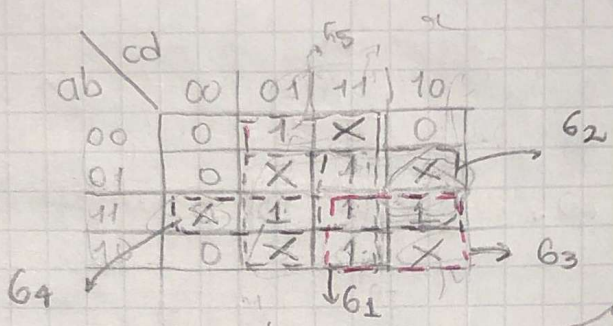
Taller Algebra Booleana

1)

a	b	c	d	W
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	X
0	1	0	0	0
0	1	0	1	X
0	1	1	0	X
0	1	1	1	1
1	0	0	0	0
1	0	0	1	X
1	0	1	0	X
1	0	1	1	1
1	1	0	0	X
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1



$W_{G1} = \bar{a}\bar{b}\bar{c}d + \bar{a}b\bar{c}d + ab\bar{c}d + a\bar{b}\bar{c}d + \bar{a}\bar{b}cd + \bar{a}bcd + abcd + a\bar{b}cd$

$W_{G2} = \bar{a}\bar{c}d + a\bar{c}d + \bar{a}cd + acd$

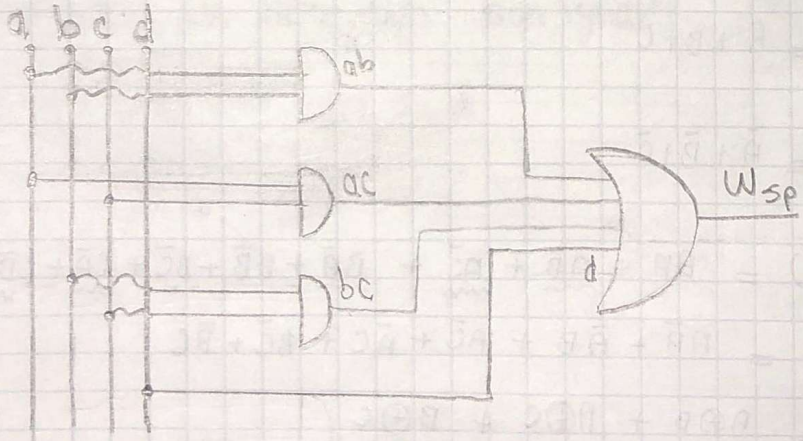
$W_{G1} = \bar{c}d + cd = d$

→ $W_{G2} = \bar{a}bcd + abcd + \bar{a}bc\bar{d} + abc\bar{d} = bcd + bc\bar{d} = bc$

→ $W_{G3} = ab\bar{c}d + a\bar{b}cd + ab\bar{c}\bar{d} + a\bar{b}c\bar{d} = acd + ac\bar{d} = ac$

→ $W_{G4} = ab\bar{c}\bar{d} + ab\bar{c}d + abcd + abc\bar{d} = ab\bar{c} + abc = ab$

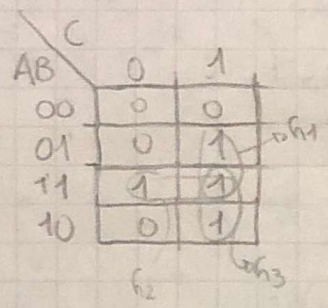
→ $W_{SP} = ab + ac + bc + d$



2)

A	B	C	X	Y
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	0

Salida X:



$X_{G1} = \bar{A}BC + ABC$

$= BC$

$X_{G2} = AB\bar{C} + ABC$

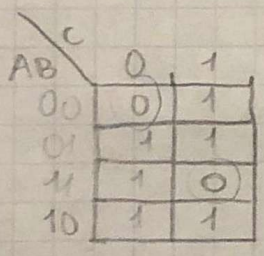
$= AB$

$X_{G3} = ABC + A\bar{B}\bar{C}$

$= AC$

$X_{SP} = BC + AB + AC$

Salida Y



$Y_{G1} = A + B + C$

$Y_{G2} = \bar{A} + \bar{B} + \bar{C}$

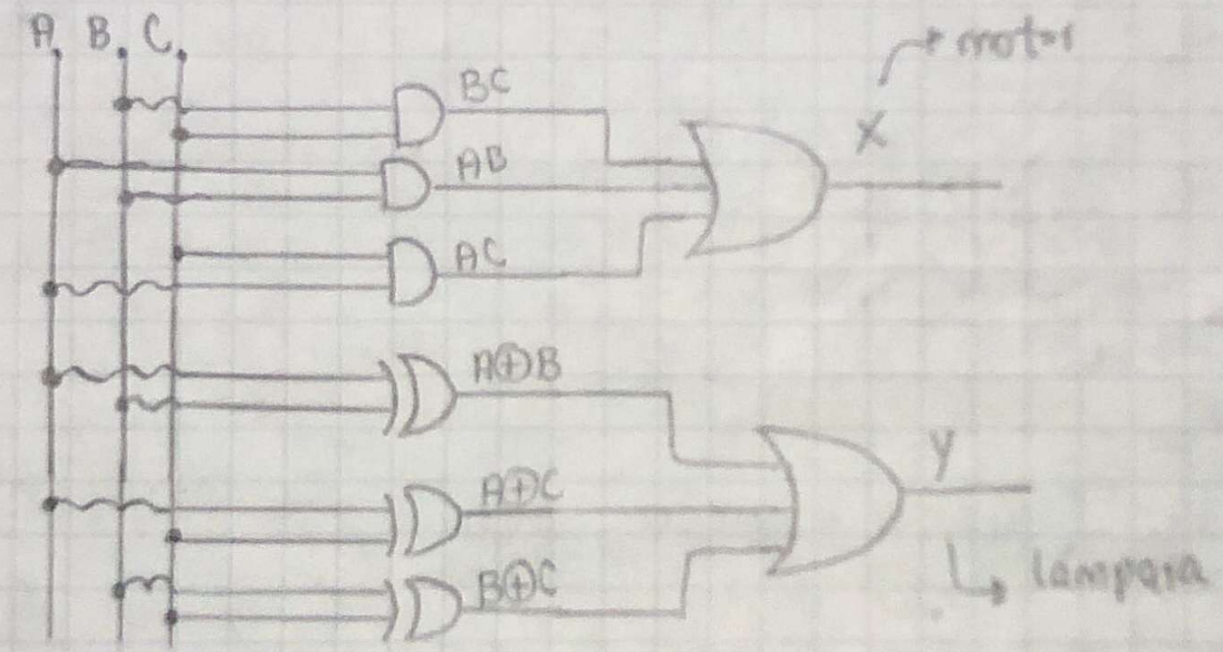
$Y_{PS} = (A+B+C)(\bar{A}+\bar{B}+\bar{C}) = A\bar{A} + A\bar{B} + A\bar{C} + B\bar{A} + B\bar{B} + B\bar{C} + C\bar{A} + C\bar{B} + C\bar{C}$

$Y_{PS} = A\bar{B} + \bar{A}B + A\bar{C} + \bar{A}C + B\bar{C} + \bar{B}C$

$Y_{PS} = A \oplus B + A \oplus C + B \oplus C$

→ diseño del circuito del punto 2)

Circuito lógico:



$9 < n \leq 15$.

variables iguales es par

4)

a	b	c	d	X
0	0	0	0	(1) //
0	0	0	1	0
0	0	1	0	0
0	0	1	1	(1) //
0	1	0	0	0
0	1	0	1	(1) //
0	1	1	0	(1) /
0	1	1	1	0
1	0	0	0	0
1	0	0	1	(1) //
1	0	1	0	(1) //
1	0	1	1	0
1	1	0	0	(1) /
1	1	0	1	0
1	1	1	0	0
1	1	1	1	(1) /

$$X_{sp} = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}cd + \bar{a}b\bar{c}d + \bar{a}bc\bar{d}$$

$$a\bar{b}\bar{c}d + a\bar{b}cd + ab\bar{c}\bar{d} + abcd$$

Simplificación por Karnaugh

ab \ cd	00	01	11	10
00	1_{G_1}	0	1_{G_2}	0
01	0	1_{G_3}	0	1_{G_4}
11	1_{G_5}	0	1_{G_6}	0
10	0	1_{G_7}	0	1_{G_8}

$X_{G_1} = \bar{a}\bar{b}\bar{c}\bar{d}$ $X_{G_2} = \bar{a}\bar{b}cd$ $X_{G_3} = \bar{a}b\bar{c}d$ $X_{G_4} = \bar{a}bc\bar{d}$

$X_{G_5} = ab\bar{c}\bar{d}$ $X_{G_6} = abcd$ $X_{G_7} = a\bar{b}\bar{c}d$ $X_{G_8} = a\bar{b}c\bar{d}$

$$X_{sp} = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}cd + \bar{a}b\bar{c}d + \bar{a}bc\bar{d}$$
$$ab\bar{c}\bar{d} + abcd + a\bar{b}\bar{c}d + a\bar{b}c\bar{d}$$

La simplificación por procedimientos algebraicos y por el método de Karnaugh son iguales

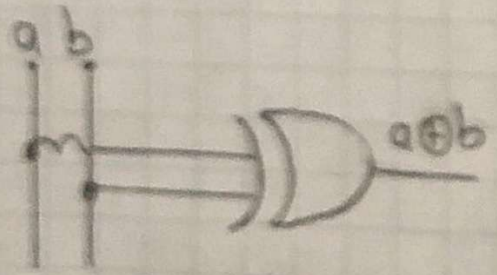
5)

a	b	L
0	0	0
0	1	1
1	0	1
1	1	0

$L_{sp} = \bar{a}b + a\bar{b} = a \oplus b$ ✓

$L_{ps} = (a+b)(\bar{a}+\bar{b}) = a\bar{a} + a\bar{b} + b\bar{a} + b\bar{b}$

$L_{ps} = a\bar{b} + \bar{a}b = a \oplus b$ ✓

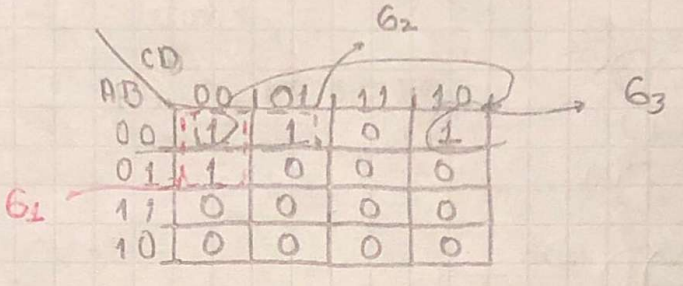


6)

A	B	C	D	P
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

0 → desactivado
1 → activado

proceso se deberá con P → 0



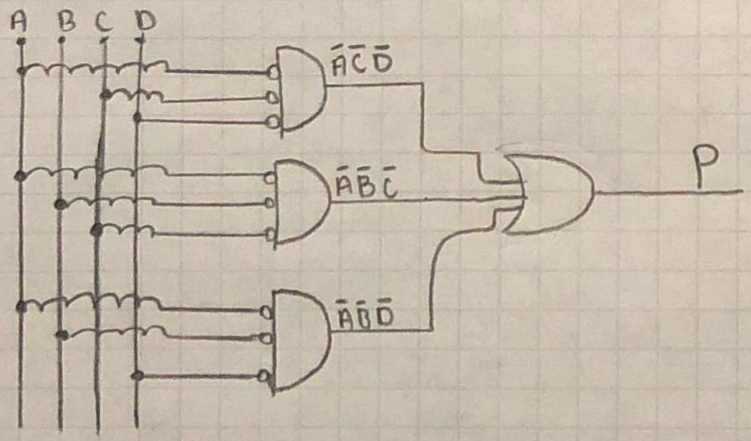
$P_{G1} = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} = \bar{A}\bar{C}\bar{D}$

$P_{G2} = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D = \bar{A}\bar{B}\bar{C}$

$P_{G3} = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} = \bar{A}\bar{B}\bar{D}$

$\Rightarrow P_{SP} = P_{G1} + P_{G2} + P_{G3}$

$P_{SP} = \bar{A}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{D}$



7)

$S_1 \rightarrow 1$ si: $9 < N <= 15$

$S_2 \rightarrow 0$ si: $N = 0$; $N \% 2 = 0$

		e	f	g	h	S_1	S_2	
*	0	0	0	0	0	0	0	
	1	0	0	0	1	0	1	
*	2	0	0	1	0	0	0	
	3	0	0	1	1	0	1	
*	4	0	1	0	0	0	0	
	5	0	1	0	1	0	1	
*	6	0	1	1	0	0	0	
	7	0	1	1	1	0	1	
*	8	1	0	0	0	0	0	
	9	1	0	0	1	0	1	
$S_1=1$	* 10	1	0	1	0	1	0	✓
	11	1	0	1	1	1	1	✓
	* 12	1	1	0	0	1	0	✓
	13	1	1	0	1	1	1	✓
	* 14	1	1	1	0	1	0	✓
	15	1	1	1	1	1	1	✓

→ S_1 por sumas de productos

$$S_1 = \bar{e}f\bar{g}\bar{h} + \bar{e}fgh + ef\bar{g}\bar{h} + ef\bar{g}h + efg\bar{h} + efgh$$

$$S_1 = ef\bar{g} + efg + efg = ef\bar{g} + ef$$

→ S_2 por sumas de productos:

$$S_2 = \bar{e}f\bar{g}h + \bar{e}fgh + \bar{e}f\bar{g}h + \bar{e}fgh + ef\bar{g}h + efgh + ef\bar{g}h + efgh$$

$$S_2 = \bar{e}f\bar{h} + \bar{e}fh + e\bar{f}h + efh$$

$$S_2 = \bar{e}h + eh = h$$

* $S_1 = ef\bar{g} + ef$

* $S_2 = h$

Comparador binario

Restricciones: S0 $A>B$, S1 $A<B$, S2 $A=B$, "1"

$A = A_1\ A_2$

$B = B_1\ B_2$

	A		B		S0	S1	S2
	A_1	A_2	B_1	B_2			
1	0	0	0	0	0	0	1
	0	0	0	1	0	1	0
	0	0	1	0	0	1	0
	0	0	1	1	0	1	0
2	0	1	0	0	1	0	0
	0	1	0	1	0	0	1
	0	1	1	0	0	1	0
	0	1	1	1	0	1	0
3	1	0	0	0	1	0	0
	1	0	0	1	1	0	0
	1	0	1	0	0	0	1
	1	0	1	1	0	1	0
4	1	1	0	0	1	0	0
	1	1	0	1	1	0	0
	1	1	1	0	1	0	0
	1	1	1	1	0	0	1

XNOR

1
0
0
1

2^N

$(A+\bar{B}+C+\bar{D})(\bar{A}+B+C+D)$

$\bar{A}B\bar{C}\bar{D} + AB\bar{C}D$

S0

A \ B	00	01	11	10
00	0	0	0	0
01	1	0	0	0
11	1	1	1	0
10	1	1	0	0

S1

A \ B	00	01	11	10
00	0	1	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	1	0

S2

A \ B	00	01	11	10
00	1	0	0	0
01	0	1	0	0
11	0	0	1	0
10	0	0	0	1

S0

$G_1 = (\bar{A}_1 A_2 \bar{B}_1 \bar{B}_2 + A_1 A_2 \bar{B}_1 \bar{B}_2) = A_2 \bar{B}_1 \bar{B}_2 (\bar{A}_1 + A_1) = A_2 \bar{B}_1 \bar{B}_2$

$G_2 = (A_1 A_2 \bar{B}_1 B_2 + A_1 A_2 B_1 B_2) = A_1 A_2 B_2 (\bar{B}_1 + B_1) = A_1 A_2 B_2$

$G_3 = (A_1 A_2 \bar{B}_1 \bar{B}_2 + A_1 A_2 \bar{B}_1 B_2 + A_1 \bar{A}_2 \bar{B}_1 \bar{B}_2 + A_1 \bar{A}_2 \bar{B}_1 B_2)$

$= (A_1 A_2 \bar{B}_1 (\bar{B}_2 + B_2) + A_1 \bar{A}_2 \bar{B}_1 (\bar{B}_2 + B_2) = A_1 A_2 \bar{B}_1 + A_1 \bar{A}_2 \bar{B}_1$

$= A_1 \bar{B}_1 (A_2 + \bar{A}_2) = A_1 \bar{B}_1$

$S0 = A_2 \bar{B}_1 \bar{B}_2 + A_1 A_2 \bar{B}_2 + A_1 \bar{B}_1$

S4

$G_4 = (\bar{A}_1 \bar{A}_2 D_1 \bar{B}_2 + \bar{A}_1 \bar{A}_2 B_1 B_2) = \bar{A}_1 \bar{A}_2 D_1 (\bar{D}_2 + B_2) = \bar{A}_1 \bar{A}_2 B_1$

$G_2 = \bar{A}_1 \bar{A}_2 D_1 B_2 + \bar{A}_1 \bar{A}_2 D_1 \bar{B}_2 + A_1 \bar{A}_2 D_1 B_2 + A_1 \bar{A}_2 D_1 \bar{B}_2$

$(\bar{A}_1 \bar{A}_2 B_1 (B_2 + \bar{B}_2) + A_1 \bar{A}_2 D_1 (B_2 + \bar{B}_2) = \bar{A}_1 \bar{A}_2 B_1 + A_1 \bar{A}_2 B_1$

$= \bar{A}_2 B_1 (\bar{A}_1 + A_1) = \bar{A}_2 B_1$

$G_3 = \bar{A}_1 \bar{A}_2 B_1 B_2 + A_1 \bar{A}_2 D_1 B_2 = \bar{A}_2 B_1 B_2 (\bar{A}_1 + A_1) = \bar{A}_2 B_1 B_2$

$S4 = \bar{A}_1 \bar{A}_2 B_1 + \bar{A}_2 B_1 + \bar{A}_2 B_1 B_2$

S2

$G_1 = \bar{A}_1 \bar{A}_2 \bar{B}_1 B_2 \quad G_2 = \bar{A}_1 A_2 \bar{B}_1 B_2 \quad G_3 = A_1 A_2 B_1 B_2 \quad G_4 = A_1 \bar{A}_2 B_1 \bar{B}_2$

$S2 = \bar{A}_1 \bar{A}_2 \bar{B}_1 \bar{B}_2 + \bar{A}_1 A_2 \bar{B}_1 B_2 + A_1 A_2 B_1 B_2 + A_1 \bar{A}_2 B_1 \bar{B}_2$