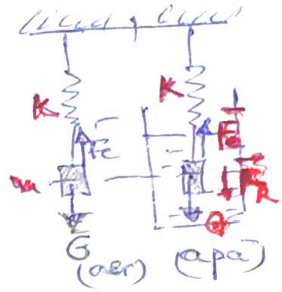
F19 _ Mișcarea Oscilatorie Amortizată

26.10.2020

aXI - pag (6-9)

Două pendule elastice identice (K, m) oscilează cu două medii $\begin{cases} \text{aer}: \vec{F_e} = -K\vec{x} \\ \text{apa}: \vec{F_R} = -r\cdot\vec{v} \text{ (med. disipativ)} \end{cases}$



P2. $\boxed{\vec{R} = m\cdot\vec{a}}$

$$\begin{cases} \vec{R_1} = \vec{F_e} = m\cdot\vec{a} \\ \vec{R_2} = \vec{F_e} + \vec{F_R} = m\cdot\vec{a} \end{cases} \qquad \begin{cases} \vec{v} = \frac{\Delta\vec{x}}{\Delta t} = \frac{d\vec{x}}{dt} = \dot{x} \\ \vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{d^2\vec{x}}{dt^2} = \ddot{x} \end{cases}$$

înlocuim: $\vec{F_R} = -r\cdot\vec{v} = -r\dot{x}$, r - coef. de rezistență

forța de rezistență

$$-Kx - r\cdot\dot{x} = m\cdot\ddot{x}$$

$$m\ddot{x} + r\dot{x} + Kx = 0 \;/ :m$$

$$\ddot{x} + \left(\frac{r}{m}\right)\cdot\dot{x} + \left(\frac{K}{m}\right)\cdot x = 0.$$

notăm $\left(\frac{K}{m}\right) = \omega_0^2$, $b = \left(\frac{r}{2m}\right)$ - coef. de amortizare

$$\left\{\left(\frac{r}{m}\right) = 2b, \; K = m\cdot\omega^2\right.$$

Soluția ec. $\ddot{x} + 2b\dot{x} + \omega^2 x = 0$ este:

$$x = A' \sin(\omega' t + \varphi_0) \quad \text{unde:} \begin{cases} \omega' = \sqrt{\omega_0^2 - b^2} = \sqrt{\left(\frac{K}{m}\right) - b^2} \\ A' = A_0 \cdot e^{-bt} = A_0 \cdot e^{-\left(\frac{r}{2m}\right)t} \end{cases}$$

Amplitudinea scade exp. cu timp. adică amortizat

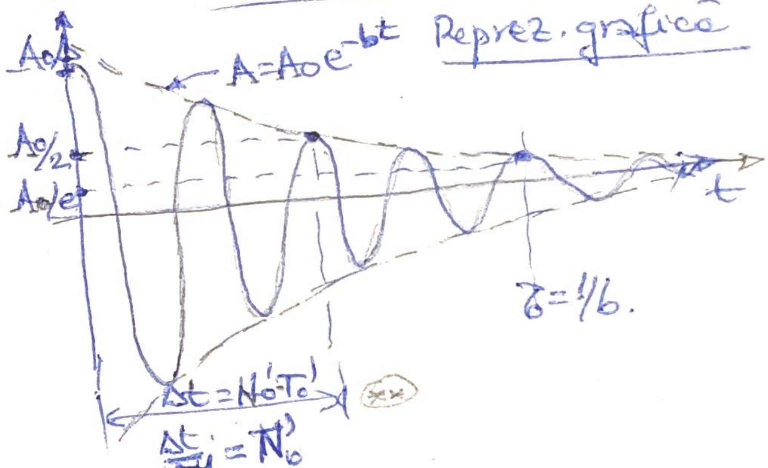
- Decrementul logaritmic, Δ

$$\boxed{\Delta \overset{def}{=} \ln\left[\frac{A(t)}{A(t+T)}\right] = bT_0' = \frac{T_0'}{\tau} = \frac{\ln 2}{N_0'}}$$

- măsoară amortizarea osc. cu timp.

$\tau = \left(\frac{1}{b}\right)$ - timpul de relaxare / viață

Reprez. grafică



(*) La $(t = \tau)$ $\boxed{A(\tau) = A_0/e}$

$$\begin{cases} A(t) = A_0 \cdot e^{-bt} \\ A(\tau) = A_0 \cdot e^{-b\tau} = A_0/e \end{cases}$$

$$\ln\left[A_0 \cdot e^{-b\tau}\right] = \ln\left[\frac{A_0}{e}\right]$$

$$\begin{cases} -b\tau \cdot \ln A_0 = -\ln A_0 \;/ :\ln A_0 \\ b\tau = 1 \rightarrow \boxed{\tau = 1/b} \end{cases}$$

deci $\Delta = \ln\left[\frac{A(t)}{A(t+T)}\right] = \ln\left[\frac{A_0 e^{-bt}}{A_0 \cdot e^{-b(t+T_0')}}\right] = \ln e^{+bT_0'} = b\cdot T_0' = T_0/\tau$

(**) La $t = \Delta t = N_0' T_0' \rightarrow \boxed{A = A_0/2}$

$A = A_0 e^{-bt}$

$$\Rightarrow \frac{A_0}{2} = A_0 e^{-b\Delta t} = A_0 e^{-bN_0' T_0'} \;/ :A_0$$

$$1/2 = e^{-bN_0' T'} \;/ \ln$$

$$-\ln 2 = -bN_0' T' = -N_0' \cdot (bT') = -N_0 \cdot \Delta$$

$$\rightarrow \boxed{\Delta = \frac{\ln 2}{N_0}}$$