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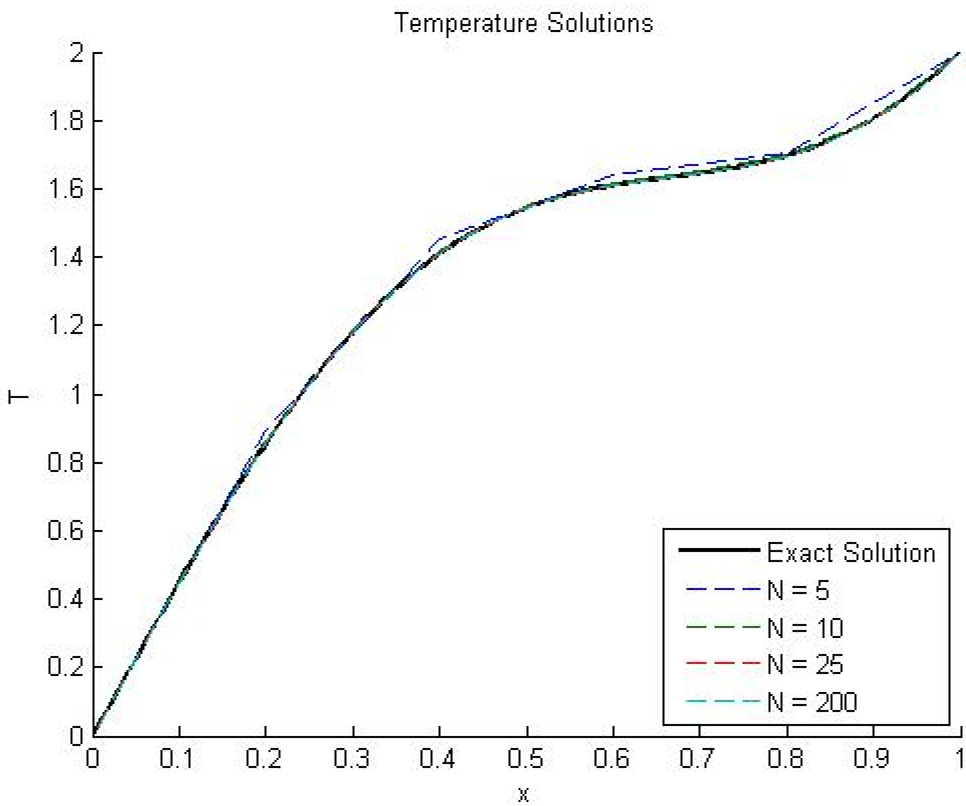
```
% HW 1, 1D heat Equation
% Corbin Foucart
```

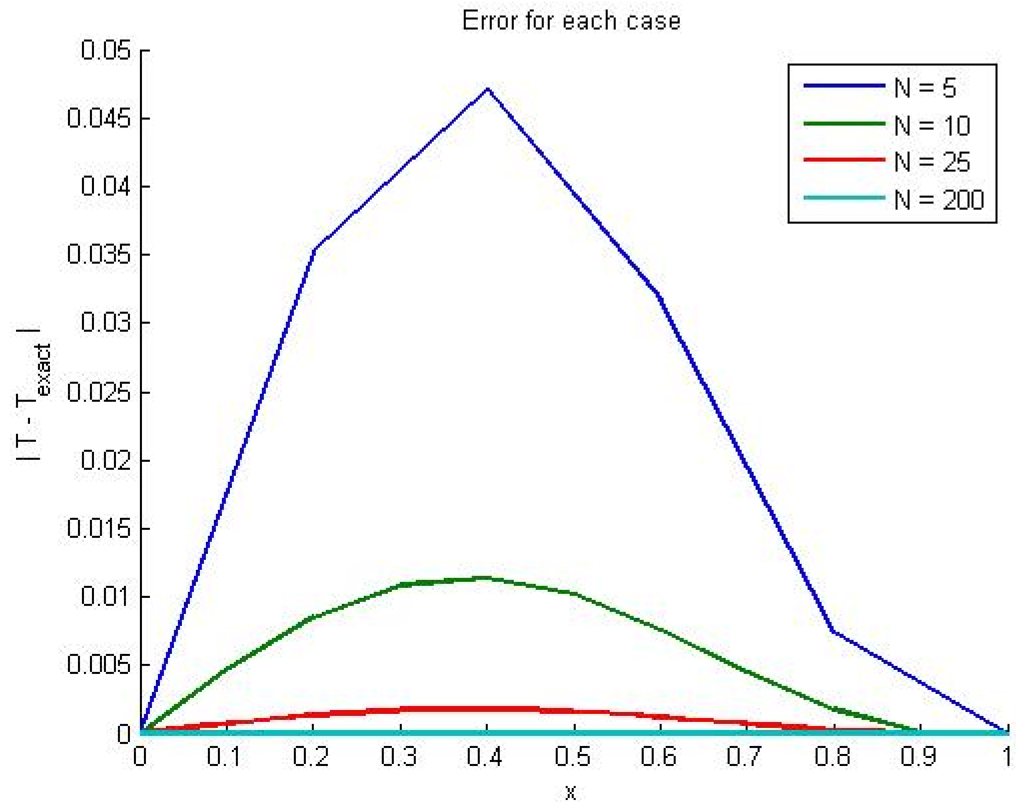
Sinusoidal Source Function, 1.d,e

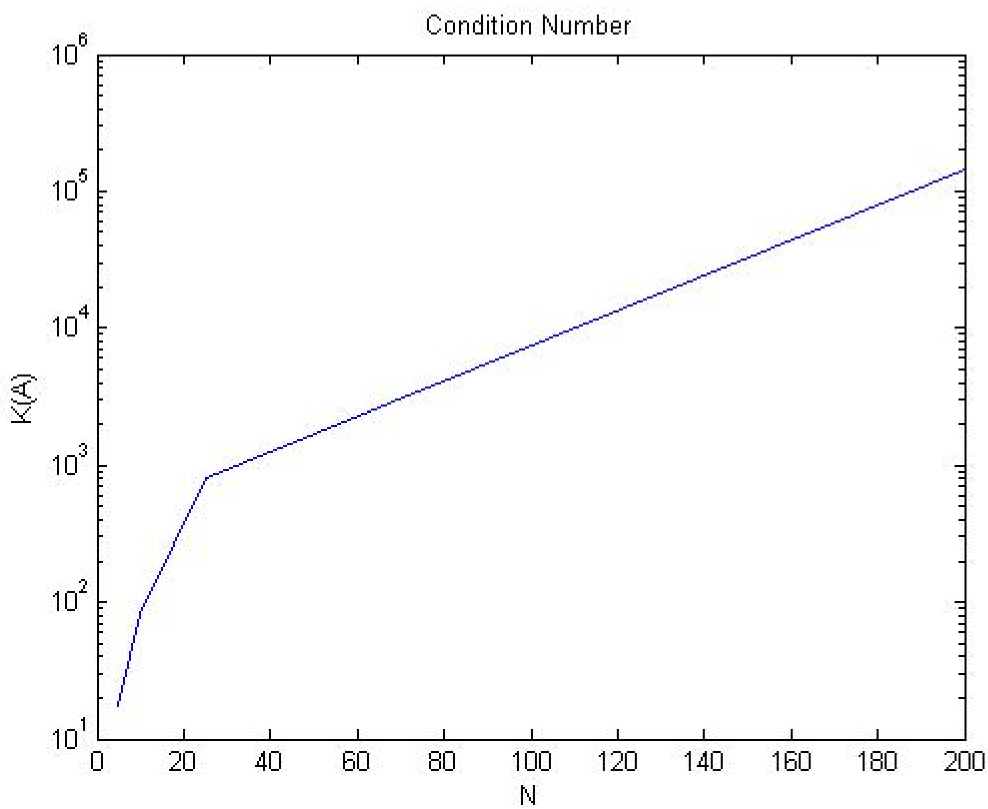
```
clear all; close all; clc;

% Source Function
source = @(x) -10.*sin(3*pi/2.*x);

% Exact Solution
exact_soln = @(x) (2 + 40/(9*pi^2)).*x + 40/(9*pi^2).*sin(3*pi/2.*x);

% boundary conditions
T_0 = 0; T_L = 2;
x_0 = 0; x_L = 1;

% ---------------------- Approximate Solution ------------------------ %
% Goal is to solve system At = b

% number of points in domain
N = [5, 10, 25, 200];

% for each number of points
for ii = 1:length(N)

num_points = N(ii);
h = 1/num_points;
x = [x_0:h:x_L]';

% assembly of A
A = zeros(num_points + 1);
for i = 2:num_points
    A(i, i - 1) = 1;
    A(i,i) = -2;
    A(i, i + 1) = 1;
end
A(1,1) = 1;
A(num_points + 1, num_points + 1) = 1;

% assembly of b (rhs)
b = zeros(num_points + 1, 1);
```

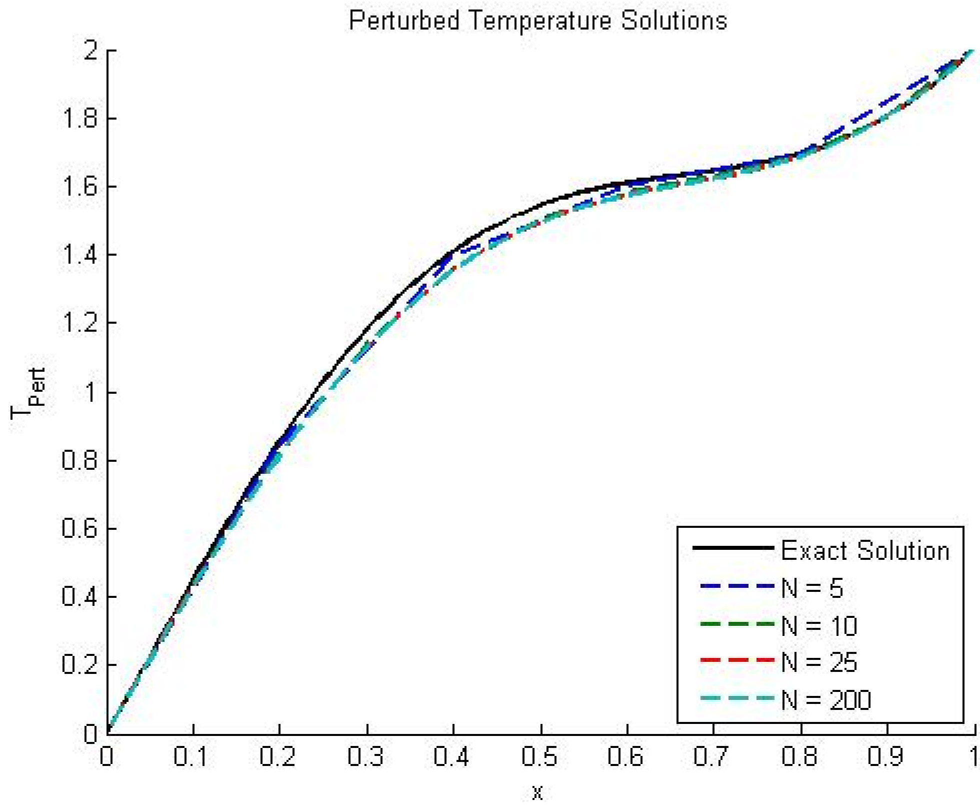
```
f_x = source(x);
for k = 2: num_points
    b(k) = f_x(k) * h^2;
end
b(1) = T_0;
b(num_points + 1) = T_L;

% temperatures vector (soln vector)

t = A\b;
t_exact = exact_soln(x);

% Error
error = abs(t - t_exact);

% Condition Numbers, Perturbation
dA = 0.1*A;
dA(1,1) = 0;
dA(num_points + 1, num_points + 1) = 0;
t_pert = (A + dA)\b;

% save the data we created for plotting
C{ii} = {A, b, t, t_exact, x, error, t_pert};

K(ii) = norm(inv(A), 'fro')*norm(A, 'fro');

end



% ------------------- Visualization, Plotting ------------------------- %

figure()
hold all
title('Temperature Solutions')
xlabel('x')
ylabel('T')
% exact solution
plot(C{1,4}{1,5}, C{1,4}{1,4}, 'k-' ,'linewidth', 2)
% for all other solutions
for plot_case = 1:length(N)
    plot(C{1,plot_case}{1,5}, C{1,plot_case}{1,3}, '--')
end
legend('Exact Solution', 'N = 5', 'N = 10', 'N = 25', 'N = 200', ...
    'Location', 'southeast')

figure()
hold all
title('Error for each case')
xlabel('x')
ylabel('| T - T_{exact} |')
for plot_case = 1:length(N)
    plot(C{1,plot_case}{1,5}, C{1,plot_case}{1,6}, '-', 'linewidth', 2)
```

```
end
legend('N = 5', 'N = 10', 'N = 25', 'N = 200', ...
    'Location', 'northeast')


figure()
semilogy(N, K)
hold on
title('Condition Number')
xlabel('N')
ylabel('K(A)')

figure()
hold all
title('Perturbed Temperature Solutions')
xlabel('x')
ylabel('T_{Pert}')
% exact solution
plot(C{1,4}{1,5}, C{1,4}{1,4}, 'k-' ,'linewidth', 2)
% for all other solutions
for plot_case = 1:length(N)
    plot(C{1,plot_case}{1,5}, C{1,plot_case}{1,7}, '--', 'linewidth', 2)
end
legend('Exact Solution', 'N = 5', 'N = 10', 'N = 25', 'N = 200', ...
    'Location', 'southeast')
```




Error for each case
N = 5
N = 10
N = 25
N = 200
$|T - T_{exact}|$
x

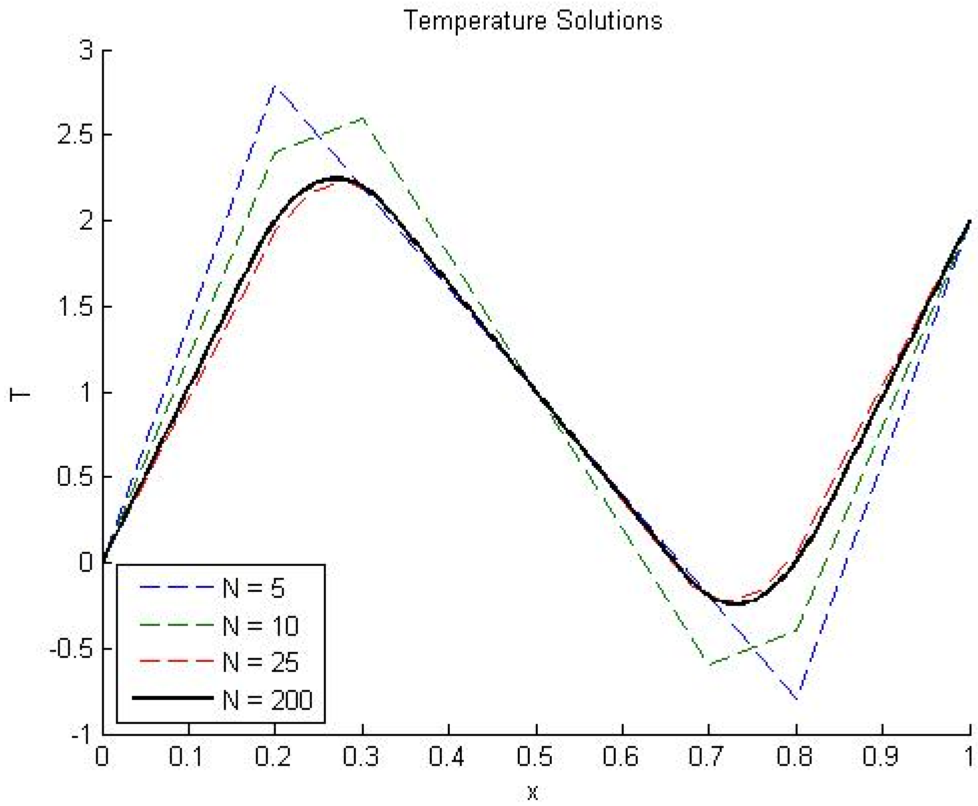

Condition Number
K(A)
N



Piecewise Source Function, 1.f

```
clear all; close all; clc;

% boundary conditions
T_0 = 0; T_L = 2;
x_0 = 0; x_L = 1;

% ---------------------- Approximate Solution ----------------------- %
% Goal is to solve system At = b

% number of points in domain
N = [5, 10, 25, 200];

% for each number of points
for ii = 1:length(N)

num_points = N(ii);
h = 1/num_points;
x = [x_0:h:x_L]';

% assembly of A
A = zeros(num_points + 1);
for i = 2:num_points
    A(i, i - 1) = 1;
```

```
    A(i,i) = -2;
    A(i, i + 1) = 1;
end
A(1,1) = 1;
A(num_points + 1, num_points + 1) = 1;

% computation of source function
f_x = zeros(length(x), 1);
for j = 1:length(x)
    if and((x(j) > 1/6),(x(j) < 1/3))
        f_x(j) = -100;
    elseif ((x(j) > 2/3) && (x(j) < 5/6))
        f_x(j) = 100;
    end
end

% assembly of b (rhs)
b = zeros(num_points + 1, 1);
for k = 2: num_points
    b(k) = f_x(k) * h^2;
end
b(1) = T_0;
b(num_points + 1) = T_L;

% temperatures vector (soln vector)

t = A\b;

% save the data we created for plotting
C{ii} = {A, b, t, x, f_x};
end



% ------------------- Visualization, Plotting ------------------------- %

figure()
hold all
title('Temperature Solutions')
xlabel('x')
ylabel('T')
% plot every solution besides last
for plot_case = 1:length(N) - 1
    plot(C{1,plot_case}{1,4}, C{1,plot_case}{1,3}, '--')
end
% plot best case thicker and in black for comparison
plot(C{1,length(N)}{1,4}, C{1,length(N)}{1,3}, 'k-', 'linewidth', 2)
legend('N = 5', 'N = 10', 'N = 25', 'N = 200', ...
    'Location', 'southwest')
```



Part 1.h

```
clear all; close all; clc;

A = zeros(6);
for i = 2:5
    A(i,i) = -2;
    A(i, i - 1) = 1;
    A(i, i + 1) = 1;
end
A(1,1) = 1;
A(6, 5) = -1;
A(6,6) = 1;

A
A_inverse = inv(A)

fprintf('We see that the inverse of A exists and A is NOT singular.');
```

```
A =

     1     0     0     0     0     0
     1    -2     1     0     0     0
     0     1    -2     1     0     0
     0     0     1    -2     1     0
```

```
     0     0     0     1    -2     1
     0     0     0     0    -1     1


A_inverse =

    1.0000         0         0         0         0         0
    1.0000   -1.0000   -1.0000   -1.0000   -1.0000    1.0000
    1.0000   -1.0000   -2.0000   -2.0000   -2.0000    2.0000
    1.0000   -1.0000   -2.0000   -3.0000   -3.0000    3.0000
    1.0000   -1.0000   -2.0000   -3.0000   -4.0000    4.0000
    1.0000   -1.0000   -2.0000   -3.0000   -4.0000    5.0000

We see that the inverse of A exists and A is NOT singular.
```

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