Compunerea Osc. perpendiculare $(x(t) \perp y(t))$ de aceeași frecvență/pulsație $(\nu, \omega = 2\pi\nu)$

Un osc. supus simultan acțiunii a două legi de oscilație, de tipul:

$$\begin{cases} x = A_1 \cdot \sin(\omega t + \varphi_1) \\ y = A_2 \cdot \sin(\omega t + \varphi_2) \end{cases} ; \quad (\varphi_2 - \varphi_1) = \Delta\varphi = \pi/2 \ (90^\circ)$$

Mișcarea osc. rezultantă se găsește prin metoda analitică/trigonom.

$$\begin{cases} \dfrac{x}{A_1} = \sin(\omega t + \varphi_1) \\ y/A_2 = \sin(\omega t + \varphi_2) \end{cases}, \text{ dezvoltând.} \quad \boxed{\begin{aligned} \sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \sin\beta\cos\alpha \\ \cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \end{aligned}}$$

$$\begin{cases} x/A_1 = \sin\omega t\cos\varphi_1 + \cos\omega t\sin\varphi_1 \\ y/A_2 = \sin\omega t\cos\varphi_2 + \cos\omega t\sin\varphi_2 \end{cases} \left| \begin{matrix} (1) \\ \cos\varphi_2 \\ \cos\varphi_1 \end{matrix} \right| \begin{matrix} (2) \\ \sin\varphi_2 \\ -\sin\varphi_1 \end{matrix} \Bigg| \text{ și le adunăm}$$

obținem:

$$(1) \quad y/A_2 \cos\varphi_1 - x/A_1 \cos\varphi_2 = \cos\omega t(\cos\varphi_1\sin\varphi_2 - \sin\varphi_1\cos\varphi_2) = \cos\omega t \cdot \sin(\varphi_2 - \varphi_1)$$

$$(2) \quad x/A_1 \sin\varphi_2 - y/A_2 \sin\varphi_1 = \sin\omega t(\sin\varphi_2\cos\varphi_1 - \sin\varphi_1\cos\varphi_2) = \sin\omega t \cdot \sin(\varphi_2 - \varphi_1)$$

$$(1)^2 + (2)^2: \quad \sin^2\Delta\varphi = \Big[\underbrace{(y/A_2)^2}_{\cos^2\varphi_1} + \underbrace{(x/A_1)^2}_{\cos^2\varphi_2} - \frac{2xy}{A_1A_2}\cos\varphi_1\cos\varphi_2\Big] + \Big[(y/A_2)^2\sin^2\varphi_1 + (x/A_1)^2\sin^2\varphi_2 - \frac{2xy}{A_1A_2}\sin\varphi_1\sin\varphi_2\Big]$$

grupând termenii asemenea și făcând $\begin{cases} \sin^2\varphi_1 + \cos^2\varphi_1 = 1 \\ \cos^2\varphi_2 + \sin^2\varphi_2 = 1 \end{cases} \Rightarrow$

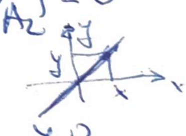
$$\sin^2\Delta\varphi = (y/A_2)^2 + (x/A_1)^2 - \frac{2xy}{A_1A_2}\cos(\varphi_2 - \varphi_1) \quad \text{sau:}$$

$$\boxed{\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1A_2}\cos\Delta\varphi = \sin^2\Delta\varphi}$$

Cazuri particulare:

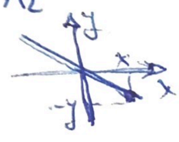
1) $\Delta\varphi = 2k\pi$, $k = 0, 1, 2 \ldots$; $\begin{cases} \cos 2k\pi = +1 \\ \sin 2k\pi = 0 \end{cases} \Rightarrow \dfrac{x^2}{A_1^2} + \dfrac{y^2}{A_2^2} - \dfrac{2xy}{A_1A_2} = 0 \rightarrow \left(\dfrac{x}{A_1} - \dfrac{y}{A_2}\right)^2 = 0$

sau: $\boxed{y = \left(\dfrac{A_2}{A_1}\right) \cdot x}$ – ec. dreptei/prima bisectoare $(C_1 C_3)$

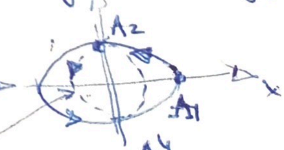


2) $\Delta\varphi = (2k+1)\pi$, $k = 0, 1, 2 \ldots$ $\begin{cases} \cos(2k+1)\pi = -1 \\ \sin(2k+1)\pi = 0 \end{cases} \Rightarrow \dfrac{x^2}{A_1^2} + \dfrac{y^2}{A_2^2} + \dfrac{2xy}{A_1A_2} = 0 \rightarrow \left(\dfrac{x}{A_1} + \dfrac{y}{A_2}\right)^2 = 0$

sau $\boxed{y = -\left(\dfrac{A_2}{A_1}\right)x}$ – ec. dreptei/a 2-a bisectoare $(C_2 C_4)$



(3) $\Delta\varphi = (\pi/2)$ $\begin{cases} \cos\pi/2 = 0 \\ \sin\pi/2 = 1 \end{cases} \rightarrow \begin{cases} \boxed{\dfrac{x^2}{A_1^2} + \dfrac{y^2}{A_2^2} - 0 = 1} \text{ – elipsă } (A_1 \neq A_2) \\ \boxed{\dfrac{x^2}{A^2} + \dfrac{y^2}{A^2} = 1} \text{ – cerc } (A_1 = A_2) \end{cases}$



4) $\Delta\varphi = \left(\dfrac{3\pi}{2}\right)$ $\begin{cases} \cos\dfrac{3\pi}{2} = 0 \\ \sin\left(\dfrac{3\pi}{2}\right) = -1 \end{cases} \rightarrow \boxed{\dfrac{x^2}{A_1^2} + \dfrac{y^2}{A_2^2} = (-1)^2 = 1}$ – elipsă inversă

$\left(\dfrac{x^2}{A^2} + \dfrac{y^2}{A^2} = 1\right)$ – cerc invers.

Obs.

1) Adunând 2 osc. circulare de sensuri opuse se obține o osc. cu ampl. dublă (2A)

2) O osc. arm. poate fi descompusă în 2 osc. circulare cu sensuri opuse și (A/2)

02.11.2020

4.11c (S8.1) Compunerea osc. perpendiculare ($x(t) \perp y(t)$) de aceeași frecvență/pulsație (ν, $\omega = 2\pi\nu$)

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Mișcarea osc. rezultantă se găsește prin metoda analitică/trigonom.

$$\boxed{\begin{aligned} \sin(\alpha \pm \beta) &= \sin\alpha \cos\beta \pm \sin\beta \cos\alpha \\ \cos(\alpha \pm \beta) &= \cos\alpha \cos\beta \mp \sin\alpha \sin\beta \end{aligned}}$$

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obținem:

(1) $y/A_2 \cos\varphi_1 - x/A_1 \cos\varphi_2 = \cos\omega t(\cos\varphi_1 \sin\varphi_2 - \sin\varphi_1 \cos\varphi_2) = \cos\omega t \sin(\varphi_2 - \varphi_1)$

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$(1)^2 + (2)^2$: $\sin^2\Delta\varphi = \left[(y/A_2)^2\cos^2\varphi_1 + (x/A_1)^2\cos^2\varphi_2 - \dfrac{2xy}{A_1A_2}\cos\varphi_1\cos\varphi_2\right] + \left[(y/A_2)^2\sin^2\varphi_1 + (x/A_1)^2\sin^2\varphi_2 - \dfrac{2xy}{A_1A_2}\sin\varphi_1\sin\varphi_2\right]$

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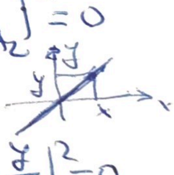

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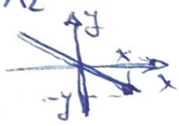

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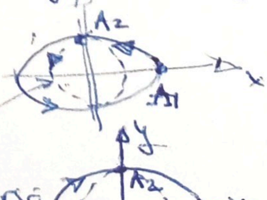

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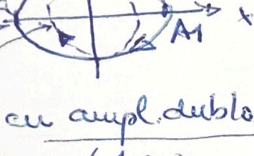
$\left(\dfrac{x^2}{A^2} + \dfrac{y^2}{A^2} = 1\right)$ — cerc invers.



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