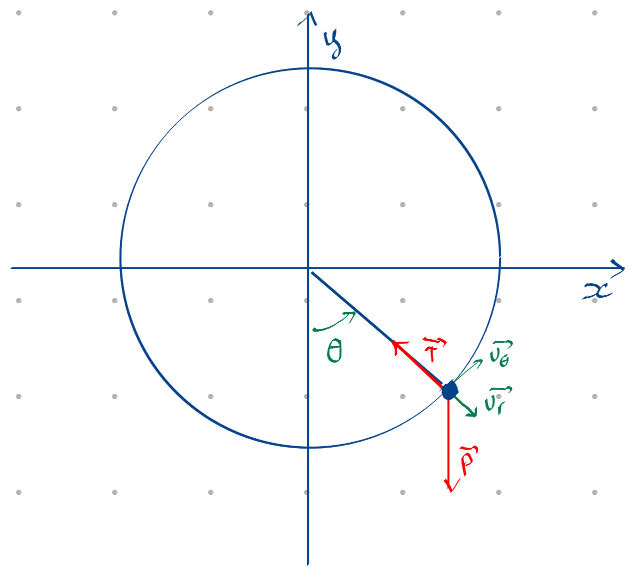

On cherche v_0 tq T soit toujours > 0 $(T = \|\vec{T}\|)$

$$\text{Pfd} \left| \begin{array}{ll} \text{sur } \vec{u_r} : & -m\ell\dot{\theta}^2 = -T + mg\cos\theta \quad (a) \\ \text{sur } \vec{u_\theta} : & m\ell\ddot{\theta} = -mg\sin\theta \quad (b) \end{array} \right.$$

Intégrons (b)

$$m\,\ell\ddot{\theta}(t)\dot{\theta}(t) = -mg\sin\theta(t)\times\dot{\theta}(t)$$

$$\frac{d}{dt}\left(\frac{1}{2}\ell\dot{\theta}(t)^2\right) = -\frac{d}{dt}\left(-g\cos\theta(t)\right)$$

$$\Rightarrow \left[\frac{1}{2}\ell\dot{\theta}^2(t)\right]_{t=0}^{t} = \left[g\cos\theta(t)\right]_{t=0}^{t}$$

$$\frac{1}{2}\ell\dot{\theta}^2 - \frac{1}{2}\frac{v_0^2}{\ell} = g\cos\theta - g \qquad \text{car } v_0 = \ell\dot{\theta}(t=0)$$

$$\Rightarrow \ell\dot{\theta}^2 = 2g\cos\theta - 2g + \frac{v_0^2}{\ell} \quad (*)$$

On injecte $(*)$ dans (a) :

$$-2mg\cos\theta + 2mg - m\frac{v_0^2}{\ell} = -T + mg\cos\theta$$

$$\Rightarrow T = 3mg\cos\theta - 2mg + m\frac{v_0^2}{\ell}$$

La corde sera toujours tendu si T est > 0 quand $\theta = \pi$.

$$T(\theta = \pi) > 0 \Rightarrow -3mg - 2mg + m\frac{v_0^2}{\ell} > 0$$

$$\Rightarrow \boxed{v_0^2 > \frac{5g}{\ell}}$$