積分小テスト

氏名＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿

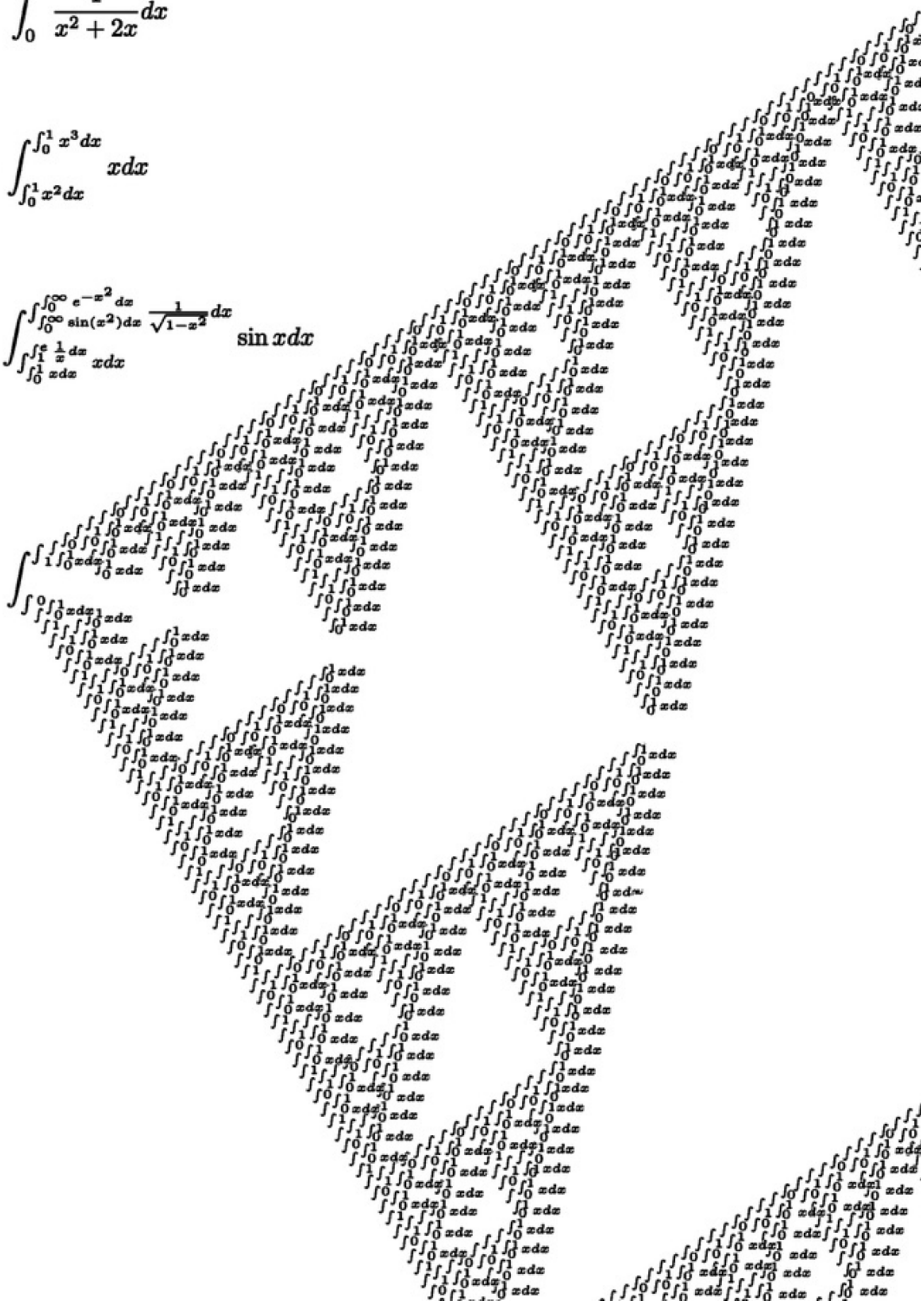
次の不定積分を求めよ．(各 25 点)

(1) $\displaystyle\int_0^1 \frac{1}{x^2+2x}dx$

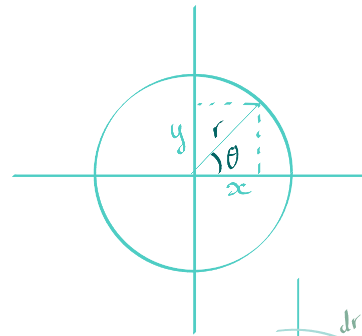
(2) $\displaystyle\int_{\int_0^1 x^2dx}^{\int_0^1 x^3dx} xdx$

(3) $\displaystyle\int_{\int_{\int_0^1 xdx}^{\int_1^e \frac{1}{x}dx} xdx}^{\int_{\int_0^\infty \sin(x^2)dx}^{\int_0^\infty e^{-x^2}dx} \frac{1}{\sqrt{1-x^2}}dx} \sin x dx$

(4)



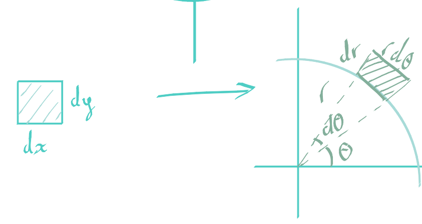
$$\int_0^\infty e^{-x^2}dx = \sqrt{\int_0^\infty e^{-x^2}dx\int_0^\infty e^{-y^2}dy} = \left(\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dx\,dy\right)^{1/2}$$

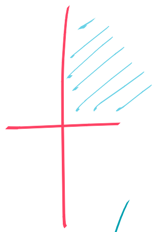


$(x,y) \to (r,\theta)$

$\Rightarrow dx\,dy = r\,dr\,d\theta$

$x^2+y^2=r^2$





$$\int_0^\infty e^{-x^2}dx = \left(\int_0^\infty\int_0^{\pi/2} e^{-r^2}\,r\,dr\,d\theta\right)^{1/2} = \left(\frac{\pi}{2}\left[-\frac{e^{-r^2}}{2}\right]_0^\infty\right)^{1/2} = \frac{\sqrt{\pi}}{2} \quad \text{et} \quad \int_0^\infty e^{-ax^2} = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$-\frac{1}{2}\left(2r\left(-e^{-r^2}\right)\right)dr$

$f' \times g'\circ f$ avec $f: r \mapsto r^2$, $g: r\mapsto e^{-r}$

$$\Rightarrow \int_0^\infty e^{-ix^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{i}} = e^{-i\frac{\pi}{4}}\frac{\sqrt{\pi}}{2} = \frac{1-i}{\sqrt{2}}\frac{\sqrt{\pi}}{2} = \sqrt{\frac{\pi}{8}} - i\sqrt{\frac{\pi}{8}}$$

(à l'arrache = on suppose sans le prouver qu'on a le droit)

Or $\int_0^\infty e^{-ix^2}dx = \int_0^\infty (\cos x^2 - i\sin x^2)\,dx$

$$\Rightarrow \int_0^\infty \sin x^2\,dx = \sqrt{\frac{\pi}{8}}$$

$$\int_{\int_0^1 x\,dx}^{\int_1^e \frac{1}{x}dx} x\,dx = 3/8 \qquad \int_1^e \frac{1}{x}dx \to 1 \qquad \int_0^1 x\,dx \to \frac{1}{2}$$

$$\int_{\sqrt{\frac{\pi}{8}}}^{\frac{\sqrt{\pi}}{2}} \frac{1}{\sqrt{1-x^2}}dx = \left[\sin^{-1}(x)\right]_{\sqrt{\frac{\pi}{8}}}^{\frac{\sqrt{\pi}}{2}} = \sin^{-1}\left(\frac{\sqrt{\pi}}{2}\right) - \sin^{-1}\left(\sqrt{\frac{\pi}{8}}\right)$$

$g\circ f' = f' \times g'\circ f$

$(\sin^{-1}\sin x)' = 1$

$\cos x \, \sin^{-1\prime}(\sin x) = 1$

$\Rightarrow \sin^{-1\prime}(\sin x) = \frac{1}{\cos x}$

$X = \sin x \quad \sin^{-1\prime} X = \frac{1}{\cos(\sin^{-1} X)}$

$= \frac{1}{\sqrt{1-X^2}}$

$$\int_{3/8} \sin x\,dx = \left[-\cos(x)\right]_{3/8}$$

$$= -\sqrt{1-\frac{\pi}{4}}\sqrt{1-\frac{\pi}{8}} - \frac{\sqrt{\pi}}{2}\sqrt{\frac{\pi}{8}} + \cos\frac{3}{8}$$

$$= \boxed{-\sqrt{\left(1-\frac{\pi}{4}\right)\left(1-\frac{\pi}{8}\right)} - \frac{\pi}{4\sqrt{2}} + \cos\frac{3}{8}}$$

$\simeq 0,014377...$

$\cos(a-b)$

$=$

$\cos a \cos b + \sin a \sin b$