## National University of Computer & Emerging Sciences, Karachi
**Spring -2018 CS-Department**
**MidTerm II**
**5th April 2018, 1:00 pm – 02:00 pm**

| Course Code: CS301 | Course Name: Theory of Automata |
|---|---|
| Instructor Name: M. Shahzad/Mrs. Shahar Bano/Subhash Sagar | |
| Student Roll No: | Section No: |

Instructions:

- Return the question paper.
- Read each question completely before answering it. There are **4 questions and 2 pages.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- All the answers must be solved according to the sequence given in the question paper.

**Time**: 60 minutes. **Max Marks**: 60 points

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# Solution Paper

Question 1a:Provide 2-3 line replies to all of the following short questions. Answer that exceeds 3 lines will not be considered. [10 points]

A) If a language can be expressed in the form of FA than why it is needed to use NFA?
NFA stands for non-deterministic FA and this sort of structure has relaxation compared with FA. So it is rather easier to represent a language using NFA.
We have methods to convert NFA into FA's so sometimes it is easier to build NFA of a given language and then convert its NFA into FA using these methods rather than directly building an FA for a language which may be very difficult.

B) Write down differences between Palindrome and Reverse function? Elaborate with example.

The language consisting of Λ and the strings s defined over Σ such that Rev(s)=s.It is to be denoted that the words of PALINDROME are called palindromes.
Reverse = w
Example: Σ={a,b},
PALINDROME={Λ , a, b, aa, bb, aaa, aba, bab, bbb, ...}

If a is a word in some language L, then reverse (a) is the same string of letters spelled backwards, called the reverse of a.
e.g
reverse (xxx) = xxx
reverse (623) = 326
reverse (140) = 041

C) what are the conditions of NFA-Null to NFA conversion to recognize the language L.
Let M1 = < Q1, $\Sigma$, q1,0, $\delta$1, A1 > be an NFA- $\Lambda$ that recognizes a language L. Then the NFA
M2 = < Q2, $\Sigma$, q2,0, $\delta$2, A2 > that satisfies the following conditions recognizes L:

Q2 = Q1,
q2,0 = q1,0,

$\delta$2( q, a ) = $\delta$1*( q, a ) = $\Lambda$( $\cup_{p\in\Lambda(q)}\delta_1(p,a)$ )

A2 = A1 $\cup$ { q1,0 } if $\Lambda$( { q1,0 } ) $\cap$ A1 $\neq \emptyset$
= A1 otherwise.

D) Intersection of two non-regular languages is always non-regular. Is it true or false? Give your statement with proof.
False.
L1=$a^n b^n$, L2=$b^n a^n$, Intersection of L1 and L2 is {}, which is regular.

E) $L_k$ = {$a^p$ : p is any prime number less than a very large given integer k}, $L_k$ is a regular language. Is it true or false? Give your statement with proof.
True.
Given integer k means a finite integer, and all prime numbers less than k will also be finite. So, it will be a regular language.

<u>Question 1b:</u> Show that, **L = {$a^n b^n c^n$ |n≥ 1}** is not regular. Use pumping lemma for at least three cases of ***y*** and where ***i* ={1,2}**
[05 points]

Solution

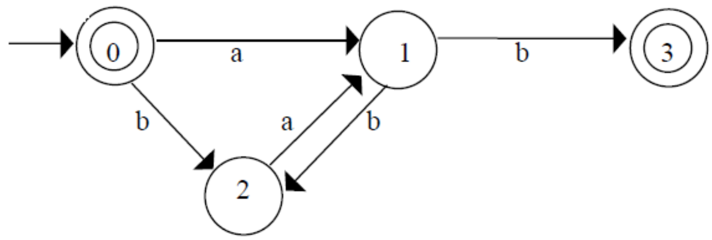
**Example** *Show that, $L = \{a^n\ b^n\ c^n \mid n \geq 1\}$ is not regular.*
**Answer** L = {*abc aabbcc aaabbbccc aaaabbbbcccc....*}

**Case 1:** Let $y = bc$
$x = a$
$z = \wedge$
$\therefore$ $w = a(bc)^i \wedge$
$w = a\,(bc)^i$
When $i = 1,\ w = abc$ which is in 'L'
When $i = 2,\ w = abcbc$ which is not in 'L'
therefore $L = \{a^n\ b^n\ c^n \mid n \geq 1\}$ is not regular.

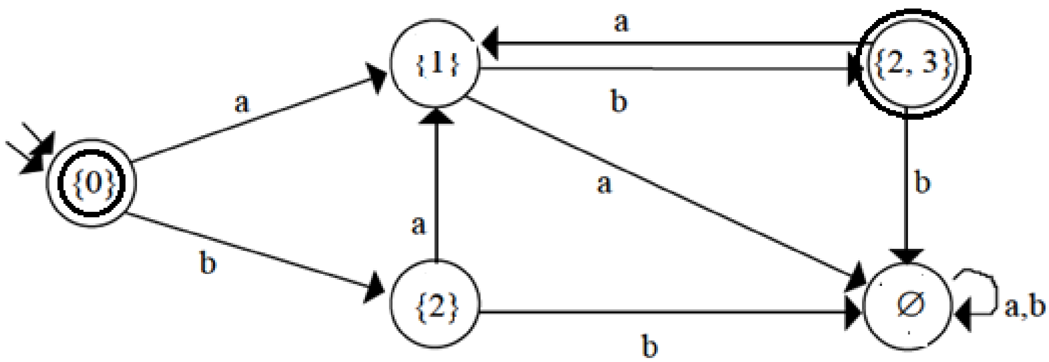
**Case 2:** Let $y = ab$
$x = \wedge$
$z = c$
$\therefore$ $w = \wedge\,(ab)^i\ c$
$w = (ab)^i\ c$
When $i = 1,\ w = abc$ which is in 'L'
When $i = 2,\ w = ababc$ which is not in 'L'
therefore, $L = \{a^n\ b^n\ c^n \mid n \geq 1\}$ is not regular

**Case 3:** Let $y = c$
$x = ab$
$z = \wedge$
$\therefore$ $w = ab\,(c)^i \wedge$
$w = ab\,(c)^i$
$i = 1,\ w = abc$ which is in 'L'
$i = 2,\ w = abcc$ which is not in 'L'
therefore $L = \{a^n\ b^n\ c^n / n \geq 1\}$ is not regular

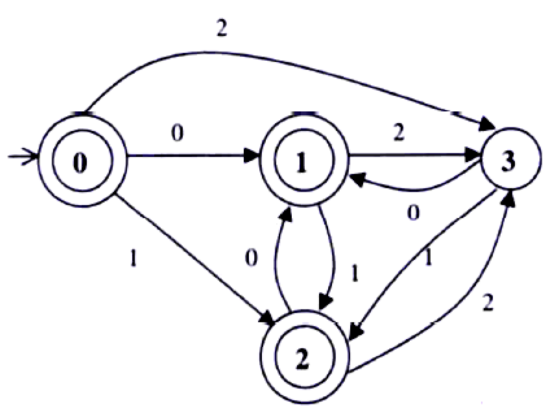
Question 2: Consider the following NFA, construct an equivalent DFA. Show all steps [15 points]




Solution:

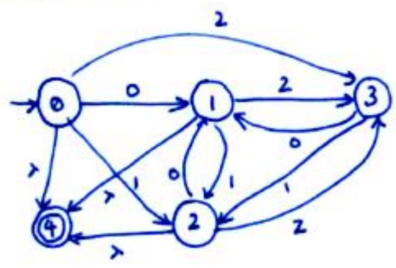



Question3: Derive the RE for the language accepted the following nfa. For full credit show all the steps clearly.
[Hint: Use approach discussed in Kleen's Theorem] [15 points]

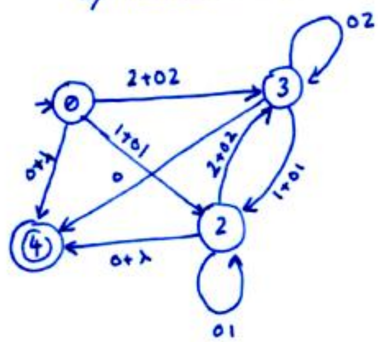



Solution

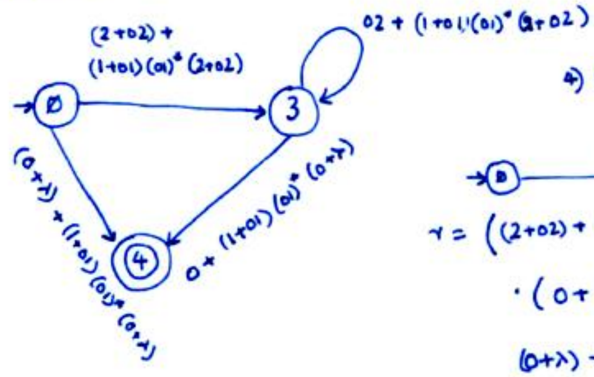
1) Single final state



2) Remove state ①



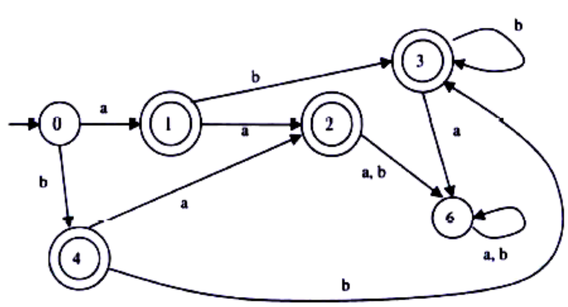
3) Remove state ②



4) Remove state 3

$$r = \left((2+02) + (1+01)(01)^*(2+02)\right) \cdot \left(02 + (1+01)(01)^*(2+02)\right)^* \cdot \left(0 + (1+01)(01)^*(0+\lambda)\right) \; + \; (0+\lambda) + (1+01)(01)^*(0+\lambda)$$

Question 4: Minimize the following DFA using partioning method:
[15 points]



Solution: