CS 3823 - Theory of Computation: Homework Assignment 1

Fall 2025 **Due:** Friday, September 12, 2025

Related Reading. Chapter 0 and Chapter 1.1
Instructions. Near the top of the first page of your solutions please list clearly **all** the members of the group (please see the syllabus for the collaboration policy) who have created the solutions that you are submitting. Listing the names of the people in the group implies their full name and their 4x4 IDs. Alternatively, you can use the space below and provide the relevant information in case you submit the solutions using this document.

Student Information for the Solutions Submitted

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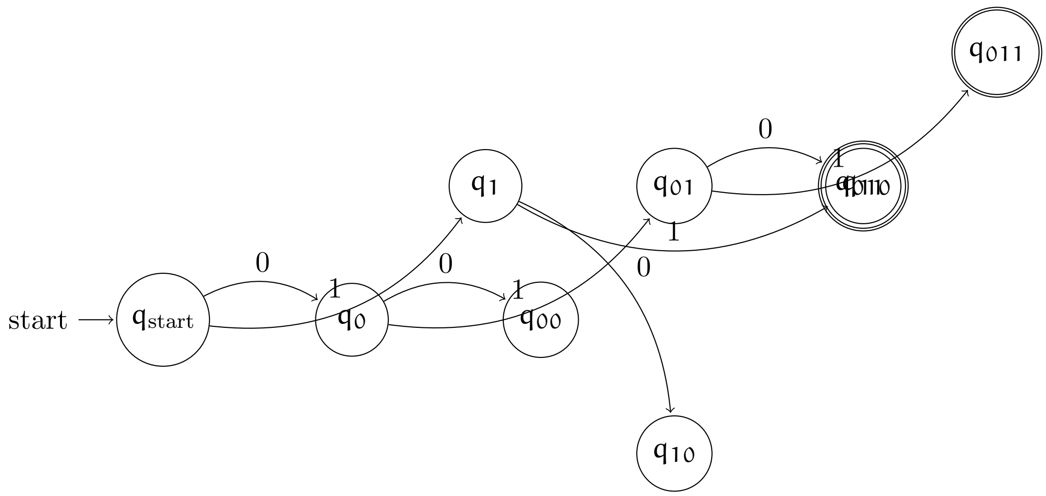
Grade

Exercise	Pages	Your Score	Max
1	2		4
2	3		4
3	4-5		12
4	6-7		12
5	8		4
6	9		4
Total	2-9		40

Additional Help and Resources. Did you use help and/or resources other than the textbook? Please indicate below.



1 Set Theory [4 points]

Let A be the set $\{x, y, z\}$ and B be the set $\{a, b, x\}$.

(i) Is A a subset of B and why?
No, A is not a subset of B because B does not contain all elements of A. Definition of subset: Set B contains all elements of set A.
For example in below, A is a subset of B, but if we added a number that isn't 4 to A, it wouldn't be a subset of B.

$$A = \{1, 2\}, B = \{1, 2, 4\}$$

(ii) What is $A \cap (B \setminus A)$?
Set Difference: All elements only in left hand set and not in right hand set.
Starting with

$$B \setminus A \to \{a, b\} = C$$

Set union: Combine elements in both sets

$$A \cap C = \varnothing$$

(iii) What is $A \times B$?

$$\{x, y, z\} \times \{a, b, x\} \to (\{x, a\}, \{x, b\}, \{x, x\}, \{y, a\}, \{y, b\}, \{y, x\}, \{z, a\}, \{z, b\}, \{z, x\}\}$$

)

(iv) What is the powerset of B? Where $B = \{a, b, x\}$, powerset provides 2^n subsets (where n is number of elements), and is all subsets (including empty and the set itself).

$$\mathcal{P}(B) \to$$
$$\{\{\}, \{a, b, x\}, \{a\}, \{b\}, \{x\}, \{a, b\}, \{b, x\}, \{a, x\}\}$$

2 Induction [4 points]

Prove by induction on k that for all integers $k \geqslant 4$ it holds that $k! > 2^k$.
Let P(k) = $k! > 2^k$ when $k \geqslant 4$

Basis step ($k = 4$):

$$P(4) = 24 > 16$$

Induction step: Let P(n) = $n! > 2^k$ when $n \geqslant 4$: $(P(n) \to P(n+1)$:
Hypothesis Step: $n! > 2^n$
Conclusion Step: $(n+1)! > 2^{n+1}$

$$2 * n! > 2^n * 2$$

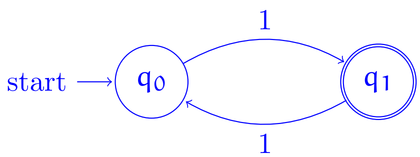
$$n!(n+1) > 2 * n! > 2^n * 2$$

$$(n+1)! > 2^{n+1}$$

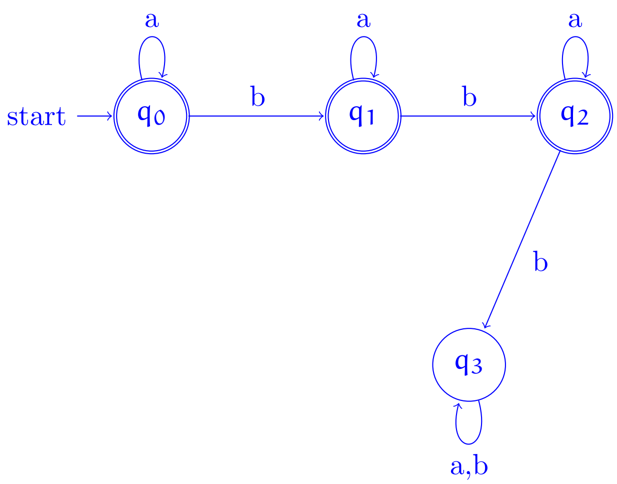
3 Drawing State Diagrams [12 points; 4 points each]

Draw state diagrams for DFAs recognizing the following languages:

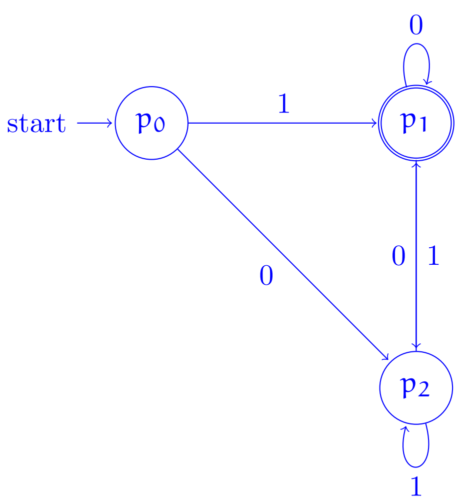
(i) $L_1 = \{w \mid \text{length of } w \text{ is odd}\}, \Sigma = \{1\}$.



(ii) $L_2 = \{w \mid w \text{ has at most two occurrences of the symbol } b\}, \Sigma = \{a, b\}$.



(iii) $L_3 = \{w \mid w \text{ starts with an } 1 \text{ or ends with a } 0\}, \Sigma = \{0, 1\}$.



4 Interpreting State Machines [12 points]

$M_1 = (\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \delta, q_0, \{q_1, q_2\})$

$L(M_1) = \{ w \mid w \text{ has string length of } 1 \text{ or } 2 \}$

δ	0	1
q_0	q_1	q_1
q_1	q_2	q_2
q_2	q_3	q_3
q_3	q_3	q_3

Any string will be accepted when added for states q_1 + q_2. But, when a 3^{rd} string is added + onwards, it's no longer acceptable.

$L(M_2) = \{ w \mid w$ has substring of "1001" or "0110" $\}$

Tried various sample string inputs and saw common patterns.

Directly saw "1001" + "0110" to be accepted.

Both transitions can keep looping until each would need to be setup for the substring.

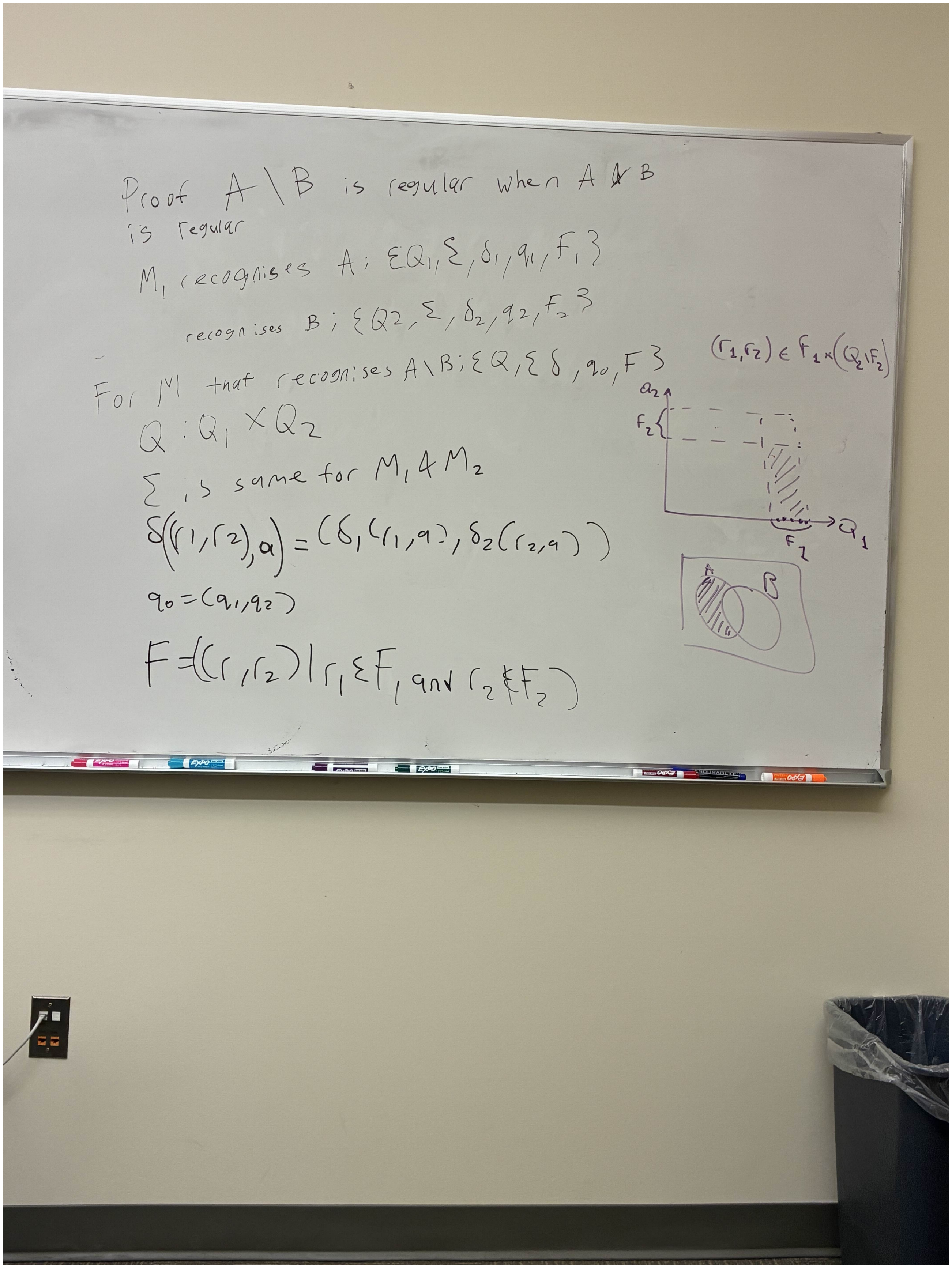
"... 1001 ...."

"... 0110 ...."

5 Closure [4 points]

Let A and B be regular languages. Show that $A \setminus B$ is also regular.
Recall that $A \setminus B = \{x \in A \mid x \notin B\}$. In other words, this operation removes from A all the strings that are also in B.

Let M1 recognize A where M1 = (Q1, Σ, δ1, q1, F1) and let M2 recognize B where M2 = (Q2, Σ, δ2, q2, F2)
Constructing M that recognizes $A \setminus B$ where M = (Q1, Σ, δ, q0, F):
Q = Q × Q
Σ is the same as in M1 and M2
δ((r1, r2), a) = (δ1(r1, a), δ2(r2, a))
q0 = (q1, q2)
F = (r1, r2) | r1 ∈ F1 and r2 ∉ F2


Proof A \ B is regular when A & B
is regular
M1 recognises A ; {Q1, Σ, δ1, q1, F1}
recognises B ; {Q2, Σ, δ2, q2, F2}
For M that recognises A \ B ; {Q, Σ δ, q0, F}
Q : Q1 × Q2
Σ is same for M1 & M2
δ((r1, r2), a) = (δ1(r1, a), δ2(r2, a))
q0 = (q1, q2)
F = {(r1, r2) | r1 ∈ F1 and r2 ∉ F2}
(r1, r2) ∈ F1 × (Q2 \ F2)
Q2
F2
F1
Q1
A
B

6 Assigned Reading Question [4 points]

Please read the Quanta article *Computation Is All Around Us, and You Can See It if You Try*, by Lance Fortnow. The article is available at the link below:

https://www.quantamagazine.org/computation-is-all-around-us-and-you-can-see-it-if-you-try-20240612/

After reading this article, please answer the question below.

Question: Does Lance Fortnow believe that randomness is unpredictable? Justify your answer.

Lance Fortnow does not believe that randomness is unpredictable. Instead, he believes that randomness is processes that are incomprehensible to us. The initial example he gave was flipping a coin. We view flipping a coin as a random 50/50 event, however the result of flipping that coin is governed by variables like air resistance, the initial force of the flip, the mass of the coin, the angle of the toss, etc. These factors influence the result of the flip, and a slight change to any could change the outcome of the flip. There are too many factors for a human to be able to calculate whether a coin toss would be heads or tails, but a being with enough intellect might be. Fortnow believes that all random events are not random and can be predicted by computations. He then talks about machine learning models and if they keep progressing at their current rate, in a few decades they might be able to predict random events.