~~Lab #1:~~ Bainbridge Tube Measurement of E/M

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THEORY

Preparing the Experiment

A Setting up the Bainbridge Experiment B~~is~~ was one of the most important steps in this entire experiment. There ~~are~~ were two important steps: aligning parallel to the Earth's magnetic field and getting the bulb rotated.

There are two steps involved in aligning to Earth's magnetic field, that is aligning on our ~~x-y~~ horizontal axis first~~,~~ and then on our ~~z~~ vertical axis. To align the ~~x-y~~ horizontal axis, firstly we ~~make sure~~ placed the Bainbridge apparatus ~~is placed~~ on a table and a ~~compass is~~ dip needle oriented horizontally was placed in near proximity to the device. Using the Bcompass, orient the bulb's ends along the magnetic field linesdip needle, the bulb was oriented to point North. Next, ~~rotate the compass or use a compass for the z-axis, to align the vertical component~~ the dip needle was rotated into a vertical orientation and the North end of the Bainbridge apparatus was raised to align the axis of the Helmholtz coils with the Earth's magnetic field. ~~Align the vertical component of the apparatus with Earth's magnetic field. We have done both of these alignments because we want no interference by the Earth's magnetic field on the measurement and aligning parallel to the Earth's field is one way of doing that.~~ This alignment was performed to satisfy the requirements for the net magnetic field as describe in Section .C

The last ~~piece~~ step in alignment of the setup ~~needed to successfully complete the Bainbridge Experiment is correctly aligning the bulb, which allows the two Hemholtz coils to create a~~ was to rotate the bulb so that the electron beam was perfectly perpendicular to the magnetic field of the Helmholtz coils, as to create a circle instead of a helix.

Derivation of ~~e/m~~ charge-to-mass ratio

The most important equation to determine the ~~mass to charge~~ charge-to-mass ratio of the electron isD~~:~~

$$\frac{e}{m} = \frac{2V_{acc}}{r^2B^2} \tag{1}$$

Since it is such an important equation I will now ~~derive where it comes from~~ go through its derivation.

We begin with the Lorentz forceD~~:~~ E

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \tag{2}$$

Since we know the Felectric current is not applied to the electrons G$\vec{\mathbf{E}}$ goes to 0, meaning the new equation isD~~:~~

$$\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}}) \tag{3}$$

Now ~~I will~~ we introduce a second equation, the one for ~~the~~ a centripetal force, given byD~~:~~ H

$$\vec{\mathbf{F}} = m\frac{v^2}{r}\hat{\mathbf{r}} \tag{4}$$

ILet us equate the two equations to create a new one:

$$q(\vec{\mathbf{v}} \times \vec{\mathbf{B}}) = m\frac{\vec{\mathbf{v}}^2}{r} \tag{5}$$

JWe know by the set up of this experiment that the magnetic field generated by the two rings of current is perpendicular to the circle made by the path of the electrons, so $\vec{\mathbf{v}} \times \vec{\mathbf{B}} = vB$. We can now include this and solve for the velocity in KEquation 5 to obtain

$$v = \frac{qBr}{m} \tag{6}$$

Let us introduce the last needed equation, that ~~of~~ for the kinetic energy ~~in relation to electron volts.~~ of the electron beam in terms of the initial electrostatic potential energy of the electrons at the cathode given byL

$$\frac{1}{2}mv^2 = eV \tag{7}$$

~~Lets plug in for velocity:~~ Substituting the velocity derived in Equation 6 yields

$$\frac{1}{2}m(\frac{qBr}{m})^2 = eV$$

~~And now~~ which is rearrange to ~~get~~ obtain the final expression~~:~~ of

$$\frac{e}{m} = \frac{2V_{acc}}{r^2B^2} \tag{8}$$

~~We see that equations 1 and 6 are the same!~~

Linearization of ~~the~~ Equation 1

MStarting with the equation for the charge to mass ratio:

$$\frac{e}{m} = \frac{2V_{acc}}{r^2B^2} \tag{9}$$

and the equation for the magnetic fields of the apparatus and the Earth:

$$B = B_0 I - B_E \tag{10}$$

~~Plugging the second in for the first, we get:~~ Substituting Equation 10 into Equation 1 gives

$$\frac{e}{m} = \frac{2V_{acc}}{r^2(B_0 I - B_E)^2} \tag{11}$$

Since the dependent variable we are looking for is the current (I), we ~~will~~ solve ~~for that:~~ Equation 11 to obtainN

$$I = \frac{\sqrt{2mV_{acc}}}{rB_0\sqrt{e}} + \frac{B_E}{B_0}$$

~~This form is correct analytically; however, this does not show the independent variables very well. The independent variables, the things we changed and fixed, were the radii and the V_{acc}. In light of this, let us separate this equation into y=mx+b form, giving us:~~

$$I = \frac{\sqrt{2m}}{B_0\sqrt{e}} \cdot \frac{\sqrt{V_{acc}}}{r} + \frac{B_E}{B_0} \tag{12}$$

~~Now,~~ In this final equation, we can clearly see ~~which variables are~~ the independent and dependent variables.

Questions

Which part of the beam should hit the post? I.e., what is the effect on the beam of hitting the gas molecules in the tube?
The part of the beam that should hit the post is the very outer edge because this is the part of the beam that carries the most energy, most closely approximating the energy the entire beam had at its inception. This is because the beam of light created before the post is using the outer, ~~brightest~~ most energetic electrons, which when imparting their energy on the Mercury gas, lose some of their initial energy and fall into the middle of the beam. Since the electrons that have ~~shone the brightest~~ interacted with Mercury atoms have lost some initial energy we do not want to use these in our ~~calculations~~ measuremetns, so we use the new fresh outer electrons at the post.

How do maximize the range over which data can be taken? What are the experimental conditions that set the minimum and maximum data point?
We maximize and minimize the range over which data can be taken by varying the voltage of V_{acc}, this moves the beam in and out with respect to the posts inside the glass tube. The experimental conditions that set the minimum and maximum data points in this experiment are the posts in the glass tube. They give a frame of reference for which to measure the radius of the beam of electrons. The minimum voltage is set by the fact you need to be able to see the beam at the farthest point in order to obtain the bending current. At lower acceleration voltages, the beam may not reach the post or the path is too dim to observe. For the maximum voltage, there is a limit set by the maximum current which can be sent through the Helmholtz coils to bend the beam to the first post. Obviously I am not making this obvious in the lab manual.

What is the optimal way to analyze the data set so that all data points are treated properly?
The optimal way to analyze the data set so that all of the data is analyzed properly is to linearize the data set and then use a linear regression to find the relation between data points. This is because linear plots are much easier to analyze to check for relations when compared to other types of relations. You want to make sure that systematic uncertainties due to knowledge of the apparatus are not diluted by including them in data points that are averaged.

COMMENTS

A This is information which needs to be moved to the "Experiment" section. At this point in your document there is no obvious reason why you are placing these restrictions on the alignment. Those restrictions come from the theoretical analysis which tells you the net magnetic field needs to point along the axis of the Helmholtz coils and that the velocity vector needs to perpendicular to the direction of the net magnetic field and lie in a plane parallel to the coils and at the midpoint between them. This is all good information, just wrongly placed.

B Verb tense is important. You are not writing a lab manual telling someone how to do the experiment. You are writing a report explaining how you did the experiment. Hence, active past tense is appropriate.

C This sentence should be moved to the end of this subsection since it is true for both aspects of the alignment.

D Punctuation around an equation should be the same as if you replaced the equation with the equivalent words. ". . . of the electron is e divided by m is equal to . . . " would not use a colon.

E I am not sure why you are using the `\va` command instead of the standard `\vec` command which also works with Greek letters. You then use `\times` instead of `\cross` which is defined in the physics package. It does make the letter bold face, but the arrow is shorter. I wouldn't make any changes, but do note that there might be conflicts using the packages.

F I do not understand your statement. The electric current in the Helmholtz coils certainly does interact withe the electrons. The is also an electric field between the anode and the cathode which causes the electrons two accelerate toward the slit to make the beam. You need to revise this sentence since your discussion relates only to the region outside of the cylindrical anode where hopefully only magnetic fields are present.

G Since E is a mathematical variable, it needs to be in math mode when placed in a sentence so that the font is consistent.

H Since $\vec{\mathbf{v}}^2 = \vec{\mathbf{v}} \cdot \vec{\mathbf{v}} = v^2$ is a scalar, the right-hand side of your equation is incorrect because you are equating a vector to a scalar. The negative sign is needed because the force is toward the center of the circular motion while the radial vector, and therefore $\hat{\mathbf{r}}$, points outward. The directions work out between Equations 3 and 4 because $q = -e$ for an electron. (Obviously the force points opposite to $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$.)

I What do you mean by equating equations? The force in Equation 4 is whatever force causes the circular motion. In this situation, the force causing the circular motion is the magnetic field component of the Lorentz force given by Equation 3. You are just making a substitution into Newton's Second Law of Motion.

J No you do not. So far you have only discussed having an electron moving in a magnetic field and the circular motion it will experience. This is true of any charged particle moving in a magnetic field, even if the magnetic field is changing. Therefore, you need to introduce aspects of the setup at the point to explain why the choices are made. First, you really do not want to deal with a spatially dependent magnetic field, therefore you need a uniform magnetic field in the region where the electrons will move. This is achieved by use of the Helmholtz coils. Second, to simplify the math you want the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$ in Equation 3 to be $90°$. This is where you bring up the constraints on the alignment. You could solve the problem with randomly oriented vectors, but the problem becomes much more complex and fraught with uncertainty.

K There are multiple problems here. First, you are referencing a specific equation, therefore it is a proper noun. Second, you are using a very poor method to reference equations. Here is the information I have sent to everyone:

- Only put numbers on equations which you will want to reference in the text.
- Use the `\label` command to add a label to the equaitons.
- Use the `\ref` command to reference the label

For example, I used the command `\label{force}` in Equation 2 and then the command `Equation~\ref{force}` in this sentence to reference that equation. Note that I included a ~ to insure that the word "Equation" and the number "2" do not get split across lines. Note that the tilde is only effective if no space comes before or after it. Referencing equations, tables, or figures in this manner is a major help if you ever add, remove, or change an equation, table, or figure resulting in modified numbers because LaTeX will automatically update the number whereever you use the `\ref` command. Since I added new equation numbers to your text, the equation you wanted to reference woudl no longer be Equation 3. Third, Equation 5 is a vector equation, so you need to work with the magnitudes of the vectors. Finally, any equation you put in the text **must** be part of a sentence, so obviously no period just before the equation.

L I am still not certain I like the organization since you do not explain how you produce the electron beam by applying a potential difference between the cathode and the anode.

M Dump the first part of this section. There is no need to repeat a formula since you can just refernce the formula you hav already written. You should start with the discussion of the net magnetic field. In Comment J, I indicated you needed to intorduce the Helmholtz coils as your means of producing a uniform magnetic field. What you do here is to indicate the need to account for other magnetic fields. The argument will eventually lead to Equation 10, but you need to put in more details.

N You need to write the equation at this point in the text so that the slope and indendent variable are obvious. You do this in the next equation, but it requires a totally useless paragraph.