Sp - 15 / 2023

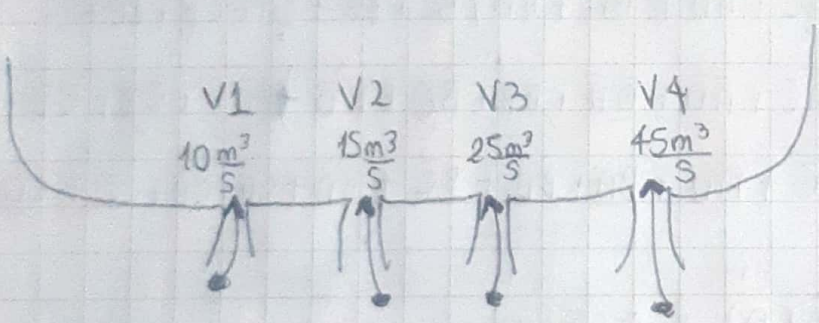
Ej. Tenemos la siguiente expresión algebraica lógica

$$M = AD + C\bar{D} + \bar{A}D + \bar{C}B$$

- a) Expandir en la forma estándar de productos de suma la expresión M.
- b) Simplificar usando solamente algebra de Boole.

Dibujar el circuito lógico obtenido en (A) y (B)

Ej. Se tiene cuatro electroválvulas para controlar los flujos de agua de un sistema de tuberías. Los flujos a través de ~~cada~~ las electroválvulas son 10, 15, 25, 45, respectivamente. (m^3/seg) (m^3/s) (V/ tiempo)



a) obtener la tabla de la verdad para ofrecer a la salida del sistema hidraulico: $10\ \frac{m^3}{s}$, $40\ \frac{m^3}{s}$, $85\ \frac{m^3}{s}$, $60\ \frac{m^3}{s}$, $35\ \frac{m^3}{s}$, $70\ \frac{m^3}{s}$

0 1 1 1 1 1

b) Determine la expresión lógica de salida en su forma estándar de productos de suma.

c) Obtener la expresión lógica mínima usando MK.

d) Dibujar exp. alg. lógica obtenida en el item c)

Powered by CamScanner

$A + A\bar{B} = A + B$

$A + A\bar{B}$

$AC + AC = AC$

$A(\bar{B} + B) =$

(1)

$$M = AB + C\bar{D} + \bar{A}D + \bar{C}B$$

- Primero expandimos: (en productos de sumas)

$A\ B\ \bar{C}\ D$ ✓ — $A\ B\ C\ \bar{D}$ — $\bar{A}\ B\ C\ D$ ✓ — ~~$A\ B\ \bar{C}\ \bar{D}$~~

~~$A\ B\ C\ \bar{D}$~~ — $A\ \bar{B}\ C\ \bar{D}$ ✓ — ~~$\bar{A}\ B\ \bar{C}\ D$~~ — ~~$A\ B\ \bar{C}\ \bar{D}$~~

~~$A\ B\ \bar{C}\ D$~~ — $\bar{A}\ B\ C\ \bar{D}$ ✓ — $\bar{A}\ \bar{B}\ C\ D$ ✓ — ~~$\bar{A}\ B\ \bar{C}\ D$~~

~~$A\ B\ \bar{C}\ \bar{D}$~~ — $\bar{A}\ B\ C\ \bar{D}$ ✓ — $\bar{A}\ B\ \bar{C}\ D$ ✓ — $\bar{A}\ B\ \bar{C}\ \bar{D}$ ✓

a)

$$M = (A+B+C+D)(A+\bar{B}+C+\bar{D})(\bar{A}+B+C+\bar{D})(\bar{A}+\bar{B}+C+\bar{D})$$

$$(\bar{A}+B+C+D)(\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+D)(\bar{A}+B+\bar{C}+D)$$

$$(A+B+C+\bar{D})(A+B+\bar{C}+D)(A+B+\bar{C}+\bar{D})(\bar{A}+B+\bar{C}+D)$$

b) Simplificar:

$$M = (A\bar{A} + A\bar{B} + A\bar{C} + A\bar{D} + BA + B\bar{B} + BC + B\bar{D} + CA + C\bar{B} + C\bar{C} + C\bar{D}$$

Powered by CamScanner

b) Simplificar:

$$M = (A\bar{A} + A\bar{B} + A\bar{C} + A\bar{D} + B\bar{A} + B\bar{B} + B\bar{C} + B\bar{D} + C\bar{A} + C\bar{B} + C\bar{C} + C\bar{D} + D\bar{A} + D\bar{B} + D\bar{C} + D\bar{D}) * (\bar{A}\bar{A} + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{A}\bar{D} + B\bar{A} + B\bar{B} + B\bar{C} + B\bar{D} + C\bar{A} + C\bar{B} + C\bar{C} + C\bar{D} + \bar{D}\bar{A} + \bar{D}\bar{B} + \bar{D}\bar{C} + \bar{D}\bar{D}) * (\bar{A}\bar{A} + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{A}\bar{D} + B\bar{A} + B\bar{B} + B\bar{C} + B\bar{D} + C\bar{A} + C\bar{B} + C\bar{C} + C\bar{D} + D\bar{A} + D\bar{B} + D\bar{C} + D\bar{D}) * (\bar{A}\bar{A} + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{A}\bar{D} + \bar{B}\bar{A} + \bar{B}\bar{B} + \bar{B}\bar{C} + \bar{B}\bar{D} + \bar{C}\bar{A} + \bar{C}\bar{B} + \bar{C}\bar{C} + \bar{C}\bar{D} + D\bar{A} + D\bar{B} + D\bar{C} + D\bar{D}) * (A\bar{A} + A\bar{B} + A\bar{C} + A\bar{D} + B\bar{A} + B\bar{B} + B\bar{C} + B\bar{D} + C\bar{A} + C\bar{B} + C\bar{C} + C\bar{D}) * (\bar{D}\bar{A} + \bar{D}\bar{B} + \bar{D}\bar{C} + \bar{D}\bar{D}) * (A\bar{A} + A\bar{B} + A\bar{C} + A\bar{D} + B\bar{A} + B\bar{B} + B\bar{C} + B\bar{D} + \bar{C}\bar{A} + \bar{C}\bar{B} + \bar{C}\bar{C} + \bar{C}D + \bar{D}\bar{A} + \bar{D}B + \bar{D}C + \bar{D}D)$$

Powered by CamScanner

$(A+\bar{B}+C+\bar{D})(A+B+C+D)$ $\bar{A}BC\bar{D}+ABCD$

3) $M = (A + A(\bar{B}+B) + AC + AC + A(\bar{D}+D) + C(\bar{B}+B) + B\bar{D} + D\bar{B} + C$
$+ C(\bar{D}+D)) \cdot (\bar{A} + \bar{A}(B+\bar{B}) + \bar{A}C + \bar{A}C + \bar{A}\bar{D} + \bar{A}\bar{D} + C(B+\bar{B}) + \bar{D}(B+\bar{B}) + C + \bar{D}$
$+ C\bar{D} + C\bar{D}) \cdot (\bar{A}(\bar{B}+B) + \bar{A} + \bar{A}C + \bar{A}C + \bar{A}D + \bar{A}D + C(\bar{B}+B) + D(B+\bar{B}) + C$
$+ CD + CD + DD) \cdot (\bar{A} + \bar{A}(B+\bar{B}) + \bar{A}\bar{C} + \bar{A}\bar{C} + \bar{A}(\bar{D}+D) + \bar{C}(\bar{B}+B) + \bar{B}\bar{D} + BD + \bar{C}$
$+ \bar{C}(\bar{D}+D)) \cdot (A + AB + AB + A(\bar{C}+C) + A(\bar{D}+D) + B + B(\bar{C}+C) + B(\bar{D}+D) + 0$
$+ CD + C\bar{D} + \bar{D}D) \cdot (0 + B(\bar{A}+A) + \bar{C}(\bar{A}+A) + AD + A\bar{D} + B + B\bar{C} + B\bar{C} + \bar{C}$
$+ B(D+\bar{D}) + \bar{C}D + C\bar{D} + \bar{D}D)$

XOR XOR XOR

Powered by CamScanner

$$M = (A + A + AC + A + C + \underbrace{D\bar{D} + D\bar{B}}_{XOR} + C + C) \cdot (\bar{A} + \bar{A} + \bar{A}C + \bar{A}\bar{D} + C + \bar{D} + C + C\bar{D} + \bar{D}) \cdot (\bar{A} + \bar{A} + \bar{A}C + \bar{A}D + \bar{C} + D + C + CD + D) \cdot (\bar{A} + \bar{A} + \bar{A}\bar{C} + \bar{A} + \bar{C} + \bar{B}\bar{D} + BD + \bar{C} + \bar{C}) \cdot (A + AB + A + A + B + B + B + CD + \bar{C}\bar{D} + \bar{D}D) \cdot (B + \bar{C} + AD + \bar{A}\bar{D} + B + B\bar{C} + \bar{C} + B + \underbrace{\bar{C}D + C\bar{D}}_{XOR} + \bar{D}D)$$

$$M = (\overbrace{A + C + AC}^{A} + B \oplus D) \cdot (\bar{A} + C + \bar{D} + \bar{A}C + \bar{A}\bar{D} + C\bar{D}) \cdot (\bar{A} + C + D + \bar{A}C + \bar{A}D + CD) \cdot (\bar{A} + \bar{C} + \bar{A}\bar{C} + \bar{B}\bar{D} + BD) \cdot (A + B + AB + CD + \bar{C}\bar{D} + \bar{D}D) \cdot (B + \bar{C} + AD + \bar{A}\bar{D} + B\bar{C} + \underbrace{\bar{C}D + C\bar{D}}_{XOR} + \bar{D}D)$$

Règles

$\bar{A} + AB = A$

$A + \bar{A}B = A + B$

$A + BC = (A + B)(A + C)$

$$M = (\overbrace{A}^{\bar{A}} + C + B \oplus D) \cdot (\overbrace{\bar{A} + \bar{A}C}^{\bar{A}} + \overbrace{\bar{D} + \bar{A}\bar{D}}^{\bar{D}} + \underbrace{C + CD}_{C}) \cdot (\overbrace{\bar{A}}^{\bar{A}} + \bar{A}C + \bar{A}D + D + C + CD) \cdot (\overbrace{\bar{A}}^{\bar{A}} + \bar{A}C + \bar{C} + \bar{B}\bar{D} + BD) \cdot (A + AB + B + CD + \bar{C}\bar{D} + \bar{D}D) \cdot (AD + \bar{A}\bar{D} + B + \overbrace{B\bar{C} + \bar{C}}^{\bar{C}} + \bar{D}D + C \oplus D)$$

Powered by CamScanner

$A + \bar{A}B = A + B$

$A + BC = (A+B)(A+C)$

$M = (A + C + B \oplus D) \cdot (\bar{A} + \bar{A}C + \bar{D} + \bar{A}\bar{D} + C + CD)$

$\cdot (\bar{A} + \bar{A}C + \bar{A}D + D + C + CD) \cdot (\bar{A} + \bar{A}C + \bar{C} + \bar{B}\bar{D} + BD) \cdot (A + AB + B + CD + \bar{C}\bar{D} + \bar{D}D)$

$\cdot (AD + \bar{A}\bar{D} + B + B\bar{C} + \bar{C} + \bar{D}D + C \oplus D)$

∴ ↳ Para $\bar{A} + \bar{A}C = \bar{A} \cdot 1 + \bar{A}C = \bar{A}(C + \bar{C}) + \bar{A}C = \bar{A}C + \bar{A}\bar{C} + \bar{A}C = \bar{A}C + \bar{A}C = \bar{A}(C + \bar{C})$

$= \bar{A}$

↳ Para $\bar{D} + \bar{A}\bar{D} = \bar{D} \cdot 1 + \bar{A}\bar{D} = \bar{D}(\bar{A} + A) + \bar{A}\bar{D} = \bar{D}\bar{A} + \bar{D}A + \bar{A}\bar{D} = \bar{D}(\bar{A} + \bar{A} + A)$

$= \bar{D}(\bar{A} + A) = \bar{D}$

↳ Para $C + C\bar{D} = C \cdot 1 + C\bar{D} = C(\bar{D} + D) + C\bar{D} = C\bar{D} + CD + C\bar{D} = C(\bar{D} + \bar{D} + D) = C$

↳ Para $\bar{A} + \bar{A}C = \bar{A}(C + \bar{C} + C) = \bar{A}\,C$

Powered by CamScanner

cambia $\bar{D}$

$$M = (A+C+B\oplus D)\cdot(\bar{A}+\bar{D}+C)\cdot(\bar{A}+D+C)\cdot(\bar{A}+\bar{C}+\overline{\bar{B}D+BD}^{\,xnor})$$
$$\cdot(A+B+\underbrace{CD+\bar{C}\bar{D}}_{xnor}+\bar{D}D)\cdot(\underbrace{AD+\bar{A}\bar{D}}_{xnor}+B+\bar{C}+\bar{D}D+C\oplus D)$$

$$M = (A+C+B\oplus D)\cdot(\bar{A}+C)\cdot(\bar{A}+\bar{C}+\overline{B\oplus D})\cdot(A+B+\overline{C\oplus D}+\bar{D}D)\cdot(\overline{A\oplus D}+B+\bar{C}+\bar{D}D+C\oplus D)$$

$$M = (A+C+B\oplus D)\cdot(\bar{A}+\bar{A}C+\bar{A}(\overline{B\oplus D})+\bar{A}C+\bar{C}+\bar{C}(\overline{B\oplus D}))\cdot(A+B+\bar{D}D+\overline{C\oplus D})$$
$$(B+\bar{C}+\bar{D}D+\overline{A\oplus D}+C\oplus D)$$

$\bar{A}(1+C)=\bar{A}$

$$M = (A+C+B\oplus D)(\bar{A}+\bar{A}C+\bar{C}+\bar{C}(\overline{B\oplus D})+\bar{A}(\overline{B\oplus D}))\cdot(A+B+\bar{D}D+\overline{C\oplus D})$$
$$(B+\bar{C}+\bar{D}D+\overline{A\oplus D}+C\oplus D)$$

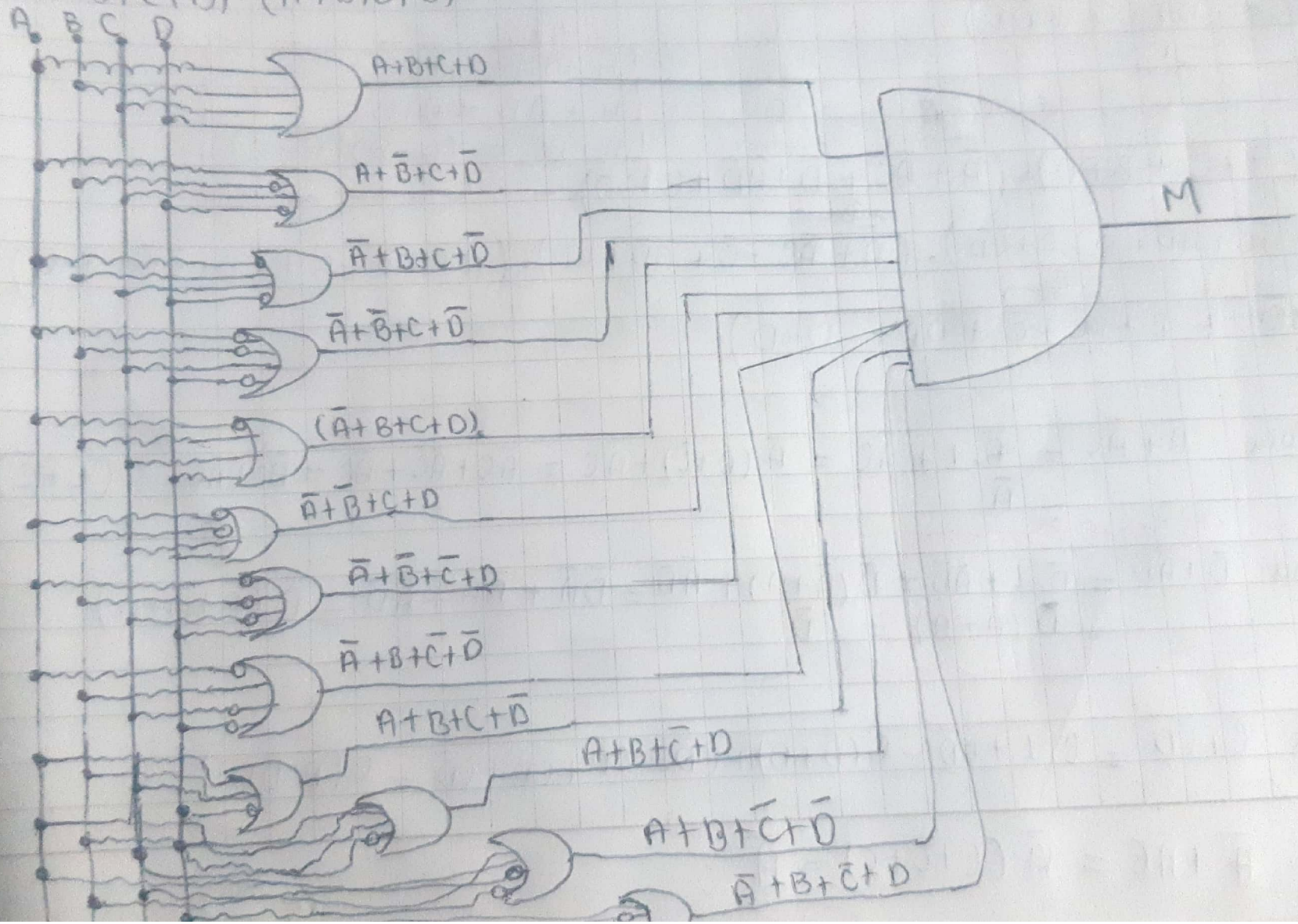
$$M = (A+C+B\oplus D)\cdot(\bar{A}+\bar{C}+\bar{A}(\overline{B\oplus D})+\bar{C}(\overline{B\oplus D}))\cdot(A+B+\bar{D}D+\overline{C\oplus D})$$
$$(B+\bar{C}+\bar{D}D+\overline{A\oplus D}+C\oplus D)$$

Powered by CamScanner

Circuito lógico de a) $M = (A+B+C+D)(A+B+C+\bar{D})(A+B+\bar{C}+D)(A+B+\bar{C}+\bar{D})$

$(\bar{A}+B+C+D)(\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+D)(\bar{A}+B+\bar{C}+\bar{D})(A+B+C+\bar{D})(A+B+\bar{C}+D)$

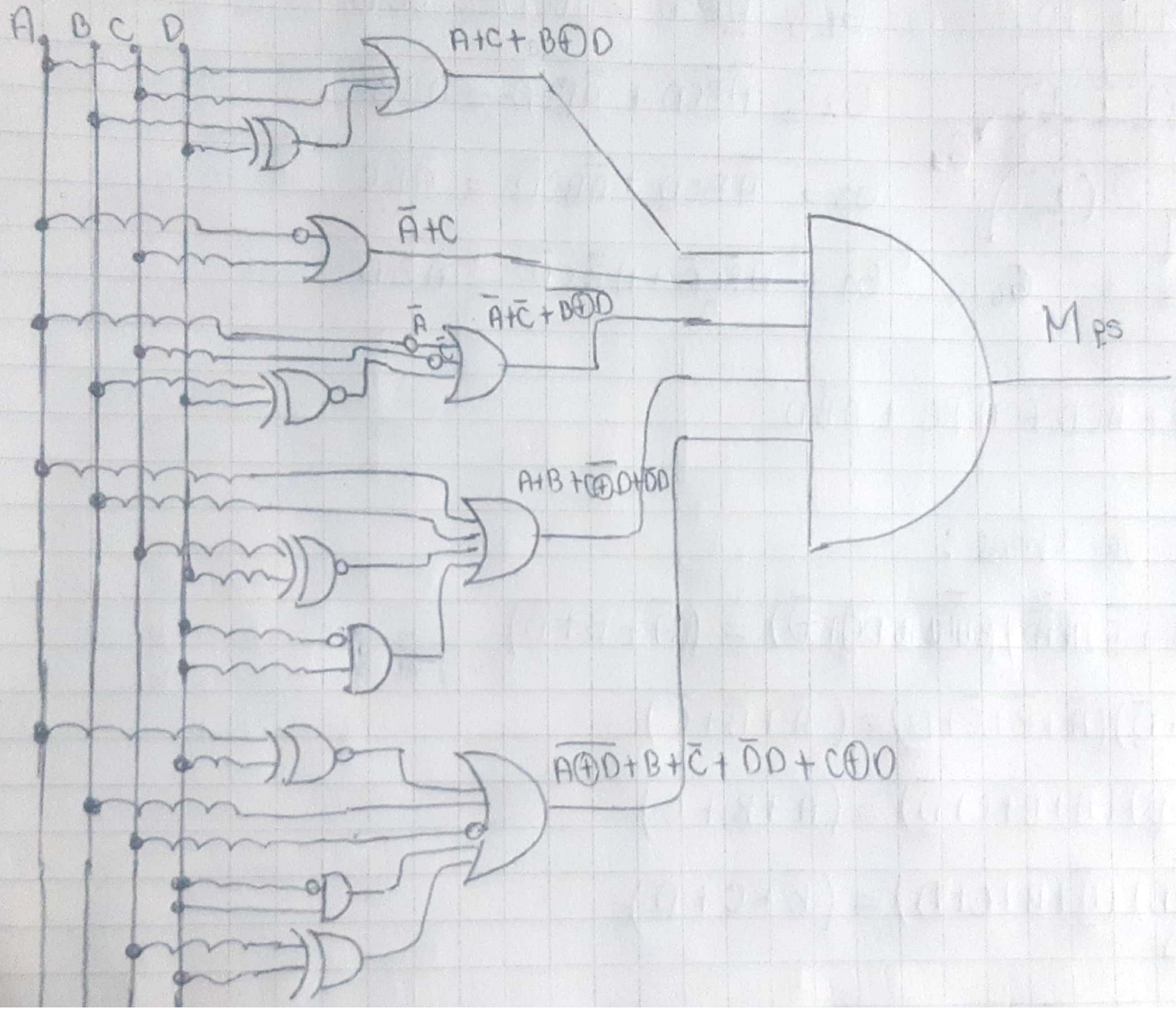
$(A+B+\bar{C}+\bar{D})(\bar{A}+B+\bar{C}+D)$



Powered by CamScanner

Circuito lógico de b)

$M = (A + C + B \oplus D) \cdot (\bar{A} + C)(\bar{A} + \bar{C} + \overline{B \oplus D}) \cdot (A + B + \overline{C \oplus D} + \bar{D}D) \cdot (\overline{A \oplus D} + B + \bar{C} + \bar{D}D + C \oplus D)$



Powered by CamScanner

2) a)

10	15	25	45	
A	B	C	D	S
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1 → $V_3+V_4 = 25+45 = 70\ m^3/s$ (6)
0	1	0	0	0
0	1	0	1	1 → $V_2+V_4 = 15+45 = 60\ m^3/s$ (4)
0	1	1	0	1 → $V_2+V_3 = 15+25 = 40\ m^3/s$ (2)
0	1	1	1	1 → $V_2+V_3+V_4 = 15+25+45 = 85\ m^3/s$ (3)
1	0	0	0	1 → $10\ m^3/s : V_1$ (1)
1	0	0	1	0
1	0	1	0	1 → $V_1+V_3 = 10+25 = 35\ m^3/s$ (5)
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1 → $V_1+V_2+V_4 = 10+15+45 = 70\ m^3/s$ (6)
1	1	1	0	0
1	1	1	1	0

Powered by CamScanner

0	1	0	0	0	
0	1	0	1	1	$\rightarrow V_2+V_4 = 15+45 = 60\ m^3/s$ (4)
0	1	1	0	1	$\rightarrow V_2+V_3 = 15+25 = 40\ m^3/s$ (2)
0	1	1	1	1	$\rightarrow V_2+V_3+V_4 = 15+25+45 = 85\ m^3/s$ (3)
1	0	0	0	1	$\rightarrow 10\ m^3/s : V_1$ (1)
1	0	0	1	0	
1	0	1	0	1	$\rightarrow V_1+V_3 = 10+25 = 35\ m^3/s$ (5)
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	1	$\rightarrow V_1+V_2+V_4 = 10+15+45 = 70\ m^3/s$ (6)
1	1	1	0	0	
1	1	1	1	0	

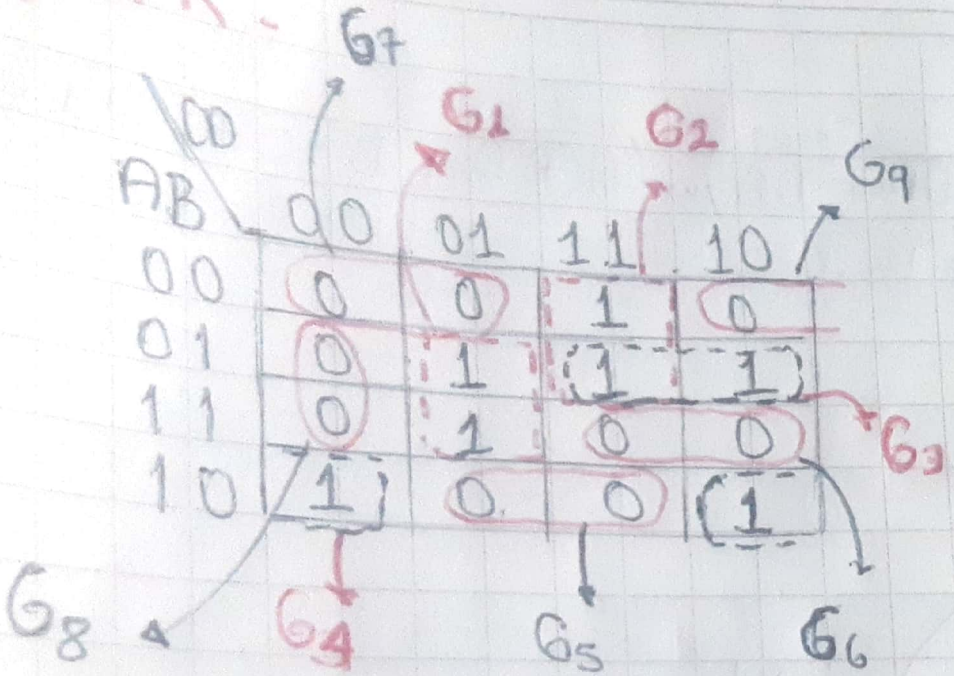
$V_3+V_4 = 25+45 = 70\ m^3/s$ (6)

b) En productos de sumas : $S_{PS} = (A+B+C+D)(A+B+C+\bar{D})(A+B+\bar{C}+D)(A+\bar{B}+C+D)$

$(\bar{A}+B+C+\bar{D})(\bar{A}+B+\bar{C}+\bar{D})(\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})$

Powered by CamScanner

c) MK:



• En sumas de productos:

$G_1 = \bar{A}B\bar{C}D + A\bar{B}\bar{C}D = B\bar{C}D$

$G_2 = \bar{A}\bar{B}CD + \bar{A}B\bar{C}D = \bar{A}CD$

$G_3 = \bar{A}BCD + \bar{A}BC\bar{D} = \bar{A}BC$

$G_4 = A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} = A\bar{B}\bar{D}$

→ $S_{SP} = B\bar{C}D + \bar{A}CD + \bar{A}BC + A\bar{B}\bar{D}$

Powered by CamScanner

$S_P = \bar{A}\bar{C}\bar{D} + A\bar{C}\bar{D} + A\bar{B}\bar{C} + A\bar{B}\bar{D}$

- En productos de sumas:

→ $G_5 = (\bar{A} + B + \underline{C} + \bar{D})(\bar{A} + B + \underline{\bar{C}} + \bar{D}) = (\bar{A} + B + \bar{D})$

→ $G_6 = (\bar{A} + \bar{B} + \bar{C} + \underline{\bar{D}})(\bar{A} + \bar{B} + \bar{C} + \underline{D}) = (\bar{A} + \bar{B} + \bar{C})$

$G_7 = (A + B + C + \underline{D})(A + B + C + \underline{\bar{D}}) = (A + B + C)$

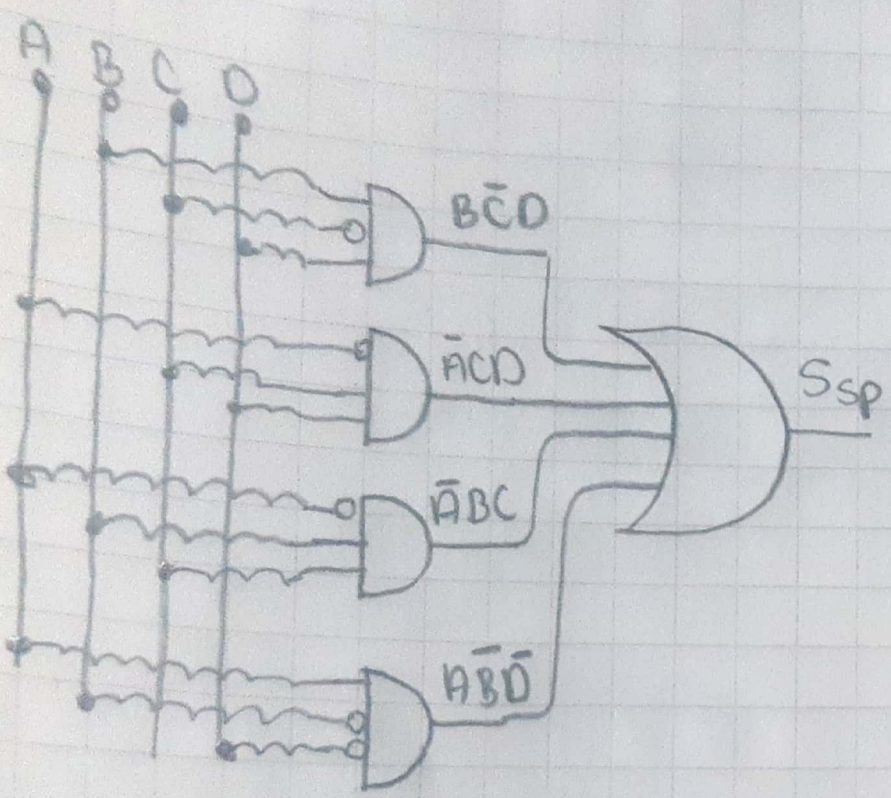
$G_8 = (\underline{A} + \bar{B} + C + D)(\underline{\bar{A}} + \bar{B} + C + D) = (\bar{B} + C + D)$

$G_9 = A + B + \bar{C} + D$

$S_{PS} = (\bar{A} + B + \bar{D})(\bar{A} + \bar{B} + \bar{C})(A + B + C)(\bar{B} + C + D)(A + B + \bar{C} + D)$

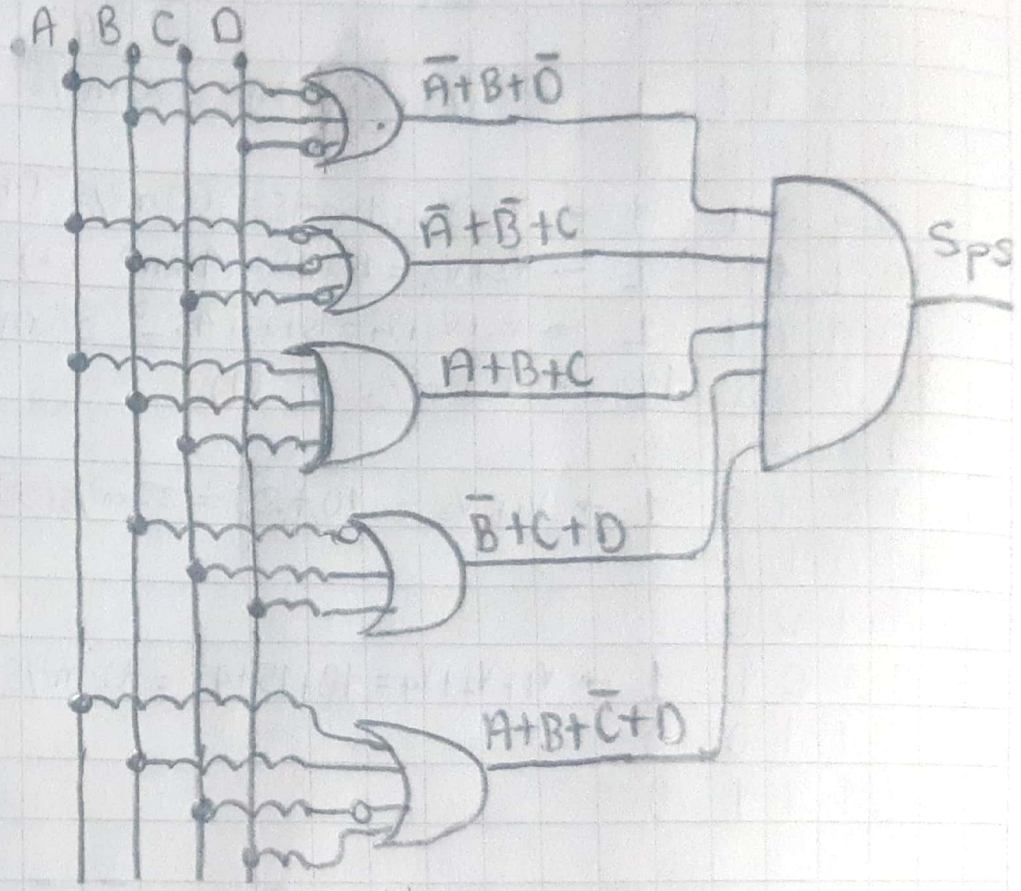
Powered by CamScanner

9) - $(A+B+D)(A+B+C)(\bar{A}+B+C)(\bar{B}+C+D)(A+B+\bar{C}+D)$



⇓

En sumas de productos



⇓

En productos de sumas.