QuC HW04

Corbin T. Rochelle

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Question 1

A.

The initial realiziation I made was that that the Eigenvectors, $|E_1\rangle$, $|E_2\rangle$, $|E_3\rangle$, and $|E_4\rangle$, were the same values for all values of `ham2factorC`; however the Eigenvalues change with a different pattern: namely $|E_1\rangle$ is the negation of $|E_2\rangle$ and $|E_3\rangle$ is the negation of $|E_4\rangle$, while $|E_2\rangle$ is calculated with $1 +$ `ham2factorC` and $|E_4\rangle$ is calculated with $1 -$ `ham2factorC`! We can explain $P_j(t)$ easily by looking at the resulting $|\psi_{final}\rangle$! For different values of `ham2factorC`'s, we see different values of $|\psi_{final}\rangle$, but if we take the norm of all of the resulting states, we get $\frac{1}{2}$. This means when we take $|\langle j|\psi_{final}\rangle|^2$, the $\langle j|$ will select one value from $|\psi_{final}\rangle$ and when taking the square of the $\frac{1}{2}$, we get $\frac{1}{4}$, which is the values we see for all $P_j(t)$.

B.

The first thing I notice is that the Eigenvalues follow the same pattern described for the system in question 1A. The second thing I notice is that although the Eigenvalues follow that pattern, the Eigenvectors do not. The $|\psi_{initial}\rangle$ is the same as $|\psi_{final}\rangle$ because the $P_j(t)$ function requires the norm to be taken of both wave functions. The norm of $|\psi_{initial}\rangle = |\psi_{final}\rangle$ for all four values, creating a straight line. For all values of t, the vectors are changing, but the norm of the values are always the same. This is because H_{total} and $|\psi_{final}\rangle$ are orthogonal to each other!

C.

I found two Hamiltonians that meet the following criteria:

$$H_{total} = (\sigma^x \bigotimes \sigma^x) + C(\sigma^x \bigotimes \sigma^x)$$
$$\text{and}$$
$$H_{total} = (\sigma^y \bigotimes \sigma^y) + C(\sigma^y \bigotimes \sigma^y)$$
$$\text{and}$$
$$H_{total} = (\sigma^z \bigotimes \sigma^z) + C(\sigma^z \bigotimes \sigma^z)$$

For all of these, the bell state is $|\psi_{initial}\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$ and $|\psi_{final}\rangle = \begin{pmatrix} x \\ 0 \\ 0 \\ x \end{pmatrix}$, where the norm of $x = \frac{1}{\sqrt{2}}$. From this we can see two of the j Cbits will select one of the x's, namely the $|00\rangle$ or $|11\rangle$, which have the $P_j(t) = 0.5$, while the other js will multiply by the zeros and have $P_j(t) = 0$. We know $|\psi_{final}\rangle$ will always have two 0s because the eigenvectors are always orthogonal to $|\psi_{final}\rangle$.

Question 2

A.

For any system of N qbits, $E_{GS} = -N$, while $E_2 = -(N-2)$. And I have checked and verified up to 6 qbits.

B.

See my Mathematica file!

C.

See my Mathematica file!

D.

See my Mathematica file!