cl. 11c – (Sg.1) – Rez. de pb. – Osc. și Compunere de osc.

pg. 64

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(5%) Na/Fe – ZnO / biroxs / Y.ZrO₂

(1.12/64)

vas U – apă

ρ, S, L

$2y$

a) ω, T, ν

b) $\omega = \omega_0 = \sqrt{g/(L/2)}$

a) $\overline{F_y = m \cdot \vec{a}_y}$ (1); $\vec{a}_y = \ddot{y} = \frac{d^2y}{dt^2}$

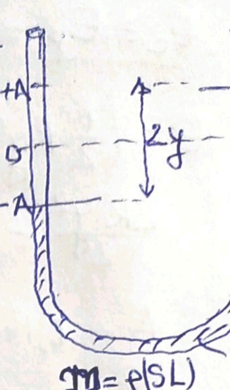
$$\begin{cases} F_y = G_y = m_y \cdot g = \rho(S \cdot 2y) \cdot g \\ m = \rho S L \end{cases}$$

deci (1): $\rho(S \cdot 2y) \cdot g = (\rho S L) \cdot \ddot{y}$

$(\rho S L)\ddot{y} - (2\rho S g) y = 0 \;/ : \rho S L$

$\ddot{y} - \underbrace{\left(\frac{2g}{L}\right)}_{\omega^2} \cdot y = 0.$

$\omega^2 = \frac{2g}{L} \rightarrow \boxed{\omega = \sqrt{\frac{2g}{L}} = \sqrt{\frac{g}{(L/2)}}}$ (1)



$\omega = 2\pi\nu = \frac{2\pi}{T}$, $(\nu = 1/T)$

$\rightarrow \boxed{\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{(L/2)}}}$; $\boxed{T = \frac{1}{\nu} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{2g}}}$

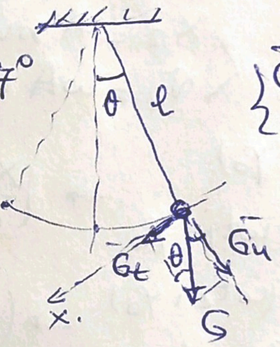
b) pendulul gravitațional / matematic

$$\begin{cases} \overline{G_t = m \cdot a} \quad , a = \ddot{x} \quad ; \theta < 5,7° \\ G_t = (m \cdot g) \sin\theta \simeq m g \theta \simeq m g \left(\frac{x}{\ell}\right) \end{cases}$$

$m g \frac{x}{\ell} = m \cdot \ddot{x}$ $\quad \sin\theta \simeq tg\,\theta \simeq \theta \simeq \frac{x}{\ell}$

$$\begin{cases} m\ddot{x} - \frac{mg}{\ell} x = 0 \;/ : m. \\ \ddot{x} - \underbrace{(g/\ell)}_{\omega_0^2} \cdot x = 0 \end{cases}$$

$\omega_0^2 = g/\ell \rightarrow \boxed{\omega_0 = \sqrt{\frac{g}{\ell}}}$ (2)



$$\begin{cases} \vec{G} = G_n \cdot \vec{n} + G_t \cdot \vec{t} \\ \vec{G} = G_x \cdot \vec{i} + G_y \cdot \vec{j} \end{cases}$$

$$\begin{cases} G_x = G_t = G \cdot \sin\theta \\ G_y = G_n = G\cos\theta \end{cases}$$

$\boxed{G^2 = G_x^2 + G_y^2 = G_n^2 + G_t^2}$

• Comparăm rel (1) cu rel (2) ⇒ $\begin{cases} \omega = \sqrt{g/(L/2)} \\ \omega_0 = \sqrt{g/\ell} \end{cases} \rightarrow \boxed{\ell = L/2}$

lung. col. de H_2O

lung. pendulului matem.

(11.45/16)

$m = 0,2\ kg$

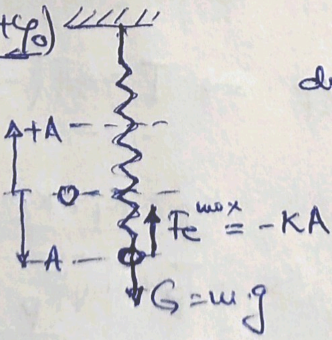
$F_e = 200\ N.$

$E = 40\ J$

$t_0 = 0, \begin{cases} y = 0; \varphi_0 = 0 \\ v = v_{max} = \omega A \end{cases}$

$y = ?$

$? = y = A\sin(\omega t + \varphi_0)$



deci: $F = K \cdot A \rightarrow K = (F/A)$. (1)

$$\begin{cases} \varphi_0 = 0 \\ E = \frac{KA^2}{2} \end{cases}$$

• prelucrăm exp. $E = \underbrace{(K \cdot A)}_{F} \frac{A}{2}$

$E = \frac{F \cdot A}{2} \rightarrow \boxed{A = \frac{2E}{F.}}$ (2)

• Introducem (2) în (1)

$K = \left(\frac{F}{A}\right) = \frac{F}{(2E/F)} = \boxed{\frac{F^2}{2E}}$ (3) ; $\boxed{\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{F^2/2E}{m}} = \sqrt{\frac{F^2}{2mE}} = \frac{F}{\sqrt{2mE}}}$ (4)

• înlocuim toate mărimile (A, ω) în ec. $y = A\sin(\omega t + \varphi_0)$ și obținem ec. osc.

$y = \underset{(2)}{A}\sin(\underset{(4)}{\omega} t + 0) = \left(\frac{2E}{F}\right)\sin\left(\frac{F}{2mE} \cdot t + 0\right)$

11.46/47

$E_c = E_p$; t.

$A = 14{,}14$ cm.

$y(t) = ?$

$$\boxed{E = (E_c + E_p) = \frac{kA^2}{2}} \qquad \begin{cases} E_c = \frac{mv^2}{2} \\ E_p = \frac{ky^2}{2} \end{cases}$$

$$E_c = E_p = \frac{1}{2} \cdot ky^2$$

$$\begin{cases} E = \left(\frac{1}{2}ky^2 + \frac{1}{2}ky^2\right) = \frac{kA^2}{2} \\ 2\left(\frac{k}{2}\right)y^2 = \left(\frac{kA^2}{2}\right) \rightarrow y^2 = \frac{A^2}{2} \rightarrow \boxed{y = \pm\left(\frac{A}{\sqrt{2}}\right)} \end{cases}$$

$$A = 0{,}1414 \text{ cm} \rightarrow y = \pm\left(\frac{A}{1{,}41}\right) = \pm\frac{0{,}1414}{1{,}41} \simeq \pm 0{,}1 \text{ m} = \pm 10 \text{ cm}$$

1.9/64

MOA

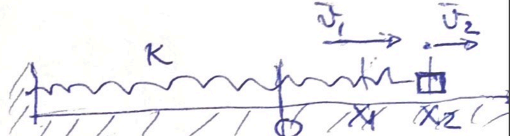
$x_1 = 2$ cm, $v_1 = 5$ m/s

$x_2 = 3$ cm, $v_2 = 4$ m/s

a) $A = ?$

b) T, ν, ω

$$\begin{cases} x = A\sin(\omega t + \varphi_0) \\ v = \omega A\cos(\omega t + \varphi_0) \end{cases} \quad (\text{MOA})$$



$$\begin{cases} x_1 = A\sin(\omega t_1 + \varphi_0) \\ v_1 = \omega A\cos(\omega t_1 + \varphi_0) \end{cases}$$

$$\begin{cases} x_2 = A\sin(\omega t_2 + \varphi_0) \\ v_2 = \omega A\cos(\omega t_2 + \varphi_0) \end{cases}$$

de unde:

$$\begin{cases} \left(\frac{x_1}{A}\right)^2 = \sin^2(\omega t_1 + \varphi_0) \\ (v_1/\omega A)^2 = \cos^2(\omega t_1 + \varphi_0) \end{cases} \rightarrow \left(\frac{x_1}{A}\right)^2 + \left(\frac{v_1}{\omega A}\right)^2 = \left[\sin^2(\omega t_1 + \varphi_0) + \cos^2(\omega t_1 + \varphi_0)\right] = 1.$$

$$\begin{cases} (x_2/A)^2 = \sin^2(\omega t_2 + \varphi_0) \\ (v_2/\omega A)^2 = \cos^2(\omega t_2 + \varphi_0) \end{cases} \rightarrow \left(\frac{x_2}{A}\right)^2 + \left(\frac{v_2}{\omega A}\right)^2 = \left[\sin^2(\omega t_2 + \varphi_0) + \cos^2(\omega t_2 + \varphi_0)\right]$$

atunci:

$$\left(\frac{x_1}{A}\right)^2 + \left(\frac{v_1}{\omega A}\right)^2 = 1 \Rightarrow (x_1^2 + v_1^2/\omega^2) = A^2 \rightarrow A = \sqrt{x_1^2 + v_1^2/\omega^2}$$

$$\left(\frac{x_2}{A}\right)^2 + \left(\frac{v_2}{\omega A}\right)^2 = 1, \Rightarrow (x_1^2 + v_1^2/\omega^2) = (x_2^2 + v_2^2/\omega^2)$$

$$\frac{1}{\omega^2}(v_1^2 - v_2^2) = x_2^2 - x_1^2 \rightarrow \boxed{\omega = \frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

prin rel. de leg.

$$\boxed{\omega = 2\pi\nu = 2\pi/T}$$

$$\nu = \left(\frac{\omega}{2\pi}\right) = \frac{1}{2\pi}\left(\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}\right) = 1/T \Rightarrow T = 2\pi\left(\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}\right) = 1/\nu = 2\pi/\omega$$

1.10/64

m

T_1, k_1

T_2, k_2

a) $T_s = ?$

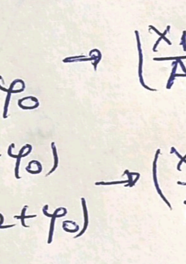
b) $T_p = ?$

$$\begin{cases} T = 2\pi\sqrt{\frac{m}{k}} \\ \omega = \sqrt{\frac{k}{m}} \end{cases}$$

$$T_1 = 2\pi\sqrt{\frac{m}{k_1}}, \; T_2 = 2\pi\sqrt{\frac{m}{k_2}}$$

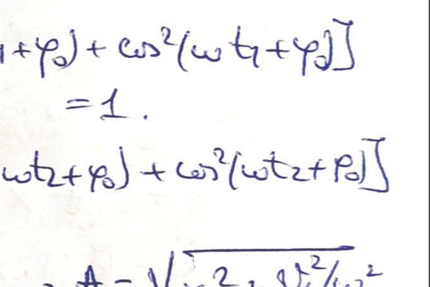
$$1/k_s = 1/k_1 + 1/k_2$$

$$k_s = \frac{k_1 \cdot k_2}{k_1 + k_2}$$



$$k_p = k_1 + k_2$$

$$\begin{cases} T_p = 2\pi\sqrt{\frac{m}{k_p}} \\ T_p = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}} = (T_1 + T_2) \end{cases}$$



$$\begin{cases} G = F_1 + F_2 = k_1x + k_2x = k_p \\ F_1 = k_1 \cdot x \; ; \; F_2 = k_2 x ; G = k \end{cases}$$

$$k_p = k_1 + k_2$$

$$T_s = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{m\left(\frac{k_1 + k_2}{k_1 \cdot k_2}\right)} =$$

$$= 2\pi\sqrt{\frac{m}{k_2} + \frac{m}{k_1}} = \sqrt{T_2 + T_1}$$

$$G = F_{e_1} = F_{e_2}$$

$$mg = k_1 x_1 = k_2 x_2 = k_s \cdot x$$

$$\begin{cases} x = x_1 + x_2 = \left(\frac{m}{k_s}\right)g = \\ x_1 = \left(\frac{m}{k_1}\right)g \; ; \; x_2 = \left(\frac{m}{k_2}\right) \cdot g \end{cases}$$