30.09.2020

Cl 11a _ S4.2 - Compunerea osc. paralele prin metoda analitică / trigonometrică

OLA - este supus simultan la două legi de oscilație (y_1, y_2) paralele
- mișcarea OLA se va efectua după o leg. rezultantă $y = y_1 + y_2$

$$\begin{cases} y_1 = A_1 \sin(\omega t + \varphi_{01}) \\ y_2 = A_2 \sin(\omega t + \varphi_{02}) \end{cases}$$

$$\begin{cases} \sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha \\ \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \end{cases}$$

deci căutăm $y = y_1 + y_2$

$$y = A_1 \sin(\omega t + \varphi_{01}) + A_2 \sin(\omega t + \varphi_{02}) =$$

$$= A_1 \sin\omega t \cos\varphi_{01} + A_1 \sin\varphi_{01} \cos\omega t + A_2 \sin\omega t \cos\varphi_{02} + A_2 \sin\varphi_{02} \cos\omega t =$$

$$= \sin\omega t (A_1 \cos\varphi_{01} + A_2 \cos\varphi_{02}) + \cos\omega t (A_1 \sin\varphi_{01} + A_2 \sin\varphi_{02}) =$$

$$= (A_1 \cos\varphi_{01} + A_2 \cos\varphi_{02}) \left[\sin\omega t + \frac{A_1 \sin\varphi_{01} + A_2 \sin\varphi_{02}}{A_1 \cos\varphi_{01} + A_2 \cos\varphi_{02}} \cdot \cos\omega t\right] =$$

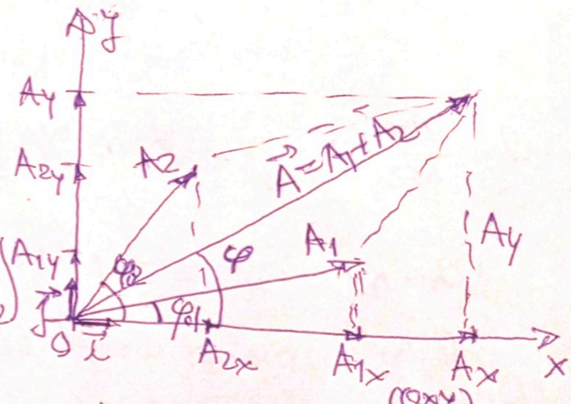
$$= \underbrace{\frac{\overset{A_{1x}}{A_1 \cos\varphi_{01}} + \overset{A_{2x}}{A_2 \cos\varphi_{02}}}{\cos\varphi}}_{(*)} \underbrace{(\sin\omega t \cos\varphi + \sin\varphi \cos\omega t)}_{= \sin(\omega t + \varphi)} \qquad \left(= \frac{\sin\varphi}{\cos\varphi}\right)$$

$$\boxed{y = A \cdot \sin(\omega t + \varphi)}$$

$$tg\,\varphi = \left(\frac{\sin\varphi}{\cos\varphi}\right) = \frac{A_1 \sin\varphi_{01} + A_2 \sin\varphi_{02}}{A_1 \cos\varphi_{01} + A_2 \cos\varphi_{02}}$$

- leg. de osc. rezultantă a OLA

unde
$$\begin{cases} A = \frac{A_x}{\cos\varphi} \overset{(**)}{=} \frac{(A_{1x} + A_{2x})}{\cos\varphi} = \boxed{\frac{A_1 \cos\varphi_{01} + A_2 \cos\varphi_{02}}{\cos\varphi}} \\ tg\,\varphi = \left(\frac{\sin\varphi}{\cos\varphi}\right) = \left(\frac{A_y}{A_x}\right) = \frac{A_{1y} + A_{2y}}{A_{1x} + A_{2x}} = \left(\frac{A_1 \sin\varphi_{01} + A_2 \sin\varphi_{02}}{A_1 \cos\varphi_{01} + A_2 \cos\varphi_{02}}\right) \end{cases}$$



(Oxy)

$$\begin{cases} A_x = (A_{1x} + A_{2x}) \\ A_y = (A_{1y} + A_{2y}) \end{cases} \qquad \begin{cases} Ox \perp Oy \\ \vec{i} \perp \vec{j} \end{cases}$$

$$\boxed{\vec{A} = A_x \vec{i} + A_y \vec{j}}$$

$$\begin{cases} A_{1x} = A_1 \cos\varphi_{01} \\ A_{2x} = A_2 \cos\varphi_{02} \end{cases} \quad \boxed{A_x = A_{1x} + A_{2x}}\ (**)$$

$$(*)\ \boxed{(A_{1x} + A_{2x})/\cos\varphi = A} \leftarrow A_x = A\cos\varphi = (A_{1x} + A_{2x})$$

$$\begin{cases} A_{1y} = A_1 \sin\varphi_1 \\ A_{2y} = A_2 \sin\varphi_2 \end{cases}$$

$$A_y = A \cdot \sin\varphi = (A_{1y} + A_{2y})$$

$$\boxed{tg\,\varphi = \left(\frac{A_y}{A_x}\right) = \left(\frac{\sin\varphi}{\cos\varphi}\right) = \frac{A_{1y} + A_{2y}}{A_{1x} + A_{2x}}}$$

30.09.2022

Compunerea osc. paralele prin metoda analitică / trigonometrică

- OLA - este supus simultan la două legi de oscilație astfel:

$$\begin{cases} y_1 = A_1 \sin(\omega t + \varphi_{01}) \\ y_2 = A_2 \sin(\omega t + \varphi_{02}) \end{cases}$$

- OLA - va oscila după o lege rezultantă astfel:

$$\begin{cases} \sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha \\ \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \end{cases}$$

Formule trigonometrice.

$$y = (y_1 + y_2) = A_1 \sin(\overset{\alpha}{\omega t} + \overset{\beta}{\varphi_{01}}) + A_2 \sin(\overset{\alpha}{\omega t} + \overset{\beta}{\varphi_{02}}) \Leftrightarrow$$

$$\Leftrightarrow [A_1 \sin\omega t \cos\varphi_{01} + A_1 \sin\varphi_{01} \cos\omega t] + [A_2 \sin\omega t \cos\varphi_{02} + A_2 \sin\varphi_{02} \cos\omega t]$$

$$= \sin\omega t \{A_1 \cos\varphi_{01} + A_2 \cos\varphi_{02}\} + \cos\omega t \{A_1 \sin\varphi_{01} + A_2 \sin\varphi_{02}\} =$$

$$= (A_1\cos\varphi_{01} + A_2\cos\varphi_{02}) \left[\sin\omega t + \left(\frac{A_1 \sin\varphi_{01} + A_2 \sin\varphi_{02}}{A_1\cos\varphi_{01} + A_2\cos\varphi_{02}}\right)\cdot\cos\omega t\right] =$$

$$= \frac{(A_1\cos\varphi_{01} + A_2\cos\varphi_{02})}{\cos\varphi} \cdot [\sin\omega t \cdot \cos\varphi + \sin\varphi \cos\omega t] \qquad tg\varphi = \left(\frac{\sin\varphi}{\cos\varphi}\right)$$

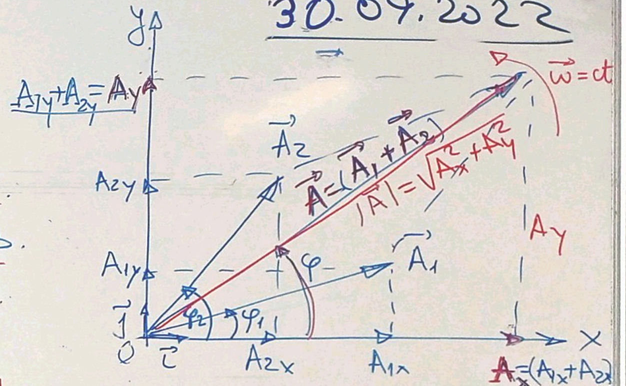
$$[\sin\omega t \cdot \cos\varphi + \sin\varphi \cos\omega t] = \sin(\omega t + \varphi)$$

$$\boxed{y = (y_1 + y_2) = A \cdot \sin(\omega t + \varphi)}$$

unde: $\boxed{A = \frac{A_1\cos\varphi_{01} + A_2\cos\varphi_{02}}{\cos\varphi}}$

$A_x = A\cdot\cos\varphi \rightarrow A = \frac{A_x}{\cos\varphi}$.

$\rightarrow A = \frac{A_x}{\cos\varphi} = \frac{(A_{1x} + A_{2x})}{\cos\varphi}$.



$$\vec{A_1} = A_{1x}\vec{i} + A_{1y}\vec{j}$$

$$\begin{cases} A_{1x} = A_1 \cos\varphi_{01} \\ A_{1y} = A_1 \sin\varphi_{01} \end{cases}; \quad |\vec{A_1}| = \sqrt{A_{1x}^2 + A_{1y}^2}$$

$$\vec{A_2} = A_{2x}\vec{i} + A_{2y}\vec{j}$$

$$\begin{cases} A_{2x} = A_2 \cos\varphi_{02} \\ A_{2y} = A_2 \sin\varphi_{02} \end{cases} \quad |\vec{A_2}| = \sqrt{A_{2x}^2 + A_{2y}^2}$$

$\vec{A} = \vec{A_1} + \vec{A_2}$ ← Fazorul / osc. rezultant.

$$\vec{A} = \vec{A_x} + \vec{A_y} = (A_{1x} + A_{2x})\vec{i} + (A_{1y} + A_{2y})\vec{j}$$

$$\boxed{tg\varphi = \left(\frac{\sin\varphi}{\cos\varphi}\right) = \frac{A_y}{A_x} = \frac{(A_{1y} + A_{2y})}{(A_{1x} + A_{2x})}}$$

Rez. de p3.

11.45/16.

$m = 0{,}2\ kg$

$F_e = 200\ N$.

$E = 40\ j$.

$t_0 = 0$, $\begin{cases} y = 0 \\ \varphi_0 = 0 \end{cases}$.

$v_{max} = \omega A$.

a) $y = ?$

b) $v = ?$

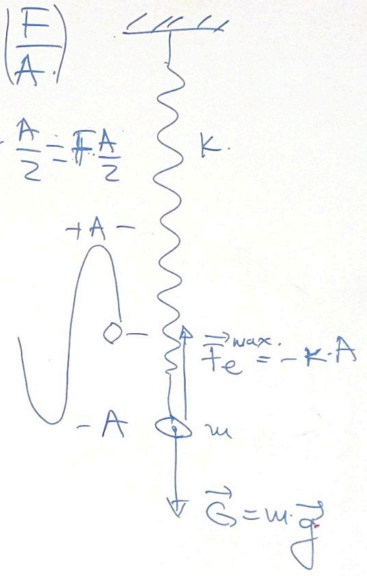
c) $a = ?$

$$\boxed{y = A \cdot \sin(\omega t + \varphi_0)}, (0)$$

$$\begin{cases} F = K \cdot A \rightarrow K = \left(\frac{F}{A}\right) & (1) \\ \varphi_0 = 0. \\ E = \left(\frac{KA^2}{2}\right) = \underbrace{(KA)}_{F} \cdot \frac{A}{2} = F \cdot \frac{A}{2} \end{cases}$$

$$E = F \cdot \frac{A}{2}$$

$$\boxed{A = \left(\frac{2E}{F}\right)}, (2)$$



ⓐ Introducem (2) in (1):

$$K = \left(\frac{F}{A}\right) = \frac{F}{(2E/F)} = \left(\frac{F^2}{2E}\right), (3)$$

$$\boxed{\omega = \sqrt{\frac{K}{m}} = \sqrt{\left(\frac{F^2}{2E}\right) \cdot \frac{1}{m}} = \frac{F}{\sqrt{2mE}}}, (4)$$

Substituim ec. (2) + (4) in ec. de osc. $y = A \sin(\omega t + \varphi_0)$

$$\boxed{y = \left(\frac{2E}{F}\right) \sin\left(\frac{F \cdot t}{\sqrt{2mE}} + 0\right) \equiv A \cdot \sin(\omega t + \varphi_0)}$$

ⓑ $$\boxed{v = v_{max} \cos(\omega t + \varphi_0) = \omega A \cos(\omega t + \varphi_0) = \frac{F}{\sqrt{2mE}} \cdot \left(\frac{2E}{F}\right) \cos\left(\frac{F \cdot t}{\sqrt{2mE}} + 0\right) = v(t)}$$

ⓒ $$\boxed{a = -a_{max} \sin(\omega t + \varphi_0) = -\omega y(t) = -\omega^2 A \cdot \sin(\omega t + \varphi_0) = -\left(\frac{F}{\sqrt{2mE}}\right)^2 \cdot \left(\frac{2E}{F}\right) \cdot \sin\left(\frac{F \cdot t}{\sqrt{2mE}} + 0\right) = a(t)}$$