

Cl 11a - §4.2 - Compunerea osc. paralele prin metoda analitică trigonometrică

OLA - este supus simultan la două legi de oscilație (y_1, y_2) paralele
- mișcarea OLA se va efectua după o leg. rezultantă $y = y_1 + y_2$

$$\begin{cases} y_1 = A_1 \sin(\omega t + \varphi_1) \\ y_2 = A_2 \sin(\omega t + \varphi_2) \end{cases}$$

$$\begin{cases} \sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \end{cases}$$

deci cautoan $y = y_1 + y_2$

$$\begin{aligned} y &= A_1 \sin(\omega t + \varphi_1) + A_2 \sin(\omega t + \varphi_2) = \\ &= A_1 \sin \omega t \cos \varphi_1 + A_1 \cos \varphi_1 \sin \omega t + A_2 \sin \omega t \cos \varphi_2 + A_2 \cos \varphi_2 \sin \omega t = \\ &= \sin \omega t (A_1 \cos \varphi_1 + A_2 \cos \varphi_2) + \cos \omega t (A_1 \sin \varphi_1 + A_2 \sin \varphi_2) = \\ &= (A_1 \cos \varphi_1 + A_2 \cos \varphi_2) \left[\sin \omega t + \frac{A_1 \sin \varphi_1 + A_2 \sin \varphi_2}{A_1 \cos \varphi_1 + A_2 \cos \varphi_2} \cdot \cos \omega t \right] = \\ &= \underbrace{A_1 \cos \varphi_1 + A_2 \cos \varphi_2}_{\cos \varphi} \underbrace{\left(\sin \omega t \cos \varphi + \sin \varphi \cos \omega t \right)}_{\sin(\omega t + \varphi)} = \sin(\omega t + \varphi) \end{aligned}$$

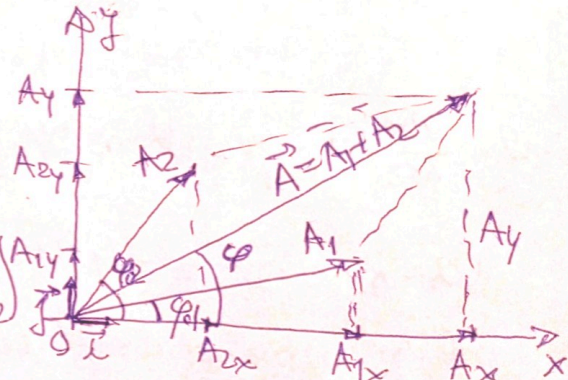
$$\tan \varphi = \frac{\sin \varphi}{\cos \varphi} = \frac{A_1 \sin \varphi_1 + A_2 \sin \varphi_2}{A_1 \cos \varphi_1 + A_2 \cos \varphi_2}$$

$$|y = A \cdot \sin(\omega t + \varphi)|$$

- leg. de osc. rezultantă a OLA

$$\text{unde } A = \frac{A_x}{\cos \varphi} = \frac{(A_1 x + A_2 x)}{\cos \varphi} = \frac{A_1 \cos \varphi_1 + A_2 \cos \varphi_2}{\cos \varphi}$$

$$\tan \varphi = \frac{\sin \varphi}{\cos \varphi} = \frac{A_y}{A_x} = \frac{A_1 y + A_2 y}{A_1 x + A_2 x} = \frac{A_1 \sin \varphi_1 + A_2 \sin \varphi_2}{A_1 \cos \varphi_1 + A_2 \cos \varphi_2}$$



$$\begin{cases} A_x = (A_1 x + A_2 x) \\ A_y = (A_1 y + A_2 y) \end{cases} \quad \begin{cases} \vec{Ox} \perp \vec{Oy} \\ \vec{A} \perp \vec{A'} \end{cases}$$

$$\vec{A} = A_x \vec{i} + A_y \vec{j}$$

$$\begin{cases} A_{1x} = A_1 \cos \varphi_1 \\ A_{2x} = A_2 \cos \varphi_2 \end{cases} \quad \begin{cases} A_x = A_1 x + A_2 x \\ A_y = A_1 y + A_2 y \end{cases}$$

$$(*) \left[\frac{A_1 x + A_2 x}{\cos \varphi} \right] \leftarrow A_x = A \cos \varphi = (A_1 x + A_2 x)$$

$$\begin{cases} A_{1y} = A_1 \sin \varphi_1 \\ A_{2y} = A_2 \sin \varphi_2 \end{cases}$$

$$A_y = A \sin \varphi = (A_1 y + A_2 y)$$

$$\tan \varphi = \frac{A_y}{A_x} = \frac{\sin \varphi}{\cos \varphi} = \frac{A_1 y + A_2 y}{A_1 x + A_2 x}$$

concl

Combinarea osc. paralele. prin metoda analitică / trigonometrică

- OA - este suprasimulat la două legi de oscilație astfel:

$$\begin{cases} y_1 = A_1 \sin(\omega t + \varphi_1) \\ y_2 = A_2 \sin(\omega t + \varphi_2) \end{cases}$$

- OA - va oscila după o lege rezultantă astfel:

$$y = y_1 + y_2 = A_1 \sin(\omega t + \varphi_1) + A_2 \sin(\omega t + \varphi_2) \stackrel{\text{P}}{=} [A_1 \sin \omega t \cos \varphi_1 + A_1 \sin \varphi_1 \cos \omega t] + [A_2 \sin \omega t \cos \varphi_2 + A_2 \sin \varphi_2 \cos \omega t]$$

$$= \sin \omega t \{ A_1 \cos \varphi_1 + A_2 \cos \varphi_2 \} + \cos \omega t \{ A_1 \sin \varphi_1 + A_2 \sin \varphi_2 \} =$$

$$= (A_1 \cos \varphi_1 + A_2 \cos \varphi_2) \sin \omega t + (A_1 \sin \varphi_1 + A_2 \sin \varphi_2) \cos \omega t =$$

$$= (A_1 \cos \varphi_1 + A_2 \cos \varphi_2) \sin \omega t + \underbrace{(A_1 \sin \varphi_1 + A_2 \sin \varphi_2)}_{A \cos \varphi} \cos \omega t$$

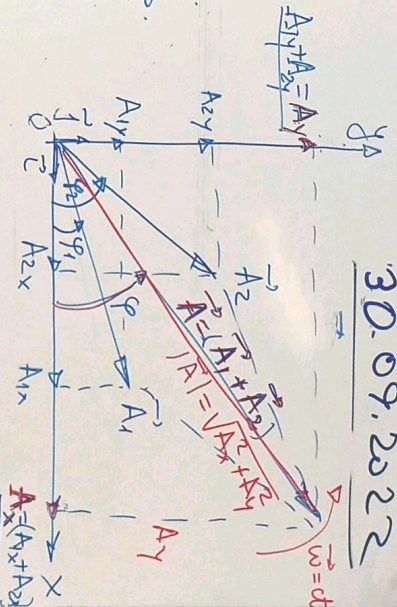
$$\stackrel{\text{P}}{=} \sin \omega t \cdot \cos \varphi + \sin \varphi \cos \omega t \quad \text{tg } \varphi = \frac{\sin \varphi}{\cos \varphi}$$

$$y = (y_1 + y_2) = A \sin(\omega t + \varphi) \quad \text{unde: } A = \frac{A_1 \cos \varphi_1 + A_2 \cos \varphi_2}{\cos \varphi}$$

$$\begin{cases} \sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \sin \beta \cos \alpha \\ \cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \end{cases}$$

Formula trigonometrică.

$$\begin{aligned} A_x &= A \cos \varphi \rightarrow A = \frac{A_x}{\cos \varphi} \\ A_y &= A \sin \varphi = \left(\frac{A_x}{\cos \varphi} \right) \sin \varphi \end{aligned}$$



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$$\vec{A}_1 = A_{1x} \vec{e}_x + A_{1y} \vec{e}_y$$

$$\begin{cases} A_{1x} = A_1 \cos \varphi_1 \\ A_{1y} = A_1 \sin \varphi_1 \end{cases} \quad |\vec{A}_1| = \sqrt{A_{1x}^2 + A_{1y}^2}$$

$$\vec{A}_2 = A_{2x} \vec{e}_x + A_{2y} \vec{e}_y$$

$$\begin{cases} A_{2x} = A_2 \cos \varphi_2 \\ A_{2y} = A_2 \sin \varphi_2 \end{cases} \quad |\vec{A}_2| = \sqrt{A_{2x}^2 + A_{2y}^2}$$

$$\vec{A} = \vec{A}_1 + \vec{A}_2 \quad \text{Formula / osc. rezultat}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(A_{1x} + A_{2x})^2 + (A_{1y} + A_{2y})^2}$$

$$\text{tg } \varphi = \frac{\sin \varphi}{\cos \varphi} = \frac{A_y}{A_x} = \frac{A_{1y} + A_{2y}}{A_{1x} + A_{2x}}$$

Ex. du p5

$m = 0,2 \text{ kg}$

$F_e = 200 \text{ N}$

$E = 40 \text{ J}$

$t_0 = 0, y_0 = 0$

$v_{\text{max}} = \omega A$

a) $y = ?$

b) $v = ?$

c) $a = ?$

$y = A \cdot \sin(\omega t + \varphi_0)$

Introduire (2) in (1):

$k = \left(\frac{F}{A} \right) = \frac{F}{(2E/F)} = \left(\frac{F^2}{2E} \right), (2)$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\left(\frac{F^2}{2E} \right) \cdot \frac{1}{m}} = \frac{F}{\sqrt{2mE}}$

Substituer ec. (2) + (1) in ec. de osc. $y = A \sin(\omega t + \varphi_0)$

$y = \left(\frac{2E}{F} \right) \sin \left(\frac{F \cdot t}{\sqrt{2mE}} + 0 \right) = A \cdot \sin(\omega t + \varphi_0)$

b) $v = v_{\text{max}} \cos(\omega t + \varphi_0) = \omega A \cos(\omega t + \varphi_0) = \frac{F}{\sqrt{2mE}} \left(\frac{2E}{F} \right) \cos \left(\frac{F \cdot t}{\sqrt{2mE}} + 0 \right) = v(t)$

c) $a = -\omega^2 A \sin(\omega t + \varphi_0) = -\omega^2 y(t) = -\omega^2 A \sin(\omega t + \varphi_0) = -\left(\frac{F}{\sqrt{2mE}} \right)^2 \cdot \left(\frac{2E}{F} \right) \cdot \sin \left(\frac{F \cdot t}{\sqrt{2mE}} + 0 \right) = a(t)$

$\begin{cases} F = k \cdot A \\ \varphi_0 = 0 \end{cases} \Rightarrow k = \left(\frac{F}{A} \right)$

$E = \left(\frac{kA^2}{2} \right) = \left(\frac{F \cdot A}{2} \right)$

$\begin{cases} E = F \cdot \frac{A}{2} \\ A = \left(\frac{2E}{F} \right) \end{cases}, (2)$

