Precise Measurement of g Using Kater's Pendulum

Introduction

This experiment uses a Kater's Reversible Pendulum to obtain an accurate measurement of the local value for the acceleration due to gravity (g).

Equipment

Kater's Reversible Pendulum, photo-gate, a computer with a *Science Workshop 750 Interface* and *DataStudio* software, measuring device, oil, and alcohol.

Background/Theory

Any measurements of the local value for the acceleration due to gravity using a simple or physical pendulum are limited by needing to accurately measure the effective length of the pendulum. Historically, the first accurate measurements of g were performed by Captain Henry Kater in 1817 using a reversible pendulum.[1] A reversible pendulum has two knife edged pivot points with moveable bobs of two different masses. Any physical pendulum can be treated the same as a simple pendulum if the radius of gyration is known.[2] However, finding the radius of gyration is a difficult task. It is also true for a physical pendulum that you will get a different period if you rotate about a different pivot point since the radius of gyration will be different. With a reversible pendulum based on Kater's design variation of period with different pivot points of a physical pendulum is used to remove the need for finding the radius of gyration. In general, if the period of oscillation of a physical pendulum about one axis a distance l_1 from the center of mass (i.e., the radius of gyration) is T_1 while the period of oscillation about the other pivot a distance l_2 from the center of mass is T_2, then the acceleration due to gravity is given by

$$\frac{8 * \pi^2}{g} = \frac{T_1^2 + T_2^2}{L} + \frac{T_1^2 - T_2^2}{l_1 - l_2} \tag{1}$$

where $L = l_1 + l_2$ is the distance between the two pivots if the center of mass is maintained between them. You can search the web for more information on this, with a good discussion given in Ref. [3]. Notice in Eqn. 1 that when the bobs are positioned so that the period of the pendulum is identical regardless of which of the two knife edges it is pivoting on, a highly accurate value of g can be obtained using the formula

$$g = \frac{4\pi^2 L}{T^2} \tag{2}$$

where T is the period of oscillation at the equal period point.[2] For our system L as indicated on the bar is $100.060(10)cm$ so that the period is about $2s$. The difficulty in the measurement comes in finding the correct position of the equal period point and a precise determination of this period.

Before leaving our discussion of the theory, it is important to point out one more import fact. In the derivation of the period for a simple pendulum it the small angle approximation of $\sin\theta \rightarrow \theta$ was used. Consequently, the period of a pendulum depends on the amplitude of the oscillation. One can come up with a proper solution for the differential equation, but that is not necessary in this case. Instead, you will simply need to determine the "zero-amplitude" period for your pendulum by observing how the period changes with amplitude.

Procedure

The distance between the two knife edges is $1.00060(10)m$ limiting the final precision of the measurement to 0.010% or about $0.10cm/s^2$. To insure that this is the dominate uncertainty in the measurement, you need the uncertainty in the period to be better than $\pm 2ms$. You should keep this in mind as you make your

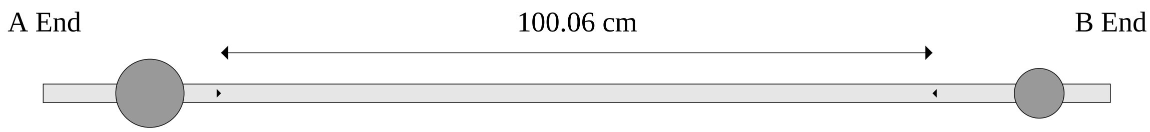

Figure 1: Schematic diagram of the Kater's Reversible Pendulum.

measurements. You still need to strive for the most precise measurement of the period, but in the end you will be limited on the precision for the value of g.

Begin by cleaning the pendulum mount and oiling its surface to prevent damaging the knife edges and interfering with the swing. You need the motion to be as close to undamped as possible so that the period you measure is correct. To allow you to make measurements with very small amplitudes, you should tape a toothpick so that it extends past the end of the pendulum. The toothpick is sufficient to block the photogate and make the measurement possible.

Position the center of the large bob $\approx 15cm$ from the A end (Fig. 1) of the bar. This distance will be fixed throughout the experiment and affects the equal period position for the smaller bob. Initially, the small bob should be positioned $\approx 30cm$ from the same end. Place the pendulum on the mount and make sure it swings freely.

Measurement of the period will be done using a photogate connected to the computer using the *Science Workshop 750 Interface* with data collection made using the *DataStudio* software. When entering *DataStudio*, choose to start a new experiment using a photogate/pendulum. In this mode, the photogate will count the total time of the swing and not just for the time between when the photogate is blocked. The photogate should be placed so that the toothpick blocks the photogate when the pendulum is at rest. To keep within the small angle approximation, all measurements should be made with an amplitude, sideways deflection for the end of the pendulum, of 5 cm or less which should be the same for all measurements. There is a minimum amplitude for the swing which must be achieved or the photogate will not operate correctly. In addition, smoothly releasing the pendulum is nearly impossible and chaotic motion is present initially. Hence, each measurement should be started after at least one minute of swinging to allow the system to stabilize.

Take a series of short measurements of the period (one minute or less total time on each) for the small mass at positions from 30.0 to 140.0 cm from the A end of the pendulum. For each measurement, the period for mounting on each pivot must be measured. It is easiest to mark the desired bob positions on the pendulum before starting the measurements so that these measurements do not need to be done at each step. A graph of the two periods versus the small bob position will give an estimate of the equal period position shown in Fig. 2.

To obtain a more precise value for the equal period position and period, begin a series of measurements for small steps (about 2.0 cm with a minimum of four positions) on either side of the estimated equal period position as well as one at your estimated equal period position. For these measurements, longer run times should be used. An example plot showing the crossing position, equal period position, is also shown in Fig. 2 In addition, a zero amplitude period is desired but not obtainable. Attempt to estimate the zero amplitude period for these measurements. This will involve studying how the period varies with amplitude (or how long it swings) and estimating how this affects the results you obtain. You will need to fit a polynomial curve to the data points for the two knife edges and then determine from the fitted lines the value for the equal period.

As an extension, you might try to determine a location of the larger bob which would allow placement of the smaller bob at a position not obstructed by a knife edge as was the case shown in Fig. refPlots. In this way you could precisely tune the equal period point. At some point the uncertainty in the distance between the knife edges becomes the limiting factor. Hence, you might look into directly measuring this distance.

References

[1] H. Kater, Philos. Trans. Roy. Soc. London **108**, 33 (1818).

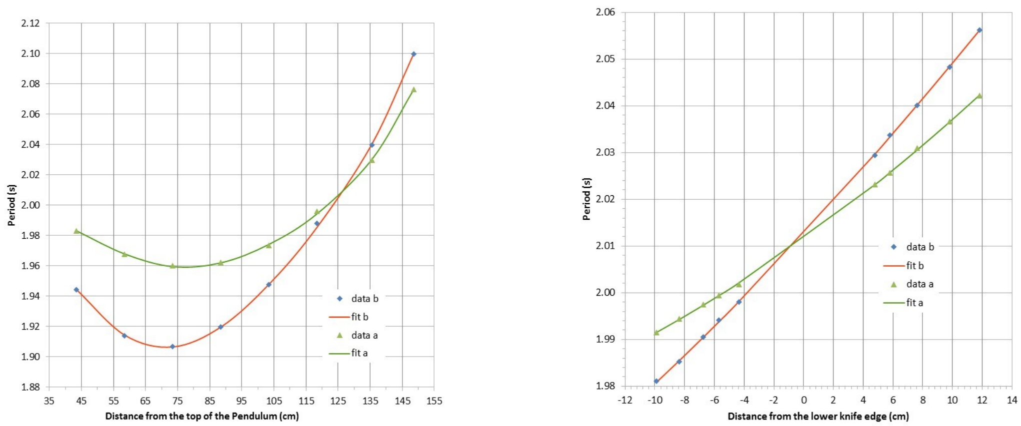

Figure 2: Plots of period about each pivot as a function of the position of the small bob for the initial large step measurements (left) and the smaller detailed steps near the equal period position (right). The reference point for measuring the position was changed for the detailed measurement.

[2] A. P. Arya, *Introduction to Classical Mechanics* (Allyn and Bacon, Needham Heights, MA) 339-342 (1990).

[3] See file http://www.hep.vanderbilt.edu/ webstems/classes/p225lab/katerpend.pdf