular          0.0281232   (   0.7840)

Cycle other parameters:
Period                 9.78101
Frequency              0.64239
Damping factor         0.98448
Order                  1.00000
Cycle variance/correlation matrix:
                 Lmink    Lmuskrat
Lmink          0.04475     -0.3049
Lmuskrat      -0.02283      0.1253
```


The estimated trends are relatively smooth, not unlike the quadratics fitted by Chan and Wallis (1978) in their initial detrending procedure. The smoothness arises because the level variances were constrained to be zero and the q-ratios for the slope are small. The cycles have a period parameter of 9.78 years. The relative importance of the cycles is indicated by State vector analysis. For data in logarithms, the amplitude of each cycle is a percentage of the trend. This is for Lmink 16% and for Lmuskrat 34%.

```
State vector analysis at period 1906
Equation Lmink

                        Value       Prob
Level                10.89963 [0.00000]
Slope                 0.00116 [0.95299]
Cycle 2 amplitude     0.16325 [      -]

Equation Lmuskrat

                        Value       Prob
Level                13.80520 [0.00000]
Slope                 0.02754 [0.51726]
Cycle 2 amplitude     0.34475 [      -]
```

When the two estimated cycles are stored into the database of MINKMUSK, OxMetrics provides graphical tools to plot the two cycles together; given in Figure 6.1. In the dialog **Test**/Components graphics, the boxes for 'Trend' and 'Irregular' should be de-activated and the box for 'Cycles and ARs' should be activated. By selecting the option 'Store selected components in database' in 'Further options...', the program will prompt you to give a name for the series to be stored. It is suggested to keep the default names. Using the **Model**/Graphics option in OxMetrics, the two series can be graphed jointly and also a cross-plot can be drawn.

Using the **Model**/Calculator option in OxMetrics, the series can be shifted backwards and forwards; this is done by taking lags of the series. After some experimenta-

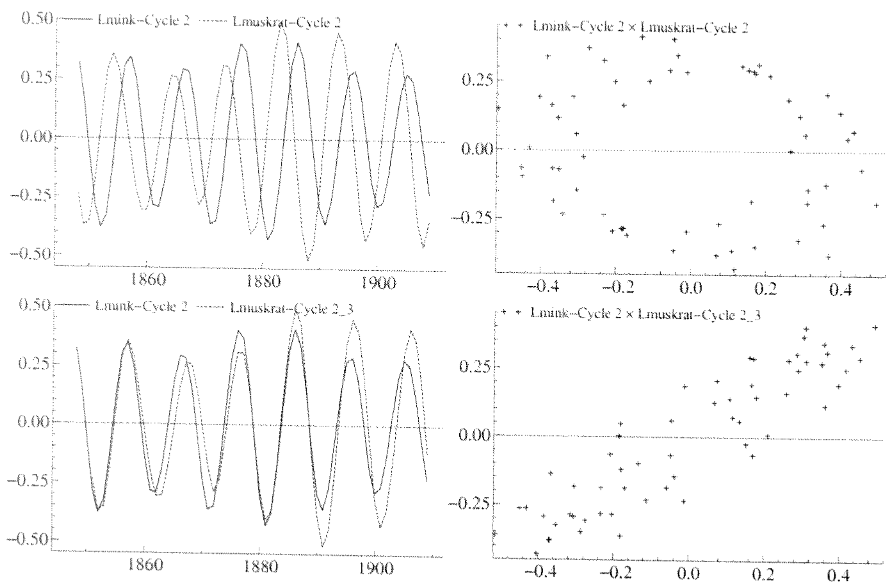


Figure 6.1 The estimated cycles for minks and muskrats.

tion it will be found that the Lmuskrat cycle leads the mink cycle by about three periods. (Take the Lmuskrat series with lag 3). The correlation between the cycle Lmink and the cycle Lmuskrat with lag 3, as shown in Figure 6.1, is 0.9. These results are consistent with the findings of Chan and Wallis.

It is interesting that the variance matrix of the slope variance is estimated to be singular; the meaning of this will be explored in §6.4. You may want to save the joint plots of 'Trend' and 'Slope' using the Graphics dialog of OxMetrics.

The predictions within the sample are very good, but there is predictive failure for the three observations outside the sample — for some reason these appear very different. To make forecasts of future observations using the last three observations, re-estimate the model using the Estimate dialog.

6.3 Autoregression

A stationary first-order VAR may be included in a model as an alternative to, or even as well as, a cycle. The possible lag orders are 1 and 2. The coefficient matrices of the lag polynomials are assumed diagonal while the variance matrix of the disturbance vector can have any specification as given at the end of Section 6.1.

An example of a VAR(2) with a particularly useful interpretation is obtained by a minor modification of the mink and muskrat model fitted in the previous section. In the Select components dialog select 'AR(2)' instead of 'Cycle medium (default 10 years)'.

Thus

$$\begin{aligned} y_t &= \mu_t + \psi_t + \epsilon_t, \qquad \text{where,} \\ \psi_t &= \Phi_1 \psi_{t-1} + \Phi_2 \psi_{t-2} + \kappa_t, \end{aligned} \tag{6.2}$$

where Φ_1 and Φ_2 are diagonal matrices. After estimating this model, the Results window gives the coefficients of the VAR; that is, the elements of Φ_1 and Φ_2. These coefficients are:

$$\widehat{\Phi_1} = \begin{pmatrix} 0.754 & 0 \\ 0 & 1.540 \end{pmatrix}, \quad \widehat{\Phi_2} = \begin{pmatrix} -0.142 & 0 \\ 0 & -0.593 \end{pmatrix}.$$

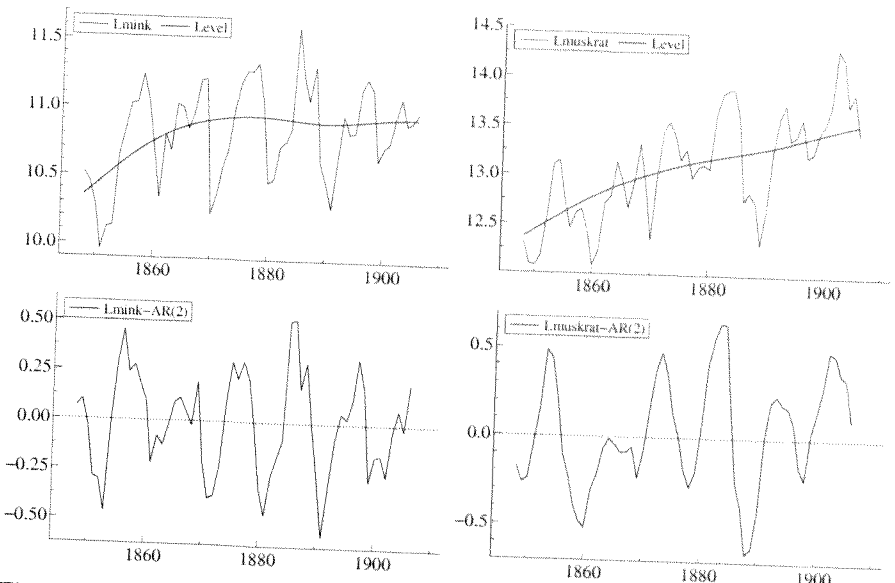


Figure 6.2 The estimated trend and autoregressive components for minks and muskrats.

The VAR(2) component is a multivariate time series process that is able to generate cycles of the form observed earlier; see Figure 6.2. The forecasts show the cycles continuing into the future, but damping down.

6.4 Common factors and cointegration

In a common factor model, some or all of the components are driven by disturbance vectors with less than N elements. *Recognition of common factors yields models which may not only have an interesting interpretation, but may also provide more efficient inferences and forecasts.* Many of the applications described in the next chapter involve common factors and their interpretation.

In terms of a SUTSE model, the presence of common factors means that the variance matrices of the relevant disturbances are less than full rank. The common factor restrictions can be imposed in the Select components dialog by selecting 'Multivariate settings...'. In the Select variance matrices and components for each equation dialog, appropriate variance matrices can be selected and their ranks can be determined.

The first subsection below describes the statistical specification of the common levels model. Common slopes, seasonals and cycles may be formulated along similar lines. Full details can be found in Chapter 9. The irregular variance matrix may also be less than full rank, though this does not, as a rule have a particularly useful interpretation, and there is rarely a case for specifying it as such on prior grounds.

6.4.1 Statistical specification of common levels

Consider the local level model, (6.1), but suppose that the rank of $\mathbf{\Sigma}_\eta$ is $K < N$. The model then contains K common levels or *common trends* and may be written as

$$\begin{aligned} \mathbf{y}_t &= \mathbf{\Theta}\boldsymbol{\mu}_t^\dagger + \boldsymbol{\mu}_\theta + \boldsymbol{\epsilon}_t, & \boldsymbol{\epsilon}_t &\sim \mathsf{NID}(\mathbf{0}, \mathbf{\Sigma}_\epsilon), \\ \boldsymbol{\mu}_t^\dagger &= \boldsymbol{\mu}_{t-1}^\dagger + \boldsymbol{\eta}_t^\dagger, & \boldsymbol{\eta}_t^\dagger &\sim \mathsf{NID}(\mathbf{0}, \mathbf{\Sigma}_\eta^\dagger), \end{aligned} \tag{6.3}$$

where $\boldsymbol{\eta}_t^\dagger$ is a $K \times 1$ vector, $\mathbf{\Theta}$ is an $N \times K$ matrix of *(correlated) factor loadings*, $\mathbf{\Sigma}_\eta^\dagger$ is a variance matrix of full rank K and $\boldsymbol{\mu}_\theta$ is an $N \times 1$ constant vector. The $\mathbf{\Theta}$ matrix consists of K rows that span the identity matrix I_K. The remaining $N - K$ rows of $\mathbf{\Theta}$ are contained in the matrix $\overline{\mathbf{\Theta}}$. The constant vector $\boldsymbol{\mu}_\theta$ has K zero values; the $N - K$ non-zero elements are contained in a vector $\overline{\boldsymbol{\mu}}$. A typical example is

$$\mathbf{\Theta} = \begin{bmatrix} \mathbf{I}_K \\ \overline{\mathbf{\Theta}} \end{bmatrix}, \qquad \boldsymbol{\mu}_\theta = \begin{pmatrix} 0 \\ \overline{\boldsymbol{\mu}} \end{pmatrix}. \tag{6.4}$$

The program allows a re-ordering of the rows of $\mathbf{\Theta}$: if a series is set to 'dependent' in the Select variance matrices ... dialog, it is assigned to the last $N - K$ rows. The same re-ordering then applies to $\overline{\boldsymbol{\mu}}$. The next sub-section provides an illustration.

The model may be recast in the original SUTSE form (6.1) by writing $\boldsymbol{\mu}_t = \mathbf{\Theta}\boldsymbol{\mu}_t^\dagger + \boldsymbol{\mu}_\theta$ and noting that $\mathbf{\Sigma}_\eta = \mathbf{\Theta}\mathbf{\Sigma}_\eta^\dagger\mathbf{\Theta}'$ is a singular matrix of rank K. In case of (6.4), we have

$$\mathbf{\Sigma}_\eta = \mathbf{\Theta}\mathbf{\Sigma}_\eta^\dagger\mathbf{\Theta}' = \begin{bmatrix} \mathbf{\Sigma}_\eta^\dagger & \mathbf{\Sigma}_\eta^\dagger\overline{\mathbf{\Theta}}' \\ \overline{\mathbf{\Theta}}\mathbf{\Sigma}_\eta^\dagger & \overline{\mathbf{\Theta}}\mathbf{\Sigma}_\eta^\dagger\overline{\mathbf{\Theta}}' \end{bmatrix}, \qquad \boldsymbol{\mu}_t = \mathbf{\Theta}\boldsymbol{\mu}_t^\dagger + \boldsymbol{\mu}_\theta = \begin{pmatrix} \boldsymbol{\mu}_t^\dagger \\ \overline{\boldsymbol{\mu}} + \overline{\mathbf{\Theta}}\boldsymbol{\mu}_t^\dagger \end{pmatrix}.$$

The $K \times K$ variance matrix $\mathbf{\Sigma}_\eta^\dagger$ is decomposed by the Cholesky decomposition

$$\mathbf{\Sigma}_\eta^\dagger = \mathbf{LDL}', \tag{6.5}$$

where $\mathbf{L}$ is a $K \times K$ unity lower triangular matrix and $\mathbf{D}$ is a $K \times K$ diagonal positive matrix. The lower coefficients in $\mathbf{L}$ and the diagonal elements of $\mathbf{D}$ are estimated

together with elements in $\overline{\mathbf{\Theta}}$ for each variance matrix associated with a disturbance vector. When there are no common trends, so $N = K$, the coefficient matrix $\overline{\mathbf{\Theta}}$ is a null matrix and matrices $\mathbf{L}$ and $\mathbf{D}$ are for the Cholesky decomposition of $\mathbf{\Sigma}_\eta$. Also, when $N = K$ vector $\overline{\boldsymbol{\mu}}$ is null and $\boldsymbol{\mu}_t = \boldsymbol{\mu}_t^\dagger$.

The presence of common trends implies what econometricians call *cointegration*. While the observations are integrated of order one; that is, they must be differenced once to make them stationary, there exists an $(N - K) \times N$ matrix of cointegrating vectors, $\mathbf{A}$, such that $\mathbf{A}\mathbf{y}_t$ is stationary. This means that $\mathbf{A}\mathbf{\Theta} = \mathbf{0}$; see the discussion in Harvey (1989, Ch. 8). Testing procedures for common trends in multivariate structural time series models have been developed by Nyblom and Harvey (2001).

6.4.2 An example of common levels: road casualties in Britain

The file SEATBQ contains quarterly time series on drivers and rear seat passengers killed and seriously injured in road accidents in Great Britain. These are labelled 'Drivers' and 'Rear' in SEATBQ. Select them both in the Formulate dialog and label them both as 'Y' (right-click Rear to set it as a 'Y' variable, the same as Drivers). A graph of the series indicates that the local level is appropriate for the trend, so deselect 'Slope' in the Select components dialog (click the 'Slope' checkbox so the tick disappears). A seasonal must also be included, but we shall ignore it in the discussion. Estimate the model with no restrictions, but for reasons which will become apparent in §6.6, only use the observations up to the end of 1982. Thus, set 'Estimation ends at' to 82(4) in the Estimate dialog. The parameter estimates of the covariance/correlation matrix for the level disturbance is estimated by

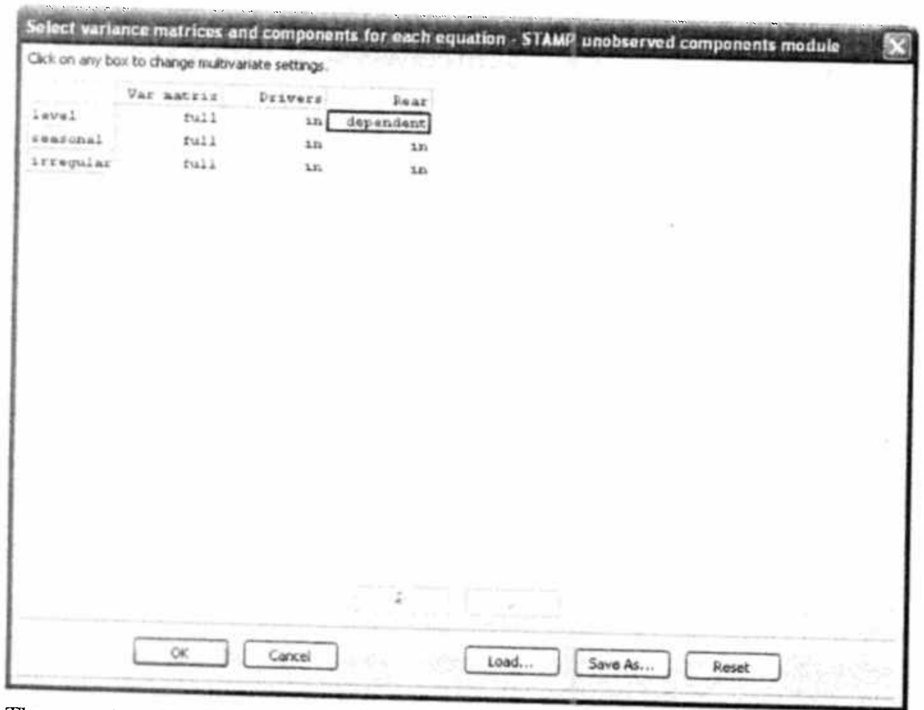
```
Level disturbance variance/correlation matrix:
              Drivers          Rear
Drivers     0.0005772        0.8908
Rear        0.0003919     0.0003353
```

The high correlation suggests that there may be a common factor. The 'Variances' option in the dialog **Test**/More written output also reveals that

```
Cholesky decomposition LDL' with L and D given by
              Drivers          Rear
Drivers         1.000        0.0000
Rear           0.6790         1.000
diag(D)     0.0005772     6.922e-005
Eigenvectors and eigenvalues are given by
                  Drivers          Rear
Drivers            0.8046        0.5938
Rear               0.5938       -0.8046
eigenvalues     0.0008664    4.611e-005
```

The small eigenvalue for Rear suggests that the rank of the Level disturbance variance matrix may be one.

Now return to the Select components dialog and ensure that the option 'Multivariate settings' is selected. In the dialog Select variance matrices and components for each equation, the level component for Rear can be made fully dependent on the trend for Front by changing the entry 'in' of level (row) and Rear (column) to 'dependent'. This can be achieved by double-clicking on this entry and selecting the appropriate label 'dependent'. In effect, the rank of the level disturbance variance matrix has changed from '2' to '1'.



Then continue by pressing OK. The covariance matrix of the level disturbances is now given by

```
Level disturbance variance/correlation matrix:
              Drivers         Rear
Drivers     0.0005690        1.000
Rear        0.0004491    0.0003544
Level disturbance factor variance for Drivers: 0.000568957
Level disturbance factor loading  for Rear: 0.789258
               Drivers         Rear
Constant        0.0000      0.08107
```


Ignoring the seasonals, the estimated model may be written as:

$$\begin{aligned} y_{1t} &= \mu_t^{\dagger} + \varepsilon_{1t}, \\ y_{2t} &= 0.789\mu_t^{\dagger} + 0.0811 + \varepsilon_{2t}, \end{aligned}$$

where $\mu_t^{\dagger}$ is a *univariate* random walk. Thus we have the following relationship between the level components as they appear in the two series:

$$\mu_{2t} = 0.789\mu_{1t} + 0.0811.$$

Since the data are in logs, the relationship between the original trends is:

$$Trend_2 = \exp(0.0811) Trend_1^{0.789}.$$

Finally note that the system is cointegrated of order $(1, 1)$, with cointegrating vector $(1, -1/.789) = (1, -1.267)$.

6.4.3 Balanced levels

In a local level model with a single common factor, the trends in the series will only be parallel (or proportional if the data are modelled in logs and anti-logs have been taken) if the elements of the standardised load matrix, Θ, are all unity. Since the first element is always unity, this implies $N-1$ restrictions. These *balanced level* restrictions may be imposed in two ways. The easiest way is go back to the Select components dialog and ensure that the option 'Multivariate settings' is selected. In the dialog Select variance matrices and components for each equation, reset the level entry under 'Rear' to 'in' and set the 'Var matrix' to 'ones' (double-click on the entry 'full' for level 'Var matrix' and select 'ones'). The estimated level disturbance variance matrix is given by

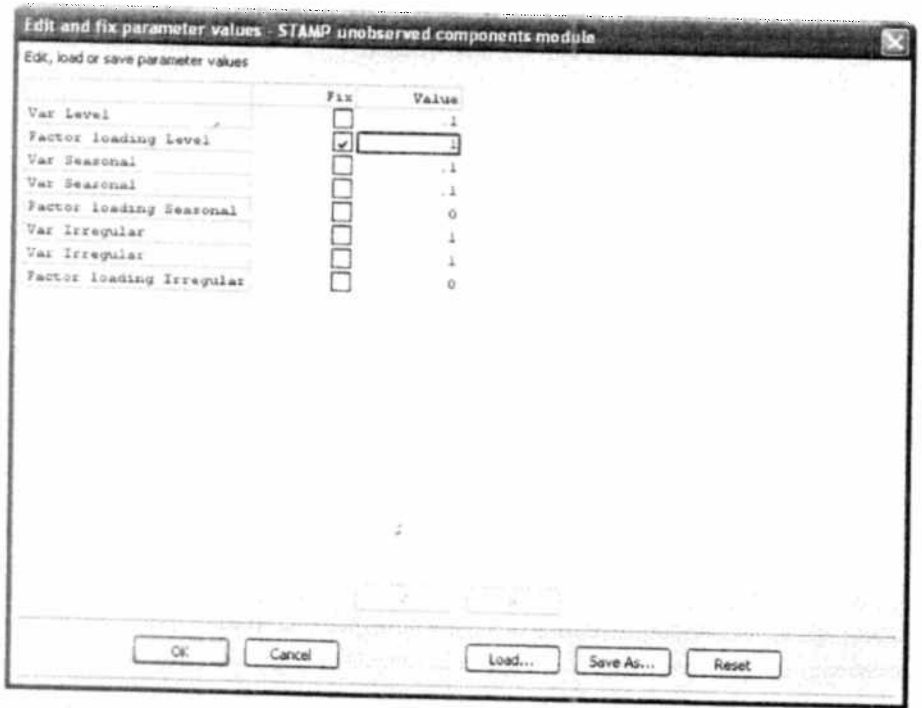
```
Level disturbance variance matrix of ones, scaled:
              Drivers         Rear
Drivers     0.0005011    0.0005011
Rear        0.0005011    0.0005011
```

The output in the Results window indicates that the fit is excellent.

```
Summary statistics
                     Drivers         Rear
T                     56.000       56.000
p                     2.0000       2.0000
std.error           0.082540      0.10451
Normality             3.9943       1.2563
H(17)                0.70313      0.74554
DW                    2.0218       2.1373
r(1)               -0.023856    -0.087228
q                     8.0000       8.0000
```

```
r(q)        -0.10296    -0.14327
Q(q,q-p)     9.2697      2.6926
Rs^2         0.49474     0.48793
```

In this case, an alternative way to enforce the restriction of a factor loading of one for the level of Rear is to keep its level as dependent and enforce the unity factor loading via the option 'Set parameters to default values and edit' in the Select components dialog (click on the appropriate radio button which can be found in the dialog below). In the dialog Select variance matrices and components for each equation we select a full 'Var matrix' for level and a 'dependent' level for Rear, as before. In the next dialog Edit and fix parameter values, click on the 'Fix' checkbox of 'Factor loading Level' and change its corresponding 'Value' from 0 to 1.



After estimation, the results are effectively the same as it should be. The level disturbance variance matrix is reported slightly differently in this case

```
Level disturbance variance/correlation matrix:
             Drivers        Rear
Drivers    0.0005011       1.000
Rear       0.0005011   0.0005011
Level disturbance factor variance for Drivers: 0.000501133
Level disturbance factor loading  for Rear: 1
```



```
              Drivers        Rear
Constant       0.0000      -1.491
```

The factor loading matrix Θ for the level is now just a column of ones while the estimate of $\overline{\mu}$ is -1.491. A plot of both trends shows them to be parallel. In the unrestricted model estimated in the previous subsection this was not the case. However, in both models the forecast functions are parallel. In the unrestricted model, the estimate of the second element in the factor loading matrix was 0.789. The likelihood ratio statistic for the hypothesis that the true value is one is $LR = 246.089 - 245.662 = 0.427$. Thus the hypothesis of a unit factor loading is easily accepted.

6.4.4 Common trends

Common levels is a special case of common trends as it arises when the trend is just a multivariate random walk. Models with common slopes may be formulated along similar lines. The full implications of different combinations of ranks of level and slope disturbances are laid out in Chapter 9. Here we just look at an important special case: *smooth trends with common slopes.*

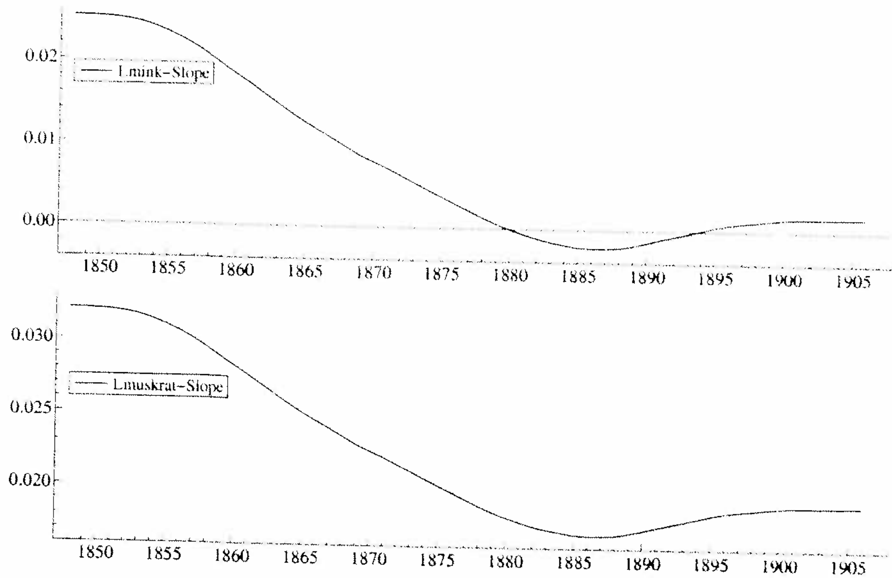


Figure 6.3 Time series plots of estimated slopes for MINKMUSK.

The simplest case to appreciate is where the variance matrix of the slope disturbances is less than full rank but the variance matrix of levels is null so that the estimated trends are relatively smooth. The mink–muskrat example of section 6.2 showed such a situation. Recompute the model (trend plus medium cycle plus irregular) imposing the constraint that the level is fixed and the slope of Lmuskrat is 'dependent' so that the rank of the slope disturbance variance matrix is one. Do not forget to change the estimation sample by typing '1906' in the entry for 'Estimation ends at' in the Estimate dialog. The output shows the standardised load matrix is $(1, \theta)' = (1, 0.555)'$ while the estimate of $\overline{\beta}$ is 0.018. Thus the growth rate in muskrats; that is, the slope in the second series, is given by :

$$\beta_{2t} = \theta\beta_{1t} + \overline{\beta} = 0.555\beta_{1t} + 0.018.$$

The graph of the slopes shows this exact linear relationship; see Figure 6.3. Note that the muskrat growth rate is higher even though its stochastic growth component is less. The fitted model may be written:

$$\begin{aligned} y_{1t} &= \mu_t^\dagger + \varepsilon_{1t}, \\ y_{2t} &= 0.555\mu_t^\dagger + 7.6 + 0.018t + \varepsilon_{2t}, \end{aligned}$$

where the constant term in the second (muskrat) equation is calculated from the levels given in the Final state; that is, since

$$\overline{\mu} = -\theta\mu_{1T} + \mu_{2T},$$

its estimate is $-0.555(10.93) + 13.67 = 7.6$.

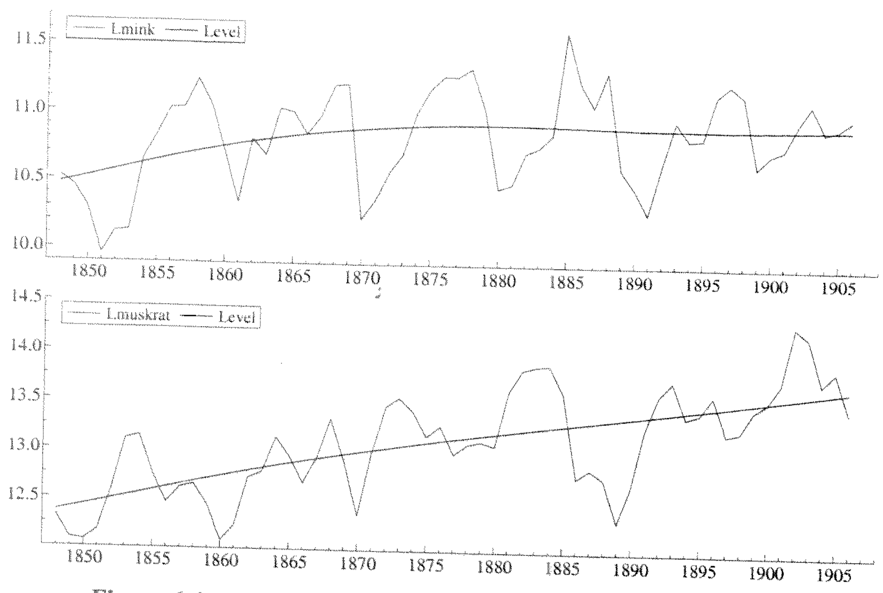


Figure 6.4 Time series plot of estimated trends for MINKMUSK.

As regards the relationship between the two trends:

$$\mu_{2t} = \theta\mu_{1t} + \overline{\mu} + \overline{\beta}t = 0.555\mu_{1t} + 7.6 + 0.018t,$$

and so one trend is a multiple of the other plus a deterministic trend component; see Figure 6.3. It is only the stochastic movements in the trends which are in common. Finally note that the system is cointegrated of order $(2, 2)$; that is, the series are integrated of order 2 and there is a combination of them which is stationary. The cointegrated trends are displayed in Figure 6.4.

Although not appropriate here, it should be noted that balanced slopes ($\theta = 1$) can be imposed by setting the slope 'Var matrix' to 'ones'.

6.4.5 Common seasonals

Common factors in seasonality imply a reduction in the number of disturbances driving changes in the seasonal patterns. A single common factor does not imply that the seasonal patterns will be proportional unless the deterministic seasonal components outside the common seasonals are all zero. What it does mean is that the changes in the seasonal patterns in the different series come from a common source.

For trigonometric seasonals it is possible, in principle, to have different disturbance variance matrices for each of the seasonal frequencies, thereby allowing common factors in some frequencies but not in others. This implies seasonal cointegration at different frequencies; see Hylleberg, Engle, Granger and Yoo (1990). The current version of STAMP only allows full seasonal cointegration ; that is, common factors at all frequencies. Testing procedures for common seasonal factors are developed by Busetti (2006).

6.4.6 Common cycles

With *common cycles* , a vector of constant terms, corresponding to the non-zero elements in $\boldsymbol{\mu}_\theta$ for the level in (6.3), is not included since the expectation of a cycle is zero.

It should be clear that common cycles embody much stronger restrictions than similar cycles. Thus with two series and one common cycle, one cycle would be proportional to the other. Common cycles are like the common feature cycles of Engle and Kozicki (1993).

A test of the null hypothesis that there are a fixed number of common cycles can be based on the LR statistic obtained from fitting the model with the common cycle restriction and the (unrestricted) similar cycle model. The distribution theory is complicated by boundary conditions, but is described in Carvalho, Harvey and Trimbur (2007). When there are only two series, the appropriate significance point for a test at the 5% level of significance is given by the 10% critical value of a chi- square with one degree of freedom, that is 2.71.

6.4.7 Factor rotations

When there is more than one common factor, they are not unique and issues of interpretation arise. This leads on to factor rotations. Consider the local level model. The first step is to formulate the multivariate local level model (6.3) in such a way that the common factors are uncorrelated with each other and have unit variance, that is

$$y_t = \Theta^* \mu_t^* + \mu_\theta + \epsilon_t, \quad \epsilon_t \sim \mathrm{NID}(0, \Sigma_\epsilon),$$

$$\mu_t^* = \mu_{t-1}^* + \eta_t^*, \quad \eta_t^* \sim \mathrm{NID}(0, \mathbf{I_K}),$$

where Θ^* is an $N \times K$ matrix offactor loadings given by $\Theta^* = \Theta \mathbf{L} \mathbf{D}^{1/2}$, the new components are $\mu_t^* = \mathbf{D}^{-1/2}\mathbf{L}^{-1}\mu_t^\dagger$ and $\mathbf{L}$ and $\mathbf{D}$ are from the Cholesky decomposition in (6.5). Since $\mathbf{L}$ is lower triangular, Θ is such that its elements, θ_{ij}, are zero for $j > i$ and $i = 1, ..., K$.

When there is more than one common factor, they are not unique and a *factor rotation* may give components with a more interesting interpretation. Let $\mathbf{H}$ be a $K \times K$ orthogonal matrix. The matrix of factor loadings and the vector of common trends can then be redefined as $\Theta^\ddagger = \Theta \mathbf{H}'$ and $\mu_t^\ddagger = \mathbf{H}\mu_t^\dagger$ yielding

$$\begin{aligned} y_t &= \Theta^\ddagger \mu_t^\ddagger + \mu_\theta + \epsilon_t, & \epsilon_t &\sim \mathrm{NID}(0, \Sigma_\epsilon), \\ \mu_t^\ddagger &= \mu_{t-1}^\ddagger + \eta_t^\ddagger, & \eta_t^\ddagger &\sim \mathrm{NID}(0, \mathbf{I_K}). \end{aligned}$$

The disturbances driving the common trends are still mutually uncorrelated with unit variance.

A number of methods for carrying out rotations have been developed in the classical factor analysis literature. These may be employed here. The program does not offer an option for computing rotations at present, but for two factors a commonly used rotating matrix is

$$\mathbf{H} = \begin{bmatrix} \cos\lambda & -\sin\lambda \\ \sin\lambda & \cos\lambda \end{bmatrix}$$

with the angle, λ, being set by a graphical method. The aim is often to give a factor significant loadings, perhaps all positive, on some variables while the other variables get loadings near zero; see Harvey, Ruiz and Shephard (1994) for a stochastic volatility application using the EXCH data.

6.5 Explanatory variables and interventions

Explanatory variables and interventions may be included in multivariate models. Thus

$$y_t = \mu_t + \gamma_t + \psi_t + \sum_{\tau=1}^{r} \Phi_\tau y_{t-\tau} + \sum_{\tau=0}^{s} \mathbf{D}_\tau \mathbf{x}_{t-\tau} + \Lambda \mathbf{w}_t + \epsilon_t, \quad t = 1, \ldots, T,$$

where $\mathbf{x}_t$ is a $K \times 1$ vector of explanatory variables and $\mathbf{w}_t$ is a $K^* \times 1$ vector of interventions. Elements in the parameters matrices, Φ, $\mathbf{D}$, and Λ may be specified to be zero, thereby excluding certain variables from particular equations.

The next section gives an example of how explanatory variables and interventions can be included in a multivariate model.

6.6 Assessing the effect of the seat belt law using a control group

The effect of an intervention on a series can be modeled in a univariate framework as described in the previous chapter. However, suppose observations are available on a second series that is highly correlated with the series of interest but is not itself affected by the intervention. In this case it is possible to construct a bivariate model and so use the second series as a *control group*. This should result in a more precise measure of the intervention effect. The multivariate capability of STAMP offers the possibility of applying this technique.

The file SEATBQ was used earlier to construct a multivariate model of front and rear seat passengers killed and seriously injured (KSI) in road accidents in cars in Great Britain. Data on the number of kilometres travelled and the real price of petrol is also included in the file and Harvey and Durbin (1986) used these data in their study of the effect of the seat belt law of the first quarter of 1983.

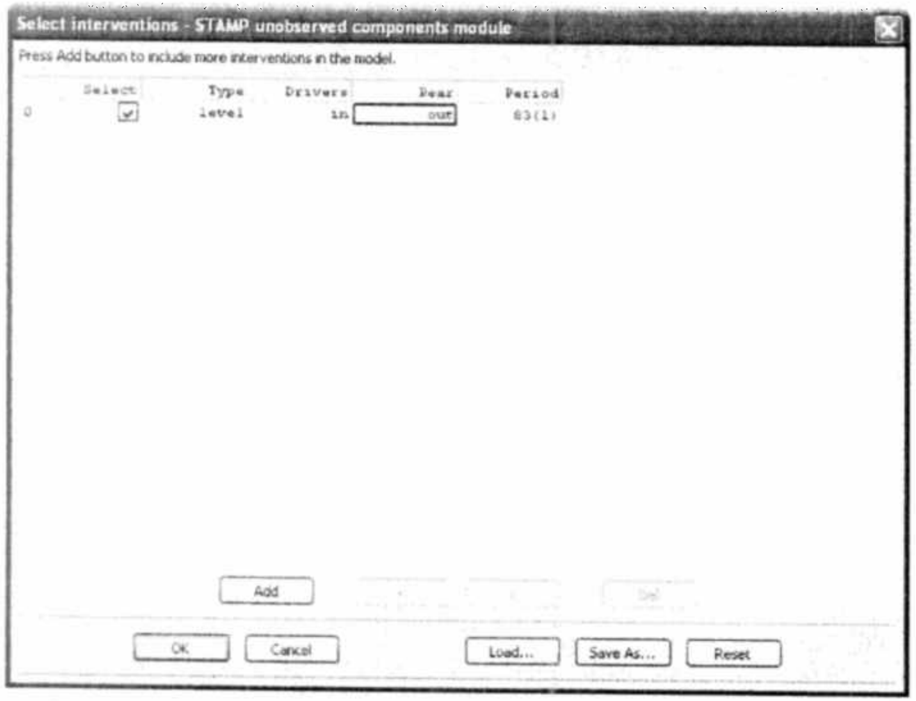
Apart from the unavailability of suitable software, the main reason why Harvey and Durbin (1986) did not carry out intervention analysis with control groups was because of the lack of a suitable control group. Here we will use rear seat passengers as a control for front seat passengers. Although rear seat passengers were not required to wear belts by the law, it has been argued that they may not be a good control since:

(1) if, as the risk compensation theory implies, drivers wearing belts drive worse, there will be more rear seat passengers injured;
(2) rear seat passengers may sustain more serious injuries because those in the front are wearing belts and hence remain in their seats on impact;
(3) passengers might have transferred, or been transferred, to the rear so that a belt need not be worn;
(4) more rear seat passengers may have followed those in the front in wearing belts.

Despite these objections, there is little statistical evidence to suggest that the seat belt law had any effect on rear seat passengers KSI (try fitting an intervention model). Nevertheless, it should be borne in mind that the first three of the points above would all result in the effect of the law on the front seat occupants being exaggerated.

The procedure described in sub-section 6.4.3 is modified only slightly. In the Select components dialog, remove the slope component (deselect by clicking on checkbox

'Slope') as before and select 'manually...' under the option 'Select interventions'. Press OK. The Select interventions dialog appears. Set it so there is a level variable at 83(1). Then click on the box below 'Rear' and select 'out'. The dummy variable then only affects Drivers.



On estimating the model with the full sample the coefficient of the level dummy is found to be −0.240 with a t-statistic of −6.172. Figure 6.5 shows the two smoothed (balanced) levels with the intervention effect clearly showing for Drivers.

Explanatory variables, Kms and Petrol, may be added. Try this with 'Front' and 'Rear'. It will be found that Petrol is not statistically significant in the Rear equation, nor is Kms in the Front. They may be excluded by checking 'Set regression coefficients...' and changing the boxes under the appropriate variables to 'out'.

6.7 Exercises

(1) Construct a joint model for Drivers and Front seat passengers in SEATBQ. Try the common trend specifications. Extend the model to include the explanatory variables, Kms and Petrol.

(2) Estimate a bivariate model for MINKMUSK with a level included and the trend

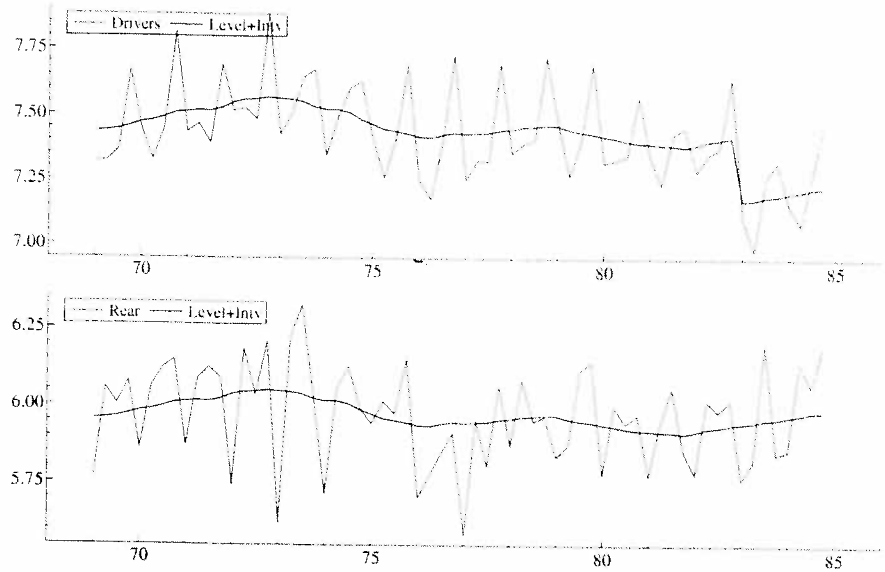


Figure 6.5 Smoothed levels of Drivers and Rear Seat Passengers killed and seriously injured in Great Britain, with allowance made for the seat belt law..

variance matrix restriction imposed of 'ones'. What is the implication of this restriction and how does the model compare with the one fitted in §6.2?

Chapter 7

Applications in Macroeconomics and Finance

This section gives some illustrations of the uses of STAMP. The focus will be on macroeconomics and finance, but the methods shown here could have easily been used in many other subjects.

7.1 Univariate trend-cycle decompositions: GDP

Model-based trend-cycle decompositions can be carried out by STAMP in different ways. The trend-cycle decomposition model is given by

$$y_t = \mu_t + \psi_t + \epsilon_t, \qquad \epsilon_t \sim \mathsf{NID}(0, \sigma_\epsilon^2), \qquad \mathsf{t} = 1, \ldots, \mathsf{T},$$

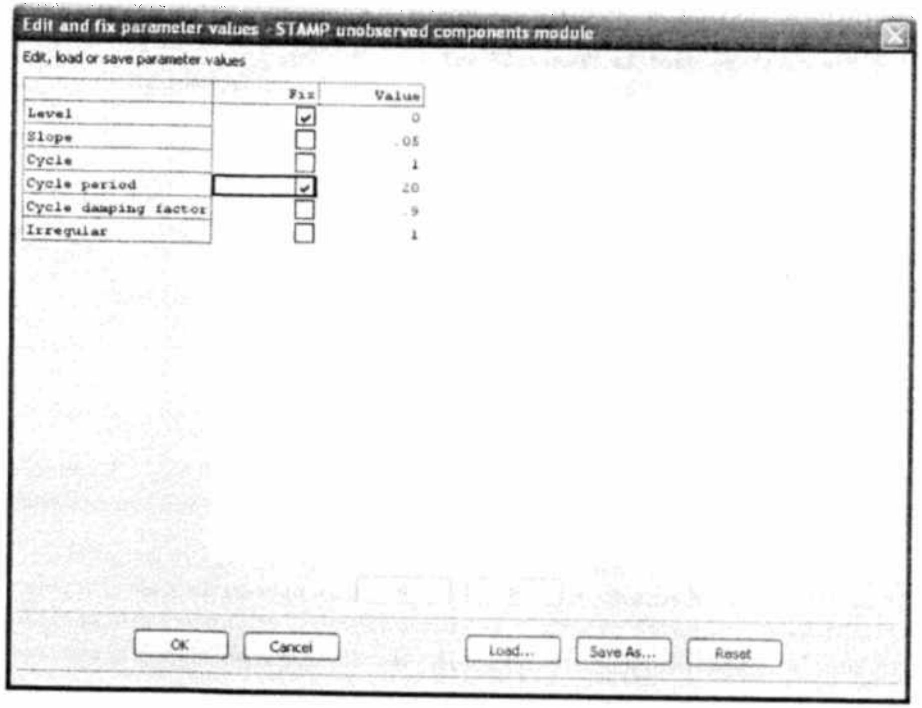
where μ_t represents the trend, ψ_t the cycle and ϵ_t the irregular component. Different specifications of the three components are considered in the literature. The following typical specifications may be considered in STAMP and can be specified in the dialog **Formulate**/Select components:

- The trend is a random walk with fixed drift ('Level, Stochastic' and 'Slope, Fixed'), the cycle is an autoregressive component of order 2 ('AR(2)') and the irregular is white noise or zero; see, for example, Clark (1989).
- The decomposition model consists of a local linear trend ('Level, Stochastic' and 'Slope, Stochastic'), a stationary trigonometric cycle at a typical business-cycle frequency ('Cycle short' or 'Cycle medium') and an irregular; see Harvey (1989) for a more detailed exposition.
- The trend is imposed to be smooth by having 'Level, Fixed', 'Slope, Stochastic' and possibly with value of 'Order of trend' larger than 1 in the trend specification (4.6); the cycle is generalised as in (4.11) where 'Order of cycle' k is typically chosen as 2 or 3 and the irregular is the standard white noise sequence; see Harvey and Trimbur (2003).

Mixtures and extensions of these specifications can also be considered. For example, the cycle component can be the sum of two cycles at different frequencies.

The parameters of the different trend-cycle decomposition models are estimated by maximum likelihood. The filtering properties are revealed by weight functions (time domain) and gain functions (frequency domain). To illustrate the facilities provided by STAMP, we consider the US GDP quarterly time series from 1947(1)-2007(2) (in logs) from the database file 'USmacro07.in7'. The three basic trend-cycle decomposition models can be estimated by maximum likelihood. However, in case the generalised cycle is included, the business-cycle frequency may need to be enforced. This is illustrated below.

The smooth trend plus generalised cycle plus irregular model is considered for logged US GDP below. Select the time series 'LGDP' from the database in the Formulate dialog, press OK and select 'Level, Fixed', 'Slope, Stochastic' and keep 'Order of trend' to its value 1. De-select 'Seasonal', keep 'Irregular' and expand the section 'Cycle(s)'. Select 'Cycle short' and increase the value of 'Order of cycle to 3. In section 'Options', select 'Edit and fix parameter values' and press OK. The Edit and fix parameter values dialog appears:



It presents the parameters that need to be estimated and their associating (starting) values. The first column has the name of the parameter, when only the name of the component appears, the parameter is the variance of this component. The value of the variances are relative to each other. In other words, the q-ratios appear where the q-ratios for the trend-cycle model are given by

$$q_\eta = \sigma_\eta^2 \,/\, \sigma^2, \quad q_\zeta = \sigma_\zeta^2 \,/\, \sigma^2, \quad q_\kappa = \sigma_\kappa^2 \,/\, \sigma^2, \quad q_\epsilon = \sigma_\epsilon^2 \,/\, \sigma^2,$$

where σ^2 is any of the variances σ_η^2, σ_ζ^2, σ_κ^2 or σ_ϵ^2, the one with the largest value is typically chosen.

By default, the largest variance is set to one and is typically σ_ϵ^2. The Edit and fix parameter values dialog also allows the user to fix parameter values. Since the dialog Select components has the level equation set by 'Level, Fixed', the variance of the 'Level' is fixed at zero here. In the Edit and fix parameter values dialog you can effectively change the level equation by changing its variance value to a nonzero value and re-activate it for estimation.

The estimation of this trend-cycle model usually requires fixing the cycle frequency to a typical business-cycle frequency of five years or twenty quarters. Therefore, the cycle frequency is fixed at 20 in this dialog by clicking on the box in the column 'Fix' and the row 'Cycle period'. Press OK to start the estimation process by pressing OK in the Estimation dialog. We obtain the following output:

```
UC( 1) Estimation done by Maximum Likelihood (exact score)
    The database used is USmacro07.in7
    The selection sample is: 1947(1) - 2007(2) (T = 242, N = 1)
    The dependent variable Y is: LGDP
    The model is:  Y = Trend + Irregular + Cycle 1
    Steady state. found

Log-Likelihood is 1119.67 (-2 LogL = -2239.33).
Prediction error variance is 8.71598e-005

Summary statistics
                 LGDP
 T             242.00
 p             5.0000
 std.error  0.0093359
 Normality     28.928
 H(80)        0.20970
 DW            1.9811
 r(1)       0.0090644
 q             19.000
 r(q)       -0.031852
 Q(q,q-p)      17.710
 Rd^2        0.096722

Variances of disturbances:
                  Value     (q-ratio)
Level          0.000000  (   0.0000)
Slope      9.27962e-007  (  0.05981)
Cycle      1.40807e-005  (   0.9075)
Irregular  1.55154e-005  (    1.000)

Cycle other parameters:
```



```
Variance                 0.00002
Period                  20.00000
Period in years          5.00000
Frequency                0.31416
Damping factor           0.60098
Order                    3.00000

State vector analysis at period 2007(2)
                          Value        Prob
Level                   9.35317  [0.00000]
Slope                   0.00670  [0.01262]
Cycle 1 amplitude       0.00326  [      -]
```

The logged GDP series is considered by all three decompositions. A graphical illustration of a decomposition is presented in Figure 7.1.

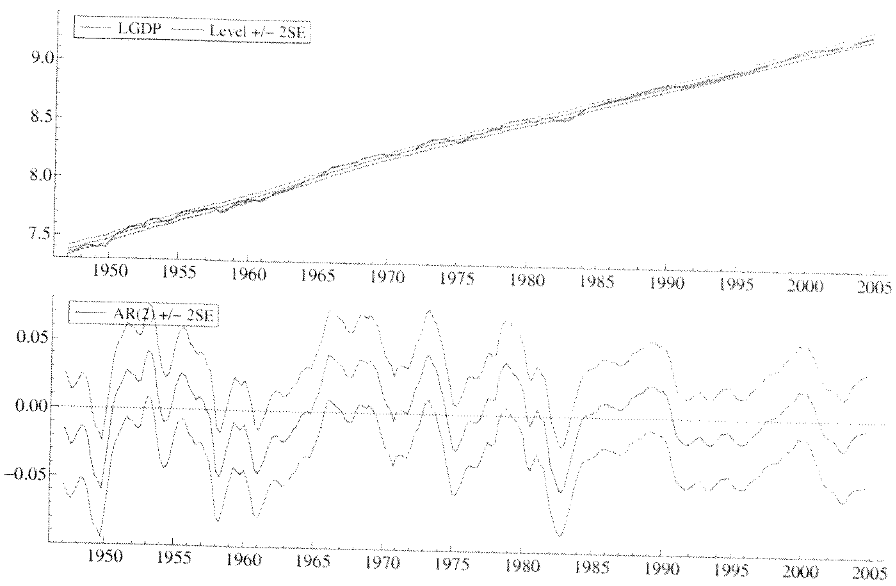
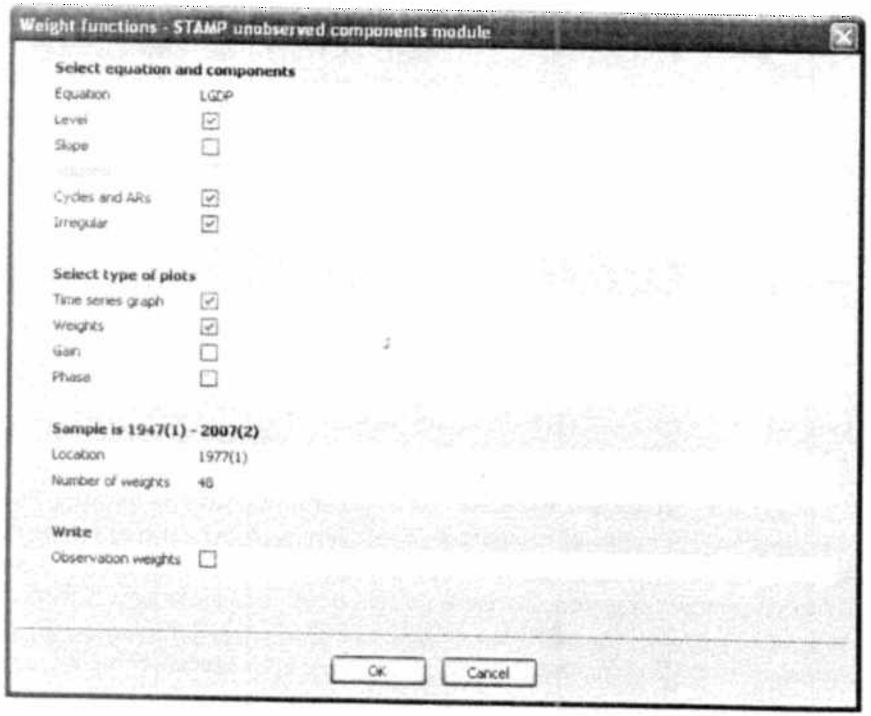


Figure 7.1 US GDP trend-cycle decompositions with a random walk plus fixed drift trend and an AR(2) cycle.

It is argued by Harvey and Koopman (2000) that it is of interest to study the observation weights of the estimated components. Since the models are linear and we assume Gaussian disturbances, the estimated components are linear functions of the observed time series. For example, consider the estimated trend component μ_t at a particular time point $t = \tau$. Observations in the neighboorhood of τ will get more weight than observations more distanced from τ for the estimate of μ_τ. In fact, it can be shown that for so-called time-invariant models, the weights have an exponentially decay as the distance of the observations from τ increases. Also the weight patterns are symmetric in the middle of the time series. However, when the value of τ approaches 1 or T, the weight pattern becomes more and more asymmetric since no observations are available either before time point 1 or after T. Finally it is noted that the weights for μ_τ sum up to one while the weights for the slope, seasonal and irregular components sum up to zero.

The frequency domain equivalence of the weight function is the frequency gain function. It indicates the relative importance of frequencies for the extracted component. In the context of business-cycle tracking, some emphasis to the frequency gain function is given in articles such as Baxter and King (1999) and Christiano and Fitzgerald (2003). The gain function of the estimated cycle component should be band-passed such that only "weight" is given to the business-cycle frequencies (typically the frequencies associated with the range of 2 to 10 years). Harvey and Trimbur (2003) show that in a model-based framework, the estimated cycle component can also possess band-pass filter properties when the cycle specification (4.11) is taken with higher values for k. An illustration of this is given below.

We return to the illustration of the US GDP trend-cycle decomposition based on a smooth trend plus generalised cycle plus irregular model. Go to the **Test/Weight** functions dialog and select the options as:



By pressing OK, Figure 7.2 appears. The weights are for the estimated components at time point τ associated with 1977 quarter 1. This is also indicated by the vertical line

in the middle of the estimated component plots in first column of graphs. The dialog allows you to choose this position for τ. The smoothed estimate of the trend component is implicitly computed by

$$\hat{\mu}_\tau = \sum_{j=1-\tau}^{T-\tau} w_j y_{\tau+j},$$

and the weights w_j are presented for index $j = 1-\tau, 2-\tau, \ldots, T-1-\tau, T-\tau$. The trend estimate is rather a smooth function and this is reflected by the distibution of the weights w_j around $j = 0$. The decay to zero is slow (it takes around 6 years !). The decay in the weighting pattern for the cycle component is also slow but the relative importance of the more distanced observations is less compared to those close to τ. Approximately 50% of the weights is determined by the weights of $\tau - 1$, τ and $\tau + 1$. This is even more so for the estimated irregular component. The frequency gain functions are close to a typical decomposition in macroeconomics: the trend is based on the low frequencies, the cycle captures the typical business-cycle frequencies and the high frequencies are for the irregular component.

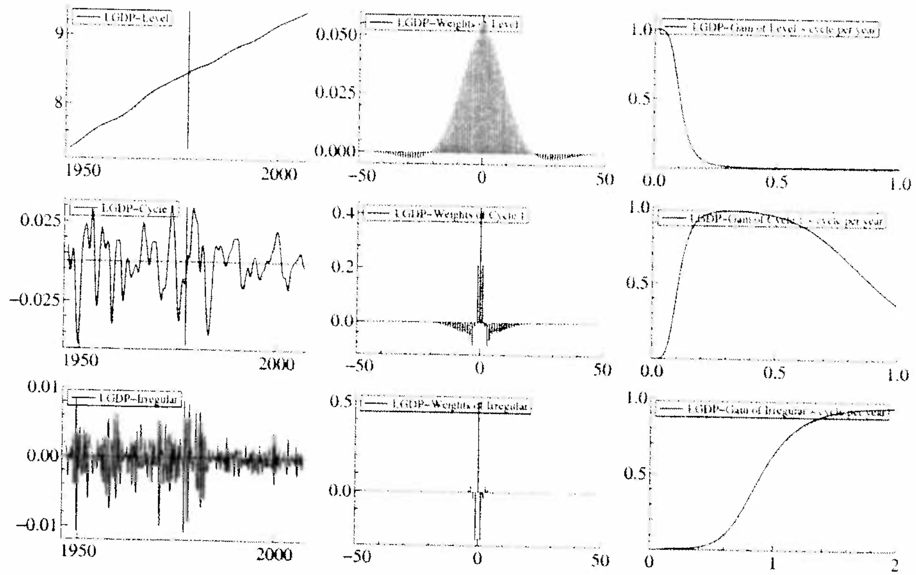


Figure 7.2 Weight and gain functions for the estimated US GDP trend-cycle decomposition based on a smooth trend plus generalised cycle plus irregular model.

Real-time business-cycle tracking focuses on the construction of the filtered estimate of the cycle component. Given that a new observation has just arrived, how do we weight the new observation and the past ones to produce the real-time indicator of the business-cycle ? To obtain some insights, the weight and gain functions can be presented for $\tau = T$. These are shown in Figure 7.3. Further, the different estimates of the cycle component (prediction, filtering and smoothing) can be obtained from the **Test**/More written output dialog by selecting the option 'Cycles and ARs' in the section 'Print recent state values' (expand this section and set value of 'Number of recent periods' to 8). In the Results window, the output is given by

```
Final state values of Cycle 1
          Coef(|t-1)    Coef(|t)     Coef(|T)   Rmse(|t-1)    Rmse(|t)     Rmse(|T)
2006(1)    0.003146    0.003376     0.006210     0.01458      0.01458     0.01038
2006(2)    0.004205    0.004176     0.005976     0.01458      0.01458     0.01086
2006(3)    0.003766    0.003617     0.003452     0.01458      0.01458     0.01158
2006(4)   0.0009131   0.0009323    0.0001707     0.01458      0.01458     0.01269
2007(1)  -0.0007951  -0.0009163    -0.002540     0.01458      0.01458     0.01396
2007(2)   -0.003155   -0.003023    -0.003023     0.01458      0.01458     0.01458
```

The results show that revision of estimates has taken place in the periods before 2007(2) but these revisions are not significant given the standard errors in the range of 0.01 and 0.015. It should also be noted that the business-cycle fluctuations in the years 2006 and 2007 are not significant, perhaps with the exception of the smoothed estimate for 2006(1).

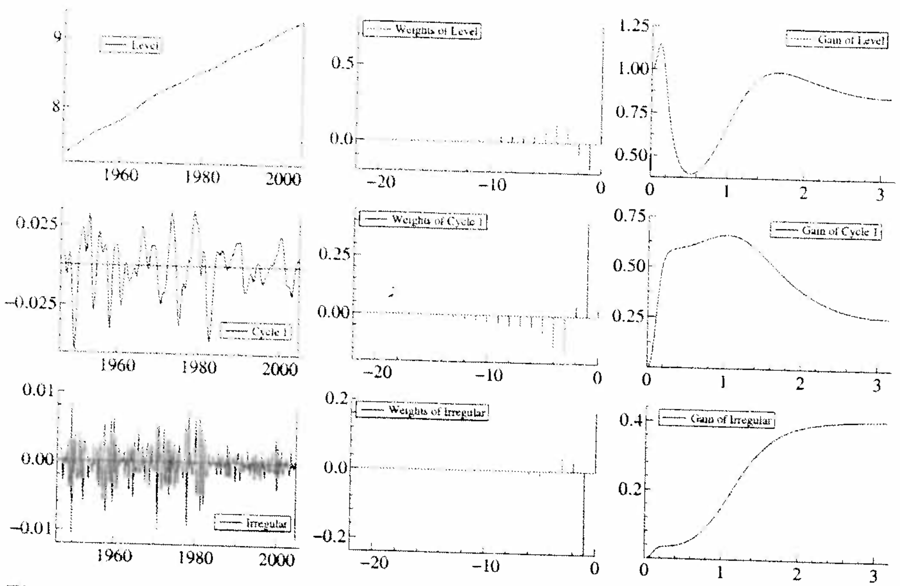


Figure 7.3 Weight and gain functions for the estimated US GDP trend-cycle decomposition based on a smooth trend plus generalised cycle plus irregular model (end-of-sample estimates: filtering).

7.2 Multivariate trends and cycles: GDP and Investment

It is shown in the previous section that US GDP can be decomposed into a trend and a cycle by setting the level variance to be 'Fixed' (though sometimes it is estimated as zero even if it is not constrained at the outset). Using the USmacro07 data over the full period gives (not forgetting to deselect the 'Seasonal' component) a plausible cycle, albeit one that is rather noisy as the irregular variance is estimated to be zero. However, it is not easy to get a good fit with a higher order cycle using this series, see previous section. Another solution is to estimate GDP jointly with Investment as the latter has relatively larger cyclical movements. As shown below, a model with balanced slopes and a fourth-order cycle works well.

To set up the model, proceed to the Formulate a model dialog and select LGDP and LINV as the two dependent variables (label them both as Y). Then proceed to the Select components dialog, deselect 'Seasonal', select 'Cycle short' in 'Cycle(s)' and change 'Order of cycle' from 1 to 4. Also ensure that 'Multivariate settings' is marked in 'Options'. Press OK. In the Select variance matrices and components for each equation dialog, change the entry for the level component of LGDP from 'in' to 'fix'. By this change, the variance associated with the level disturbance of LGDP is fixed at zero. This leads to a smooth trend component for LGDP while the trend for LINV is specified as in a local linear trend model. The slope component is balanced by having its disturbance variance matrix as a scaled variance matrix of ones. This is achieved by changing the 'Var matrix' entry of 'slope' from 'full' to 'ones'. Press OK.

After estimation, the Results window reveals the following output.

```
Level disturbance variance  for LINV: 0.000125089
Level disturbance factor variance for LINV: 0.000125089
Level disturbance factor loading  for LGDP: 0
                  LGDP        LINV
Constant         9.357      0.0000

Slope disturbance variance matrix of ones, scaled:
             LGDP         LINV
LGDP   1.226e-006   1.226e-006
LINV   1.226e-006   1.226e-006

Cycle disturbance variance/correlation matrix:
             LGDP         LINV
LGDP   1.733e-005       0.8944
LINV   8.030e-005    0.0004650

Irregular disturbance variance/correlation matrix:
             LGDP         LINV
```

```
LGDP  1.523e-005      0.6962
LINV  6.574e-005   0.0005854

Cycle other parameters:
Period                31.30762
Period in years        7.82690
Frequency              0.20069
Damping factor         0.47241
Order                  4.00000

State vector analysis at period 2007(2)
Equation LGDP
                       Value       Prob
Level                9.35689 [0.00000]
Slope                0.00763 [0.00300]
Cycle 1 amplitude    0.00659 [    .NaN]

Equation LINV
                       Value       Prob
Level                7.55961 [0.00000]
Slope                0.00939 [0.00057]
Cycle 1 amplitude    0.04895 [    .NaN]
```

As can be seen the cycles are strongly correlated (0.89). The slopes are balanced and therefore perfectly correlated. However, there is a non-zero deterministic difference between them which is why the actual estimates of the slopes for LGDP and LINV at the end of the sample, 0.0076 and 0.0094 respectively, are not the same. The forecasts will therefore diverge from each other. Note that the model is co-integrated of order (2,1). It is clear from figure 7.4 that the series do not have a single common trend and the role of the stochastic level in the LINV equation is to allow for changes in the long-run proportion of GDP that is taken up by Investment.

7.3 Inflation

7.3.1 Expected inflation

There are many reasons for wanting to estimate the expected rate of inflation. At the most basic level it is important to have a good estimate of the underlying rate of inflation for policy purposes. Governments will typically estimate this as the percentage change in the price level over the past year. However, as shown in Harvey (1989, p. 363), this estimator is inefficient. What is needed is the filtered estimator at the end of the series.

To illustrate, we consider the US inflation quarterly time series from 1947(1)-

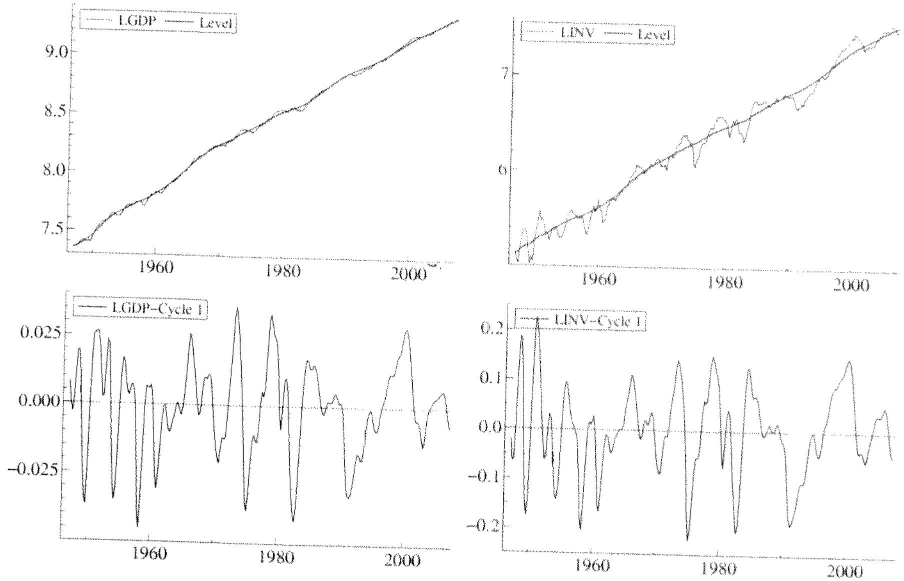


Figure 7.4 Trends and (fourth-order) cycles in LGDP and LINV from database US-macro07.

2007(2) (in logs, $\times 100$) from the database file 'USmacro07.in7'. The inflation series 'INFLcpi' is modelled as a local level model and is estimated as in §4.2.2. Part of the standard output in the Results window is:

```
Variances of disturbances:
                  Value    (q-ratio)
Level           1.85233  (  0.8066)
Irregular       2.29655  (   1.000)

State vector analysis at period 2007(2)
          Value       Prob
Level   5.29714 [0.00001]
```

STAMP can plot and save estimates of the current level of inflation and the predicted level at all points in the series. These can be obtained from the dialog **Test**/Component graphics by first selecting 'Level' in the section 'Select components'. De-select the other components. Next, expand section 'Prediction, filtering and smoothing', select 'Predictive filtering' and 'Filtering' but de-activate 'Estimates in different plots'. The last action ensures that the estimated components are presented in one graph and not (as is the default option) in different graphs. Press OK and Figure 7.5 should appear. It is interesting to compare the estimates from (contemporaneous) filtering and predictive filtering with the smoothed estimates.

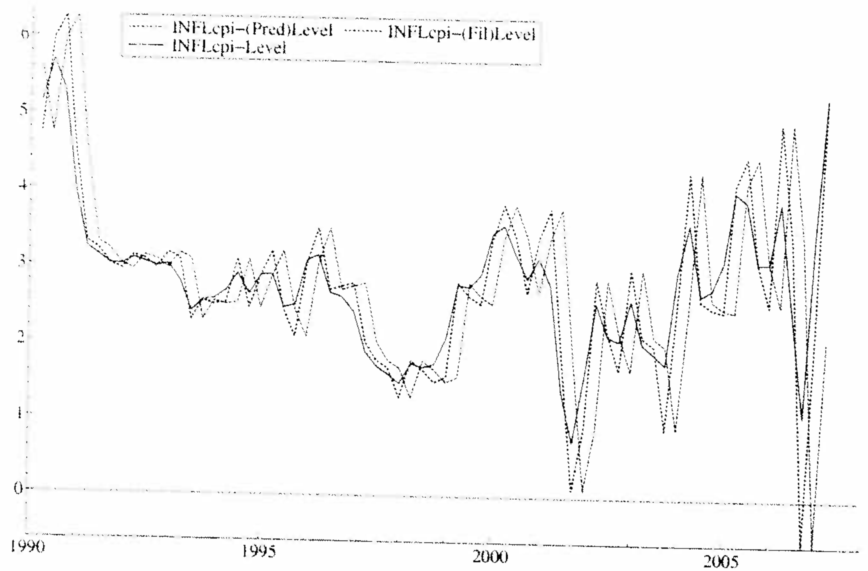


Figure 7.5 Predicted, filtered and smoothed estimates of quarterly inflation (from 1990 onwards).

7.3.2 Inflation and the output gap

Univariate time series models consisting of a random walk plus a stationary component as in the previous section are often used to model inflation. Indeed, Cogley and Sargent (2007, section 2) argue that 'A consensus has emerged that trend inflation is well approximated by a driftless random walk'. The role of the random walk component is to capture the underlying level of inflation. Since its current expected value is the long-run forecast it satisfies the usual definition of core inflation; see Bryan and Cecchetti (1994). The difference between inflation and core inflation is sometimes called the *inflation gap*. Figure 7.6 shows the smoothed components from a UC model for the annualized rate of inflation (INFLcpi) provided in the USMACRO07 data set. A stochastic cycle and a seasonal have been added to a local level model estimated in the previous section. If the inflation gap is estimated by the cycle it is somewhat smoother than the detrended series because the irregular has been filtered out.

The inflation gap can be related to the output gap obtained for US GDP in section 7.1. Figure 7.7 shows them plotted together (with inflation divided by 100). There is clearly some co-movement, but the graph suggests that insisting on a model with time invariant dynamics may be unwise. Output leads inflation in the 1970s, primarily at the time of the two oil crises, but this is not the case later on.

The STAMP program can be used to model the relationship between inflation and output in two different ways. The first is to treat the output gap as an exogenous variable

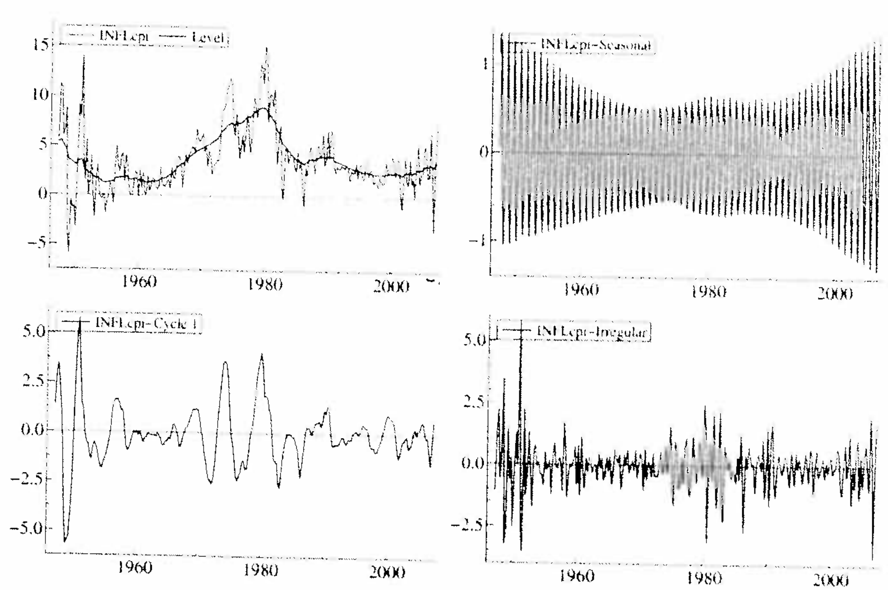


Figure 7.6 Smoothed components in inflation (INFLcpi in USMACRO07 database).

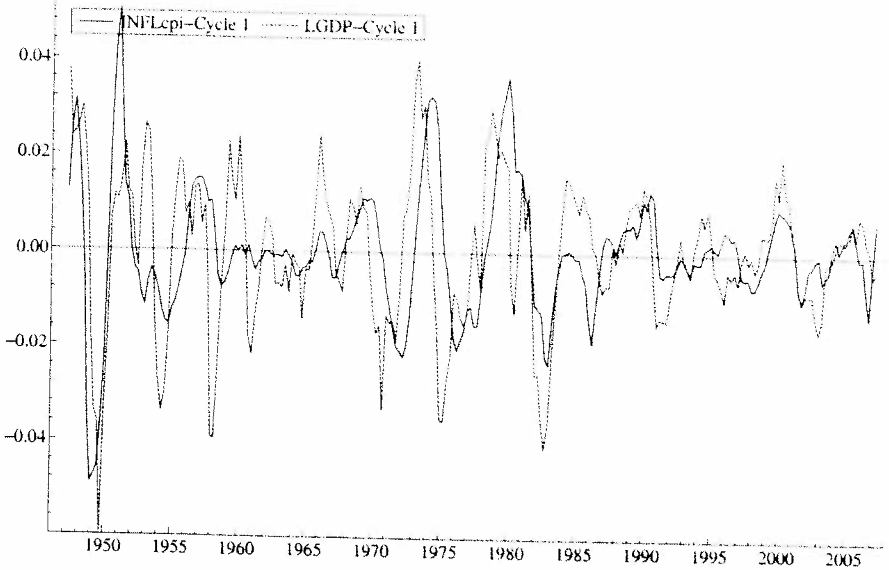


Figure 7.7 Smoothed estimates of the cycles obtained from univariate models for LGDP and INFLcpi in USMACRO07 database.

in a single model. A simple version of the Phillips curve relationship is

$$\pi_t = \mu_t + \gamma_t + \beta x_t + \varepsilon_t, \quad \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right), \quad t = 1, \dots, T \tag{7.1}$$

where π_t is inflation, μ_t is the random walk, γ_t is the seasonal component, x_t is a measure of the output gap such as the displayed LGDP cycle in Figure 7.7. The output gap x_t can be lagged. In fact a better fit is obtained over the full sample with x_{t-1}. It is also the case that x_{t-4} is significant but this turns out to be due to the 1970s and is not reproduced later.

The diagnostics are not satisfactory, but given the erratic movements in the 1970s and the subsequent sharp fall in the early 1980s this is not surprising. However, if we start in 1986(1), the model still passes the tests with flying colours. There is no evidence for lags beyond one. The output gap of lag 1 seems to give the best fit, the estimate of β being 44.5 with a t-statistic of 1.96. Figure 7.8, produced from the dialog **Test**/Components graphics is informative in that it shows the underlying level and the effect of the output gap on the level.

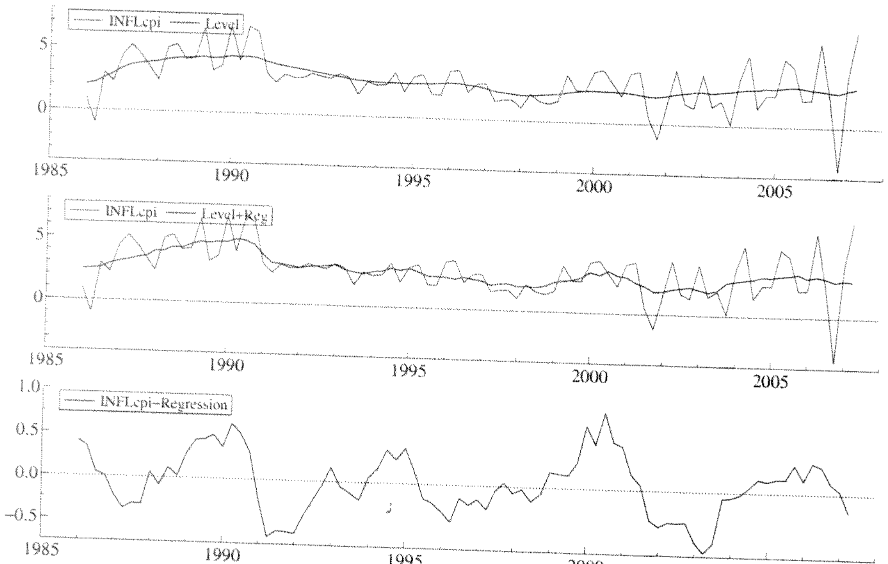


Figure 7.8 Inflation and the output gap as an explanatory variable.

```
Variances of disturbances:
                  Value      (q-ratio)
Level          0.122330   (  0.08105)
Seasonal      0.0183967   (  0.01219)
Irregular       1.50934   (    1.000)

State vector analysis at period 2007(2)
                          Value       Prob
Level                   3.10086  [0.00000]
```



```
Seasonal chi2 test          32.22345 [0.00000]

Equation INFLcpi: regression effect in final state at time 2007(2)
                Coefficient        RMSE      t-value       Prob
LGDP-Cycle 1_1     44.50634    22.70598      1.96012  [0.05342]
```

Multi-step forecasts from the end of 1997 are shown in Figure 7.9. They were obtained by going to 'Prediction graphics', moving the radio button to 'Multi-step ahead' and setting 'Post-sample size' to 38. The movements, which are conditional on the output gap, are not big but the higher inflation around 2000 is picked up. The volatility of the series in recent years has made accurate forecasting of any one quarter difficult.

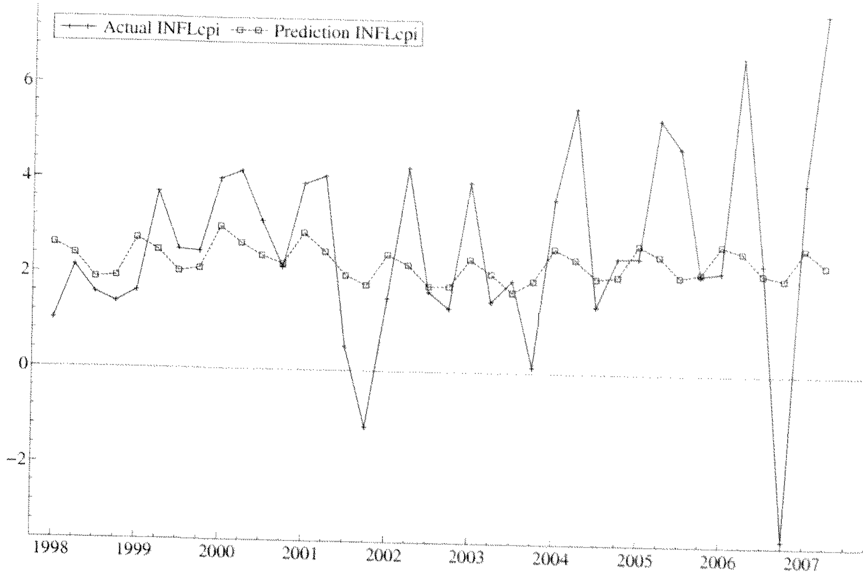


Figure 7.9 Multi-step predictions from end of 1997.

7.3.3 Bivariate modeling

Rather than first estimating the output gap from a univariate model for GDP, inflation and GDP may be modeled jointly as

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} \mu_t^\pi \\ \mu_t^y \end{bmatrix} + \begin{bmatrix} \psi_t^\pi \\ \psi_t^y \end{bmatrix} + \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^y \end{bmatrix} \tag{7.2}$$

where μ_t^π is a random walk and μ_t^y is an integrated random walk. These two stochastic trends are independent of each other. A seasonal component can be added to the model and in the estimates reported seasonal effects were included in the equation for inflation.

The stochastic cycles are modeled as similar cycles. The paper by Harvey, Trimbur and van Dijk (2007) reports estimates of a model that has the same form as (7.2) except that both trends are integrated random walks. STAMP 8 allows the random walk and integrated random walk restrictions to be imposed on the model.

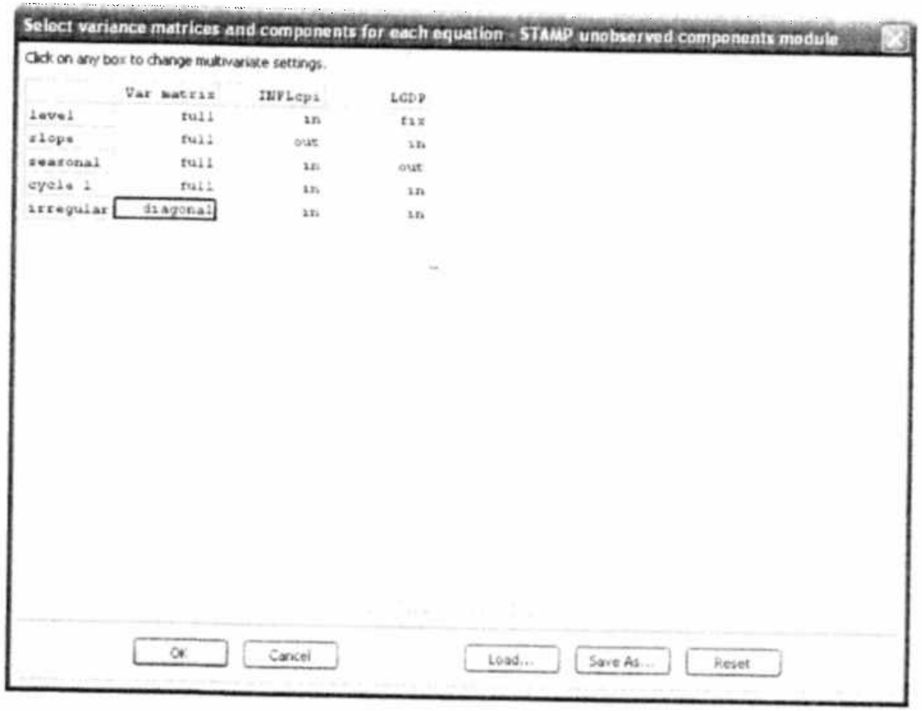
A simple transformation of the similar cycle model allows the cycle in inflation to be broken down into two independent parts, one of which depends on the GDP cycle, that is $\psi_t^\pi = \beta\psi_t^y + \psi_t^{\pi\dagger}$, where $\beta = Cov(\psi_t^\pi, \psi_t^y) = Cov(\kappa_t^\pi, \kappa_t^y)$. Substituting in the inflation equation in (7.2) gives

$$\pi_t = \mu_t^\pi + \beta\psi_t^y + \psi_t^{\pi\dagger} + \varepsilon_t^\pi \tag{7.3}$$

If the cycle disturbances κ_t^π and κ_t^y are perfectly correlated, this corresponds to (7.1) if ψ_t^y is set to x_t. However, model (7.1) could itself be extended to include a stochastic cycle.

The model estimated over the full period works quite well - but the case for a contemporaneous relationship is undermined by the lag structure identified from Figure 7.7. With data from 1986(1), the diagnostics are much better. But more to the point, a contemporaneous relationship is reasonable.

To set up the model as outlined above, proceed to the Formulate a model dialog and select INFLcpi first and LGDP second as the two dependent variables (label them both as Y). Then proceed to the Select components dialog, accept the default setting but also select 'Cycle short' in 'Cycle(s)' and keep 'Order of cycle' to 1. Also ensure that 'Multivariate settings' is marked in 'Options'. Press OK. In the Select variance matrices and components for each equation dialog, change the entries as follows.



The estimation results are given by

```
Estimation done by Maximum Likelihood (exact score)
The databased used is USmacro07.in7
The selection sample is: 1986(1) - 2007(2) (T = 86, N = 2)
The dependent vector Y contains variables:
      INFLcpi          LGDP
The model is:  Y = Trend + Seasonal + Irregular + Cycle 1
Component selection: 0=out, 1=in, 2=dependent, 3=fix
               INFLcpi      LGDP
Level                1         3
Slope                0         1
Seasonal             1         0
Cycle                1         1
Irregular            1         1
Steady state. found

Log-Likelihood is 409.231 (-2 LogL = -818.461).
Prediction error variance/correlation matrix is
             INFLcpi        LGDP
INFLcpi      2.36221     0.06736
```

```
LGDP        0.00048     0.00002

Summary statistics
                  INFLcpi        LGDP
 T                 86.000       86.000
 p                 6.0000       6.0000
 std.error         1.5369    0.0046125
 Normality         10.436     0.015023
 H(28)             1.2445      0.80649
 DW                1.6742       2.0429
 r(1)             0.12071    -0.030650
 q                 14.000       14.000
 r(q)            0.086068    -0.017939
 Q(q,q-p)          13.882       15.587
 Rs^2             0.36832      0.11933

 Variances of disturbances in Eq INFLcpi:
                     Value    (q-ratio)
Level            0.0279077  (  0.01768)
Seasonal         0.0171006  (  0.01083)
Cycle            0.0369929  (  0.02344)
Irregular          1.57852  (    1.000)

 Variances of disturbances in Eq LGDP:
                     Value    (q-ratio)
Slope         3.09023e-007  (   0.2295)
Cycle             0.000000  (   0.0000)
Irregular     1.34651e-006  (    1.000)

Level disturbance variance  for INFLcpi: 0.0279077
Level disturbance factor variance for INFLcpi: 0.0279077
Level disturbance factor loading  for LGDP: 0
              INFLcpi        LGDP
Constant       0.0000       9.346

Slope disturbance variance  for LGDP: 3.09023e-007

Seasonal disturbance variance  for INFLcpi: 0.0171006

Cycle disturbance variance/correlation matrix:
              INFLcpi        LGDP
```



```
INFLcpi      0.03699        1.000
LGDP       0.0006950  1.306e-005

Irregular disturbance diagonal variance matrix:
             INFLcpi         LGDP
INFLcpi        1.579       0.0000
LGDP          0.0000   1.347e-006

Cycle other parameters:

Period                 29.09415
Period in years         7.27354
Frequency               0.21596
Damping factor          0.95921
Order                   1.00000

Cycle variance/correlation matrix:
             INFLcpi         LGDP
INFLcpi       0.4629        1.000
LGDP        0.008697    0.0001634

State vector analysis at period 2007(2)
Equation INFLcpi
                                Value       Prob
Level                         2.79756 [0.00000]
Seasonal chi2 test           30.83048 [0.00000]
Cycle 1 amplitude             0.34816 [   .NaN]

Equation LGDP
                          Value       Prob
Level                   9.34643 [0.00000]
Slope                   0.00610 [0.00011]
Cycle 1 amplitude       0.00654 [   .NaN]
```

If there are no restrictions on the irregular, the correlation is minus one. A diagonal variance matrix for the irregular is imposed but it makes virtually no difference to the goodness of fit.

The correlation matrix of the cycle gives an estimate of β equal to $8.697/0.1634 = 53.23$. The perfect correlation means that the implied equation for π_t is effectively as in (7.1). The effect of the output gap on core inflation is shown in Figure 7.10.

If a common factor is imposed on the cycle, by setting inflation to be 'dependent' in Multivariate settings, the results is very similar. The cycle covariance matrix is now

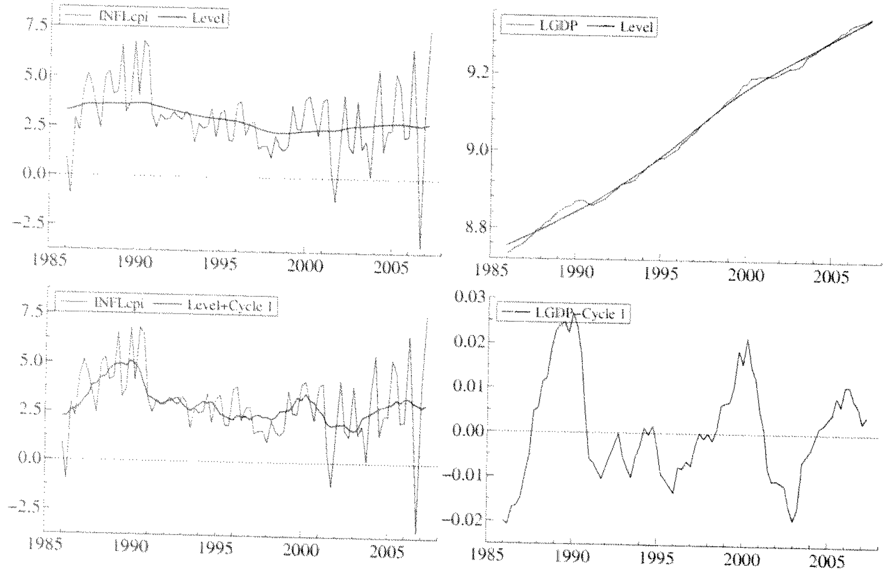


Figure 7.10 Smoothed components from a bivariate model for Inflation and GDP.

as shown below. The factor loading gives the estimate of β.

```
Cycle disturbance variance/correlation matrix:
              INFLcpi          LGDP
INFLcpi      0.03699          1.000
LGDP      0.0006950   1.306e-005
Cycle disturbance factor variance for LGDP: 1.30572e-005
Cycle disturbance factor loading  for INFLcpi: 53.2273
               INFLcpi          LGDP
Constant -1.665e-016        0.0000
```

The changes in the behaviour of GDP have been less dramatic than for inflation, which suggests using all the GDP data from 1947 and combining it with inflation data from 1986 onwards. The missing observations in inflation can be handled within a state space framework.

Finally we follow Harvey, Trimbur and van Dijk (2007) in fitting second-order cycles. Do this by changing 'Order of cycles' on the Components menu to 2. It will be seen that the second-order cycles are smoother and so may give better measures of the underlying output and inflation gaps. The correlation between them is 0.98 but if the common cycle constraint is imposed it makes very little difference to the fit. (The LR statistic is only 0.08; as shown in Carvalho, Harvey and Trimbur (2007), the 5% critical value is 2.71). The implied value of β is 57, whereas without the common cycle constraint it is 53.

7.4 Stochastic volatility

Let y_t be a stock series returns or the difference of logged exchange rates. Such a series will normally be approximately white noise. However it may not be independent because of serial dependence in the variance. This can be modelled by

$$y_t = \sigma_t \epsilon_t = \sigma \epsilon_t \exp(h_t/2), \quad \epsilon_t \sim \mathsf{IID}(0,1), \quad \mathsf{t} = 1, \ldots, \mathsf{T} \tag{7.4}$$

where

$$h_{t+1} = \phi h_t + \eta_t, \quad \eta_t \sim \mathsf{NID}(0, \sigma_\eta^2), \quad |\phi| \leq 1. \tag{7.5}$$

The term σ^2 is a scale factor, ϕ is a parameter, and η_t is a disturbance term which in the simplest model is uncorrelated with ϵ_t; literature reviews are given by Shephard (1996, 2005) and Ghysels, Harvey and Renault (1996). This *stochastic volatility*, (SV) model has two main attractions. The first is that it is the natural (Euler) discrete time analogue of the continuous time model used in papers on option pricing, such as Hull and White (1987). The second is that its statistical properties are easy to determine. The disadvantage with respect to the conditional variance models of the GARCH class is that likelihood based estimation can only be carried out by a computer intensive technique such as that described in Kim, Shephard and Chib (1998) and Sandmann and Koopman (1998). However, a quasi-maximum likelihood (QML) method is relatively easy to apply and is often reasonably efficient. This method is based on transforming the observations to give:

$$\log y_t^2 = \kappa + h_t + \xi_t, \quad t = 1, \ldots, T \tag{7.6}$$

where

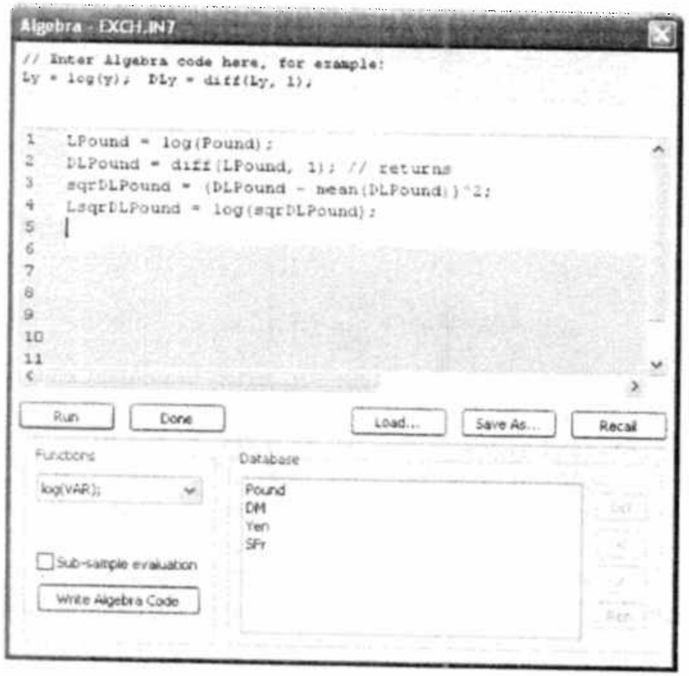
$$\xi_t = \log \epsilon_t^2 - E(\log \epsilon_t^2)$$

and

$$\kappa = \log \sigma^2 + E(\log \epsilon_t^2). \tag{7.7}$$

As shown in Harvey, Ruiz and Shephard (1994), the state space form given by equations (7.5) and (7.6) provides the basis for QML estimation via the Kalman filter and also enables smoothed estimates of the variance component, h_t, to be constructed and predictions made. One of the attractions of the QML approach is that it can be applied without the assumption of a particular distribution for ϵ_t.

In Harvey, Ruiz and Shephard (1994), the volatility in the daily exchange rate of the US dollar against four currencies is examined; see also Mahieu and Schotman (1998). The data are in the file EXCH.IN7. After loading EXCH, carry out the transformations of the data for QML estimation. This can be done using the Algebra facility in OxMetrics:



Subsequently you may fit an 'AR(1)' plus 'Irregular' plus 'Level, Fixed' model. Some of the STAMP output is given below for the British pound against the dollar:

```
Summary statistics
 std.error          2.1865
 Normality          181.73
 H(314)             1.0790
 r(1)            -0.033869
 r(29)            0.038984
 DW                 2.0646
 Q(29,27)           13.431
 R^2              0.039738

 Variances of disturbances.

Component                 Value     (q-ratio)
Level                   0.00000   ( 0.0000)
AR(1)                   0.73785   ( 0.1592)
Irregular                4.6351   ( 1.0000)

 Parameters in AR(1)

Variance                0.73785
AR1 coefficient         0.99598
```

There are a number of things to notice. First, the normality statistic is high. This is inevitable because the transformed model is not Gaussian. It should not worry us.

Second the estimate of ϕ is around 0.996.

The smoothed estimate of the volatility process, h_t, may be extracted in the usual way inside the **Test**/Component graphics dialog. Marking the 'Anti-log analysis' box in the 'Further options' section gives the exponent of the smoothed volatility. This may be interpreted as the ratio of the volatility to the underlying level. It may be preferable to consider the variations in the standard deviation, $\exp(\frac{1}{2}h_{t|T})$, in which case the Calculator in OxMetrics must be used to take the square root; see Figure 7.11.

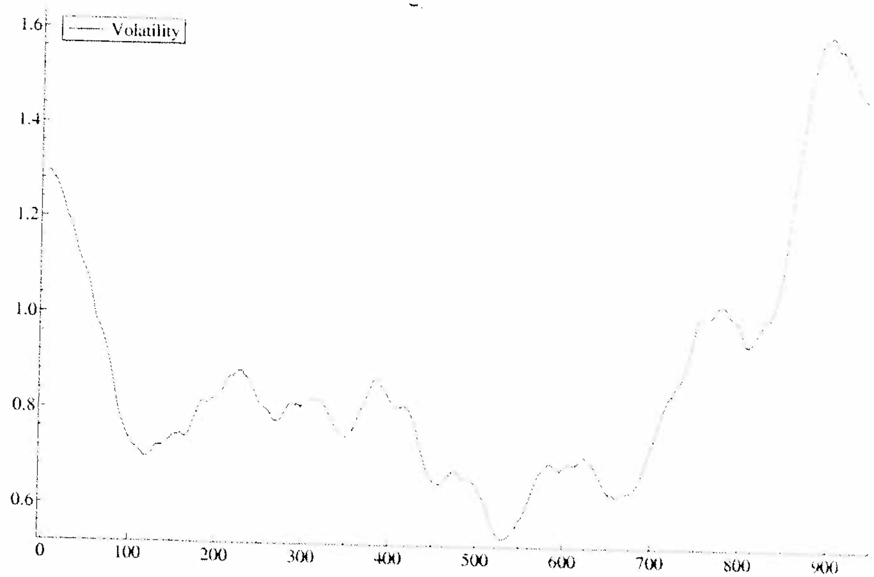


Figure 7.11 Estimated $\exp(h_{t|T}/2)$ for the Pound series.

To estimate σ^2, compute the heteroskedasticity corrected observations:

$$\tilde{y}_t = y_t \exp(-h_{t|T}/2),$$

and compute the variance $\tilde{\sigma}^2$. A plot of $\tilde{\sigma}\exp(\frac{1}{2}h_{t|T})$ against y_t shows how the standard deviation changes with the observations. It may be worth focusing on a shorter period.

7.5 Seasonal adjustment and detrending

One of the attractions of structural time series models is that the trend and seasonal is estimated as part of the overall model. Nevertheless there may be occasions on which the complexity of the full model is such that it cannot be estimated within STAMP. In such circumstances it may be helpful to work with detrended and/or seasonally adjusted data.

7.5.1 Seasonal adjustment

STAMP offers the option of constructing a seasonally adjusted series. This may be saved within the **Test**/Components graphics dialog using the option 'Store selected components in database' or within the **Test**/Store in database dialog. The adjusted series is obtained by extracting the seasonal component in the optimal way from the fitted model. *Thus, given the model specification, it is the best estimator of the non-seasonal part of the series at all time periods, including the beginning and the end.* Of course as more observations become available, the estimates of the seasonal component will change, particularly near the end of the series. This is a natural consequence of model-based seasonal adjustment.

For most purposes, seasonal adjustment based on the basic structural model is recommended. However, it is worth noting that the estimates of the seasonal effects seem, in practice, to be relatively insensitive to the specification of the trend and the inclusion of cycles.

Finally, when working with monthly data, it may sometimes be desirable to allow for calendar effects; see Harvey (1989, pp. 333–7). This may be done by including appropriately formulated explanatory variables in the model. An example of the inclusion of a trading day effect in a structural time series model can be found in Kitagawa and Gersch (1984). The effect of moving festivals — primarily Easter — can be modelled by including a variable which gives the number of days in each month affected by the festival. Other references on the use of structural models in seasonal adjustment include den Butter and Mourik (1990), Maravall (1985) and Harvey (1989, Ch. 6).

7.5.2 Detrending

To illustrate the detrending options in STAMP, consider logged US GDP quarterly time series from 1947-2004 (in logs) from the database file 'USmacro07.in7'. Simple detrending can be carried out within Algebra by fitting a smooth spline using the function `smooth_hp()`:

```
smooth_hp(LGDP, 1600, hp);
DetrendedLGDP = LGDP - hp;
```

The value of $\lambda = 1600$ of the spline function is recommended by Hodrick and Prescott (1980) for quarterly data. It is interesting to note that the so-called Hodrick and Prescott filter can be replicated using the smooth local linear trend model with the signal-to-noise ratio of the slope variance fixed at λ^{-1}; see Harvey and Jaeger (1993). To illustrate this, specify in STAMP a smooth local linear trend model (only select 'Level, Fixed', 'Slope, Stochastic' and 'Irregular') and fix parameter values in the Edit and fix parameter values dialog. Then fix $\sigma_\eta^2 = 0$ and $\sigma_\zeta^2 = 0.000625 = 1/1600$:

Edit, load or save parameter values

	Fix	Value
Level	☑	0
Slope	☑	.000625
Irregular	☐	1

The smooth estimates of the level and irregular components are numerically equivalent to the Hodrick and Prescott trend and detrended series, respectively, see Figure 7.12.

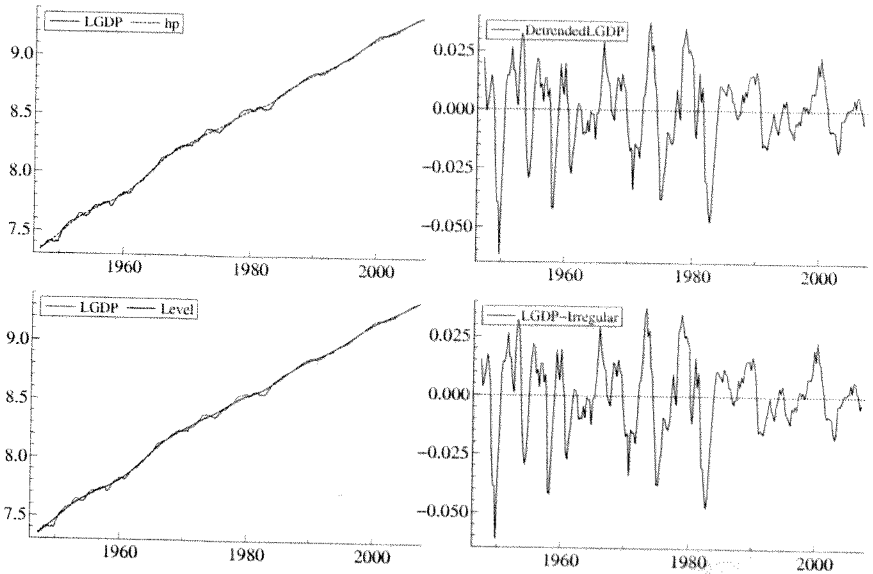


Figure 7.12 Hodrick and Prescott detrending using Algebra in OxMetrics and using local linear trend model in STAMP.

However, detrending with any *ad hoc* procedure should always be used with care because of the danger of creating spurious cycles; see Harvey and Jaeger (1993). A better way to proceed is to fit a model with a trend plus cycle(s) plus irregular, see earlier section. This kind of model-based detrending is quite appealing, though it is important to realize that the properties of the detrended series will not be the same as the properties of the unobserved components in the model which are not part of the trend.

If the series is seasonal, the user may wish to detrend and seasonally adjust. This may be done within STAMP by substrating simultaneously the trend and seasonal unobserved components.

7.6 Missing values

In empirical research, missing observations are often encountered and need to be dealt as part of an econometric time series analysis. The most obvious examples are situations where parts of the time series are not available to the researcher. In other cases, missing observations can occur when, for example, quarterly observations are available for a flow variable until 1990 and monthly observations after 1990. A practical solution is to treat all obervations as monthly and take the first two consecutive months within each three months period as missing until 1990. Further, missing observations can also be systematic when calendar time is the natural time index for a variable that is observed at irregular time intervals. For example, trades in a financial market occur irregular over time but the time-of-day is still relevant for intra-daily seasonality. In this situation, it is preferred to work with a fixed time index where observations are missing at times when no trades occur.

7.6.1 Some missing observations in the time series

We consider the logged electricity consumption (other final users) time series in the database 'ENERGY.IN7' but have treated some observations as missing. The resulting time series with missing values is presented in Figure 7.13. The basic structural time series model is considered for modelling and the estimation output of STAMP is given below.

```
UC( 1) Modelling ofuEL1 by Maximum Likelihood (using ENERGYmiss.in7)
    The selection sample is: 1960(1) - 1986(4)
    with 31 missing observation(s)
    The model is:  Y = Trend + Seasonal + Irregular

Log-Likelihood is 189.831 (-2 LogL = -379.661).
Prediction error variance is 0.00368006

Summary statistics
 std.error        0.060663
 Normality          46.791
 H(24)             0.70541
 r(1)             0.091327
 r(7)             0.082782
 DW                 1.7591
 Q(7,4)             5.6323
 Rs^2              0.94665

Variances of disturbances.

Component                Value      (q-ratio)
Level               0.00038704   ( 0.7381)
Slope              1.8795e-006   ( 0.0036)
Seasonal            0.00017306   ( 0.3300)
Irregular           0.00052441   ( 1.0000)
```



```
State vector analysis at period 1986(4)
 - level is 6.34676 with stand.err 0.0469008.
 - slope is 0.0110814 with stand.err 0.00599756.
 - joint seasonal chi2 test is 29.0685 with 3 df.
 - seasonal effects are
period        value      stand.err
     1     0.164786       0.044871
     2    -0.074543       0.051597
     3    -0.152833       0.049351
     4     0.062590       0.045979
```

The output is as usual with the only exception that it is explicitly reported at the beginning that 31 observations were found missing in this analysis. The fact that the output is as usual does support the fact that STAMP can deal with missing observations. To obtain estimates for the missing observations based on this model and on all available data, go to the dialog **Test**/More written output and activate the option 'Missing observation estimates':

```
Missing observation estimates (using full sample)

                        Value      stand.err
1960(1)               5.20814      0.0599084
1960(2)               4.97227      0.0602317
1960(4)               4.98654      0.0673372
1961(4)               5.11206        0.02854
1962(1)               5.46235      0.0400834
1963(2)               5.32565      0.0399539
...
1984(1)                6.3593      0.0393742
1986(2)               6.25005      0.0577701
1986(3)               6.18285      0.0592114
1986(4)               6.40935      0.0622762
```

More evidence of STAMPs ability to handle missing observations is given by the graphical output. The default graphical output from the dialog Component graphics is presented in Figure 7.14. However, the treatment of missing observations in STAMP becomes most clear when the graphical output of the Weight functions is investigated. The graphs are presented in Figure 7.15 and we learn that weight functions explicitly take account of the missing observations by not giving any weight to missing observations. Time points with missing observations are indicated by black dots in the plots of estimated components. This is done to make it clear at what time points observations are missing. Since the smoothed components are estimated for all time points (interpolation), it needs to be made explicit which observations are missing. The exception is the irregular component from which missing observations can be detected directly since their corresponding estimated irregular values are zero. Other output options in the **Test** menu can be used without problems. The effect that missing observations can have on the statistical output in STAMP can sometimes be quite revealing and interesting.

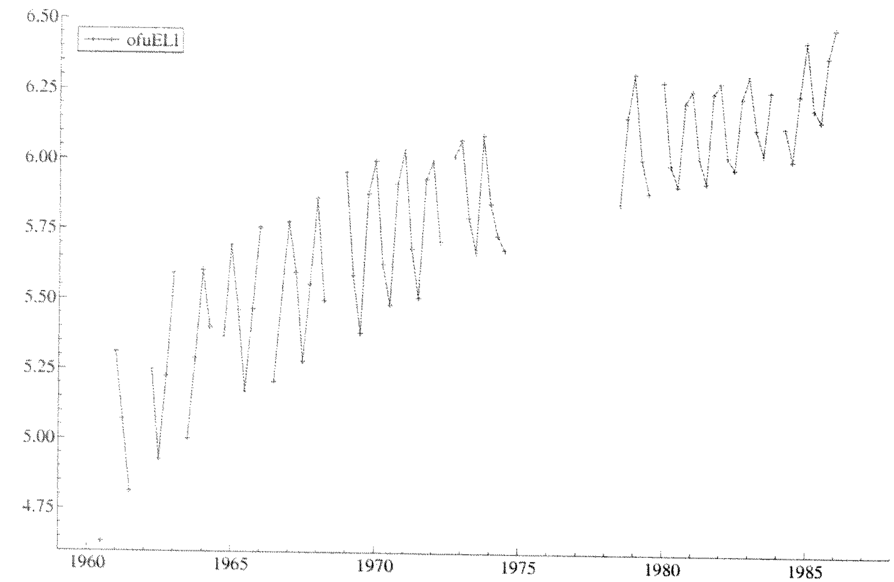


Figure 7.13 Quarterly electricity consumption (other final users) with missing observations.

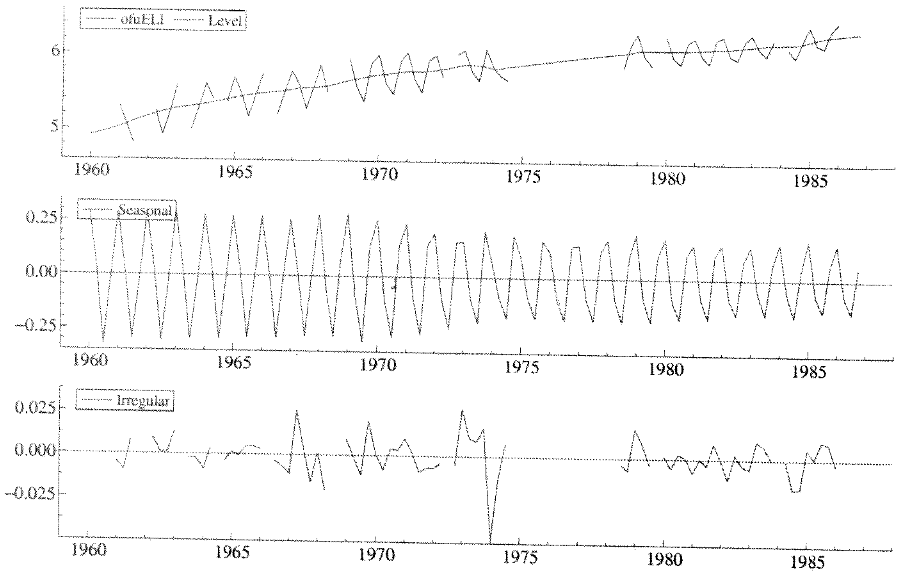


Figure 7.14 Estimates of components when observations are missing.

7.6.2 High-frequency trade prices and many missing observations

High-frequency time series of traded stock prices are usually irregularly spaced. When it is preferred to keep the calendar time and the time stamp of trades is recorded in

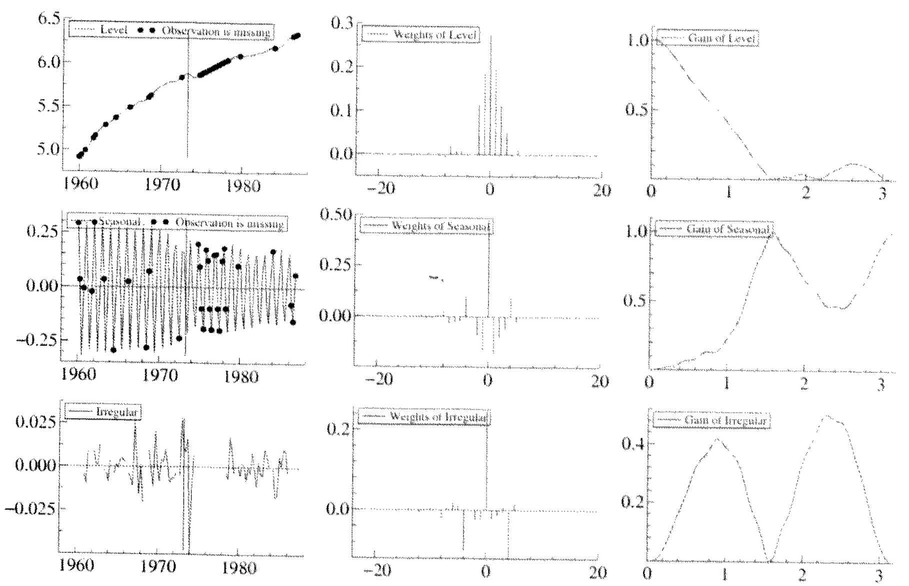


Figure 7.15 Weight and gain functions for components when observations are missing.

seconds, we can have the time index in seconds and treat seconds for which no trades take place as missing. Such a time series (prices in logs) is presented in Figure 7.16 and can be found in the OxMetrics database 'TRADEPRICE.IN7'.

Financial prices are usually modelled as a random walk process. In efficient markets, all relevant information is accounted for in the latest observed price. Since the traded prices in Figure 7.16 are observed at a high frequency, they can be subject to so-called micro-structure noise, see Hansen and Lunde (2006). A basic model to capture the random walk price and the micro-structure noise is the random walk plus noise model (the local level model). Given the irregular spacing of observed prices and the possible wide gaps (in seconds) between observations, we introduce more smoothness to the price process by considering a smooth trend model ('Level, Fixed', 'Slope, Stochastic' and 'Irregular') and estimate this model in STAMP. The deafult output is

```
UC( 1) Modelling LPrice by Maximum Likelihood
    The selection sample is: 1 - 23400
    with 20012 missing observation(s)
    The model is:  Y = Trend + Irregular

Log-Likelihood is 26716.7 (-2 LogL = -53433.4).
Prediction error variance is 5.40634e-008

Summary statistics
 std.error      0.00023252
```

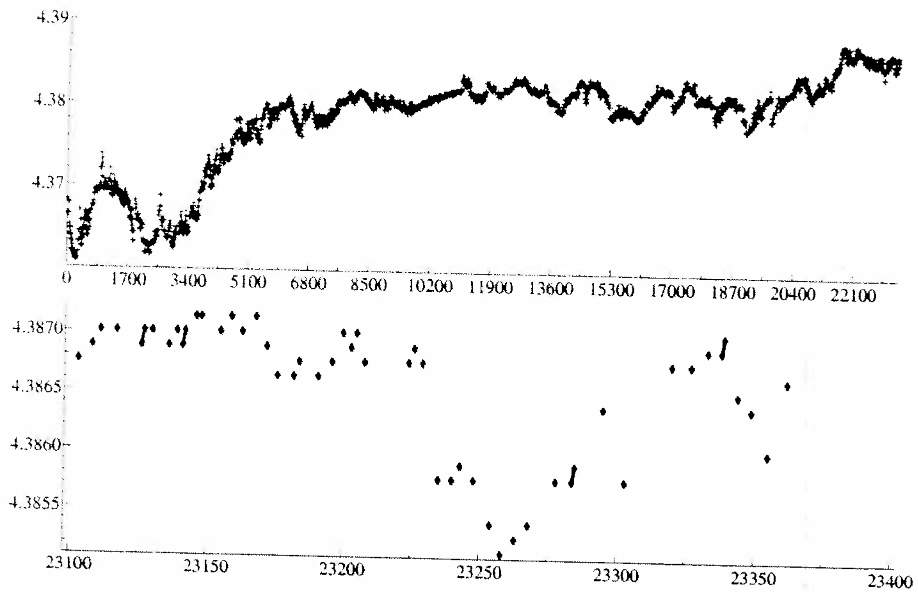


Figure 7.16 Trade prices (logs) for one day and for the last five minutes (second by second).

```
Normality          769.26
H(1128)           0.48463
r(1)             0.013652
r(57)          -0.0029337
DW                 1.9718
Q(57,55)           465.03
Rd^2              0.57049

Variances of disturbances.

Component                 Value    (q-ratio)
Level                   0.00000   ( 0.0000)
Slope               8.9082e-011   ( 0.0022)
Irregular           3.9721e-008   ( 1.0000)

State vector analysis at period 23400
 - level is 4.38665 with stand.err 0.0016858.
 - slope is 6.77663e-006 with stand.err 6.41375e-005.
```

We note the high number of missing observations in this long time series and the bad diagnostic statistics although the fit is reasonable. These results show that there is much more to the modelling of such time series than this simple model and hence the recent interest in the analysing and modelling of financial high-frequency time series. Nevertheless, it is interesting to show that high-frequency time series with many missing observations can be handled by STAMP. The results of signal extraction is presented

in Figure 7.17.

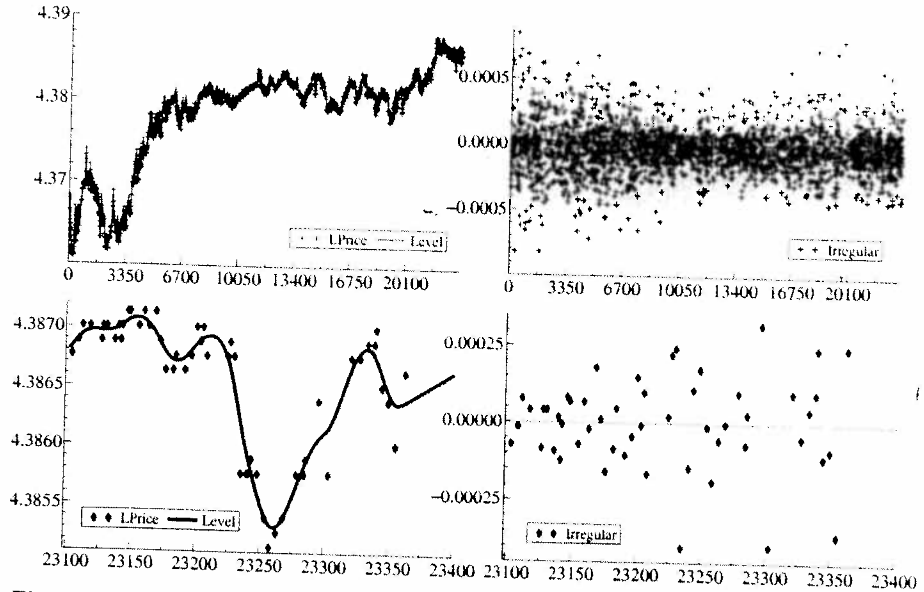


Figure 7.17 Trade price decomposition for one day and for the last five minutes (second by second).

7.7 Exercises

(1) Fit a univariate stochastic volatility model to the DM series in the EXCH.IN7 database. Compare the fit to that recorded above for the pound. Fit a bivariate model for the DM and the pound. What is the quasi-likelihood improvement for the bivariate model over the two univariate models? Is it significant? How might you extract a common cycle from the multivariate model?

(2) Suppose we wish to estimate the underlying rate of inflation with seasonal data. Take first differences of the price level, p, in UKCYP and fit a local level plus seasonal plus irregular. Seasonally adjust the series, store it and make a graphical comparison with the seasonal differences of p divided by 4, that is $\Delta_4 p_t/4$. Does one series lag behind the other? Which method of estimating the deseasonalised rate of inflation do you prefer? See exercise 6.3 of Harvey (1989, p. 363).

(3) Fit a trivariate model to LGDP, LINV and LCONS in 'USmacro07.in7'. You may want to use the data from 1960 onwards and focus on a trend-cycle decomposition.