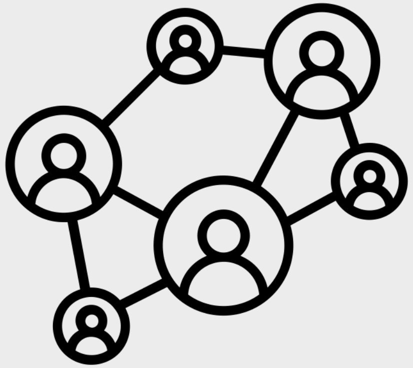
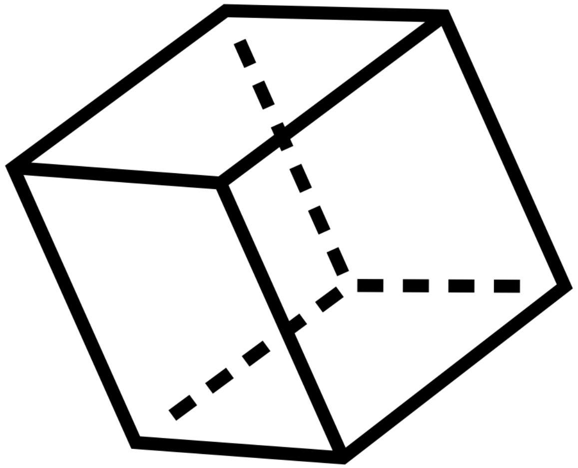


ULSAN NATIONAL INSTITUTE OF SCIENCE AND TECHNOLOGY

Fourth Week

Introduction to the Network



1.Introduction to the Matrix

4.1 Basic terminology

Definition 4.1.1 (Binary operation)

Let F be a set. Then, the function $* : F \times F \to F$ is called by **binary operation**

Definition 4.1.2 (Field)

Let F be a set with the binary operation $+$ and $\times$. F is a **field** if F satisfies following conditions for all $a, b, c \in F$.

i) $a + (b + c) = (a + b) + c$ (**Associative for addition**)

ii) $\exists 0 \in F$ such that $a + 0 = 0 + a = a$ (**Identitiy for addition**)

iii) $\forall a \in F,\ \exists x \in F$ such that $a + x = x + a = 0$ (**Inverse for addition**) especially we write x as $-a$.

iv) $a + b = b + a$ (**Commutative for addition**)

v) $a \times (b \times c) = (a \times b) \times c$ (**Associative for multiplication**)

vi) $\exists 1 \in F$ such that $a \times 1 = 1 \times a = a$ (**Identity for multiplication**)

vii) $a \times (b + c) = a \times b + a \times c$ and $(a + b) \times c = a \times c + b \times c$ (**Distributive law**)

viii) $a \times b = b \times a$ (**Commutative for multiplication**)

ix) $\forall a \in F,\ \exists \hat{x} \in F$ such that $a \times \hat{x} = \hat{x} \times a = 1$ (**Inverse for multiplication**) especially we write $\hat{x}$ as a^{-1}.

4.1 Basic terminology

Definition 4.1.3 (A Vector space)

A **vector space** V over field F is a set with the addition $+$ and scalar multiplication $\cdot$ satisfying following conditions with for any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$ and $a, b \in F$

i) $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$ (**Associative for addition**)

ii) $\exists 0 \in F$ such that $\mathbf{u} + 0 = 0 + \mathbf{u}$ (**Identity for addition**)

iii) $\forall \mathbf{u} \in V, \exists \mathbf{x} \in V$ such that $\mathbf{u} + \mathbf{x} = \mathbf{x} + \mathbf{u} = \mathbf{0}$ (**Inverse for addition**)

iv) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ (**Commutative for addition**)

v) $1\mathbf{u} = \mathbf{u}$

vi) $a(b\mathbf{u}) = (ab)\mathbf{u}$

iv) $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$ and $(a + b)\mathbf{u} = a\mathbf{u} + a\mathbf{u}$

And, the element of the vector space is called by **vector**

4.1 Basic terminology

Example 4.1

Our main exampel of the vector space is $\mathbb{R}^n$. the vector $\mathbf{v} \in \mathbb{R}^n$ is give by

$$\mathbf{v} = \begin{bmatrix} v_1, v_2, \cdots, v_n \end{bmatrix} \text{ or } \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}, \, v_i \in \mathbb{R}, \, i = 1, \cdots, n$$

4.2 Matrices and Matrix Operation

Definition 4.2.1 (Matrix)

A **matrix** is a rectangular array of numbers, e.g, $A \in \mathbb{R}^{m \times n}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \text{ or } A = [a_{ij}]_{m \times n}$$

row

column

An, i, j th entry of a matrix A is denoted by $A_{i,j} = a_{i,j}$.

⋆ We can consider the row vector with n entries as $1 \times n$ matrix and the column vector with m entries as $m \times 1$ matrix.

⋆ The **size** of a matrix is describe in terms of the number of rows and columns it contains.

4.2 Matrices and Matrix Operation

Definition 4.2.2 (Matrix Sum and Difference)

Let A and B are matrices of the same size.

sum $A + B$ is defined by $(A + B)_{ij} = A_{ij} + B_{ij}$

difference $A - B$ is defined by $(A - B)_{ij} = A_{ij} - B_{ij}$

Definition 4.2.3 (Matrix Multiple)

Let A is matrix and for any scalar c, then $cA = (cA)_{ij} = cA_{ij}$. The matrix cA is called a **scalar multiple** of A.

Let A is a $m \times r$ matrix and B is a $r \times n$ matrix, then the **product** AB $m \times n$ matrix is defined by

$$(AB)_{ij} = \sum_{k=1}^{r} A_{ir} B_{rj}$$

Definition 4.2.4 (Matrix Transpose)

Let A is any $m \times n$ matrix. the **transpose** of A, denoted by $A^\top$, is defined by $(A^\top)_{ij} = A_{ji}$.

4.2 Matrices and Matrix Operation

Properties 4.2.1

Let A and B are any matrices with the same size.

i) $(A^\top)^\top = A$

ii) $(AB)^\top = B^\top A^\top$

iii) $(A+B)^\top = A^\top + B^\top$

Definition 4.2.5 (Special Matrices)

$\star$ A $n \times n$ matrix A is called a **square matrix of order** n. And the $A_{ii}, i = 1, \cdots, n$ are said to be on the **main diagona** of A. Then entries which are not in the main diagonal is called the **off diagonal entries**.

$\star$ a **diagonal matrix** is a matrix in which the entries outside the matin diagonal are all zero.

$\star$ A square matrix with 1's on the main diagonal and zeros elsewhere is called an **identity matrix.** the $n \times n$ identity matrix is denoted by I_n.

$\star$ Let A is a square matrix. A matrix A is **symmetric** if $A = A^\top$.

$\star$ Let A is a square matrix. A matrix A is **orthogonal** if $AA^\top = I = A^\top A$.

4.2 Matrices and Matrix Operation

Definition 4.2.6 (Trace)

If A is a square matrix, the **trace** of A, denoted by $\operatorname{tr}(A)$, is defined to be the sum of the entries on the main diagonal of A.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \Rightarrow \operatorname{tr}(A) = \sum_{i=1}^{n} a_{ii}$$

Definition 4.2.7 (Inverse Matrix)

If A is a sqare matrix, and if a matrix B of the same size can be found such that $AB = BA = I$, then A is said to be **invertible** (or **nonsingular**) and B is called an **inverse** and is denoted by $B = A^{-1}$. If no such matrix B can be found, then A is said to be **singular**

4.2 Matrices and Matrix Operation

Definition 4.2.8 (Determinant)

$\star$ The **determinant** of empty matrix $A = \begin{bmatrix} & \end{bmatrix}$ is

$$\det(A) = 1$$

$\star$ The **determinant** of constant matrix $A = \begin{bmatrix} a \end{bmatrix}$ is

$$\det(A) = a$$

$\star$ The **determinant** of 2×2 matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is

$$\det(A) = a_{11}a_{22} - a_{12}a_{21}$$

4.2 Matrices and Matrix Operation

Definition 4.2.8 (Determinant)

$\star$ The **minor of entry** a_{ij} is dentoed by M_{ij} and is defined to be the determinant of the submatrix that remains after the ith and jth column are deleted from A. The number $(-1)^{i+j}M_{ij}$ is denoted by C_{ij} and is called the **cofactor of entry** a_{ij}

$$M_{ij} = \det\left(\begin{bmatrix} a_{11} & \cdots & a_{1j-1} & a_{1j+1} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{i-11} & \cdots & a_{i-1j-1} & a_{i-1j+1} & \cdots & a_{in} \\ a_{i+11} & \cdots & a_{i+1j-1} & a_{i+1j+1} & \cdots & a_{i+1n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj+1} & \cdots & a_{nn} \end{bmatrix}\right) \text{ and } C_{ij} = (-1)^{i+j}M_{ij}$$

$\star$ The **determinant** of a $n \times n$ matrix A is denoted by $\det(A)$ or $|A|$ and is defined by the sum of the cofactors of any row or column of the matrix multiplied by the entries that generated them.

$$\det(A) = \sum_{i=1}^{n} a_{ij}C_{ij} \text{ or } \det(A) = \sum_{j=1}^{n} a_{ij}C_{ij}$$

4.2 Matrices and Matrix Operation

Properties 4.2.2

i) $\det(A^\top) = \det(A)$

ii) $\det(AB) = \det(A)\det(B)$

iii) $\det(A^{-1}) = \dfrac{1}{\det(A)}$

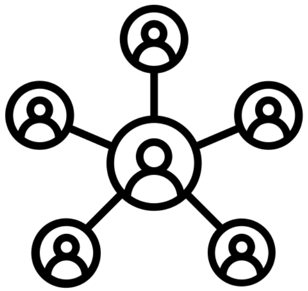
Theorem 4.2.1

Inverse of a Matrix Using its Adjoint If A is an invertible matrix, then

$$A^{-1} = \frac{1}{\det(A)}\operatorname{adj}(A)$$

where $\operatorname{adj}(A)$ is the **adjoint of** A is defined by

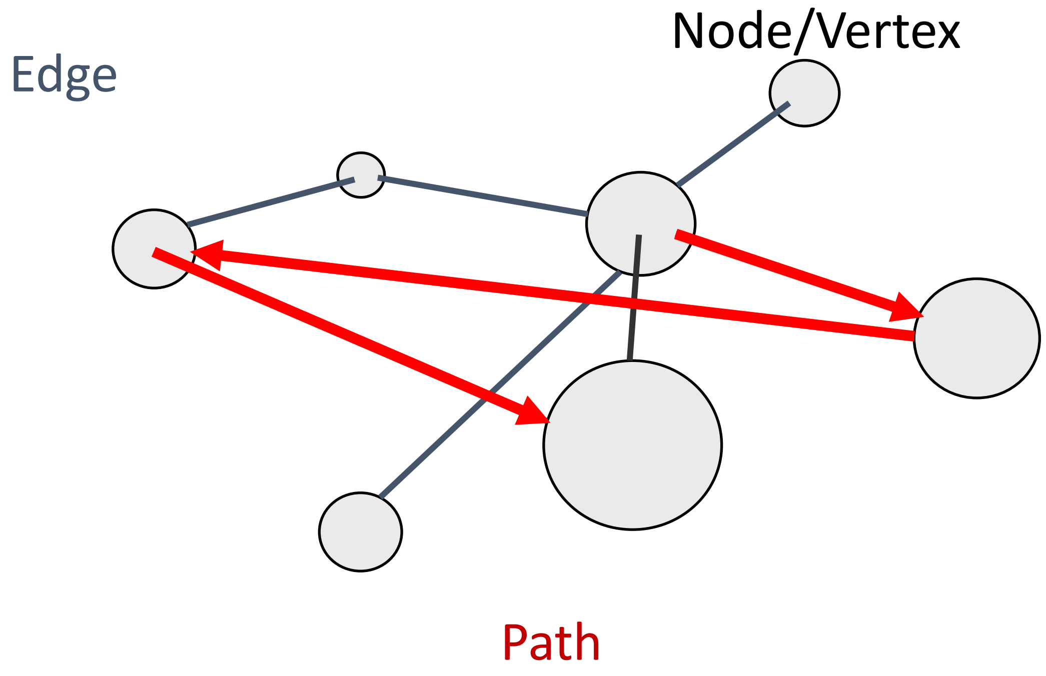
$$\operatorname{adj}(A) = [C_{ij}]_{n\times n},\ C_{ij} = \text{the cofactor of } a_{ij}.$$



2. Network Theory

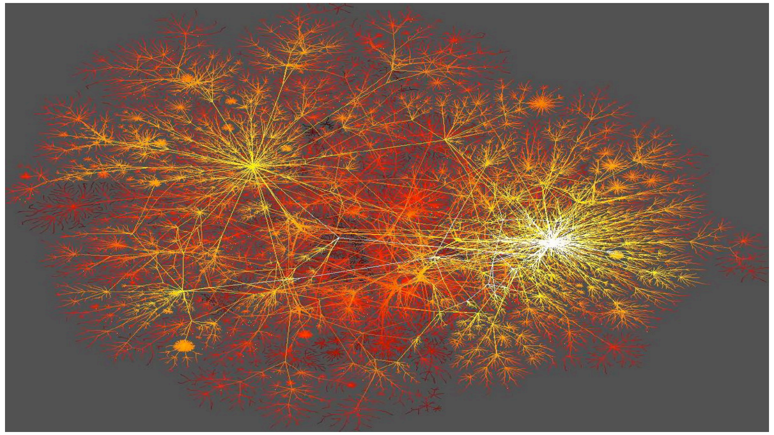
Network ?

"A collection of objects connected to each other in some fashion"
[Watts, 2002]

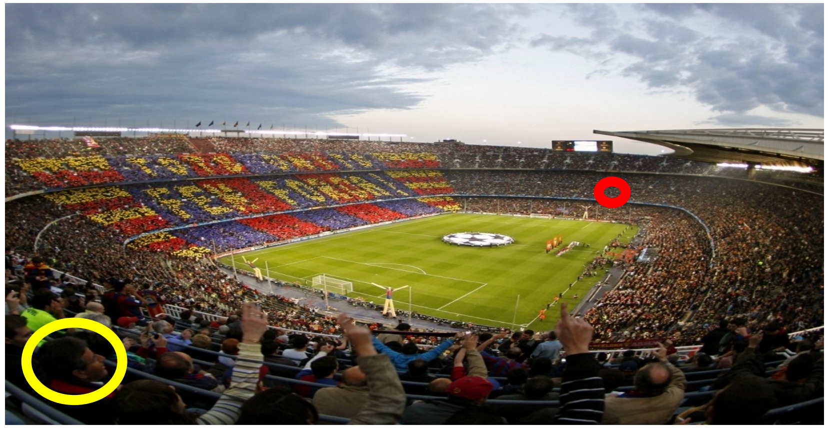


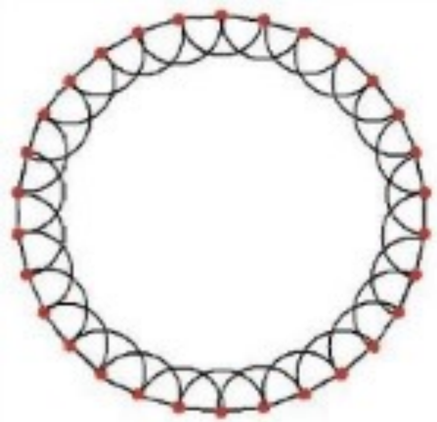
Data networks

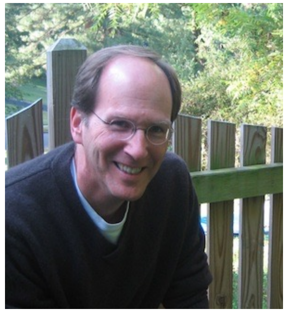
Air traffic network



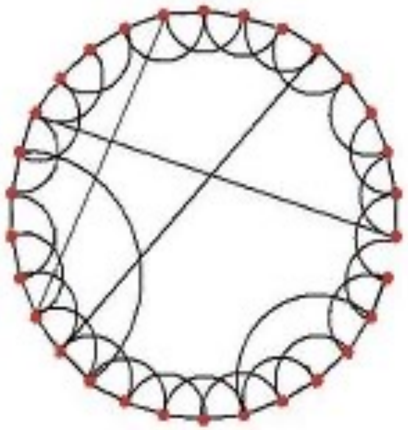
Network : Small World Network





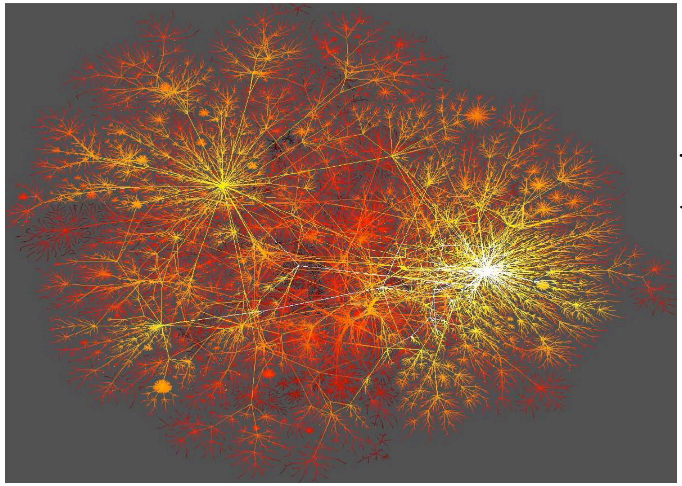


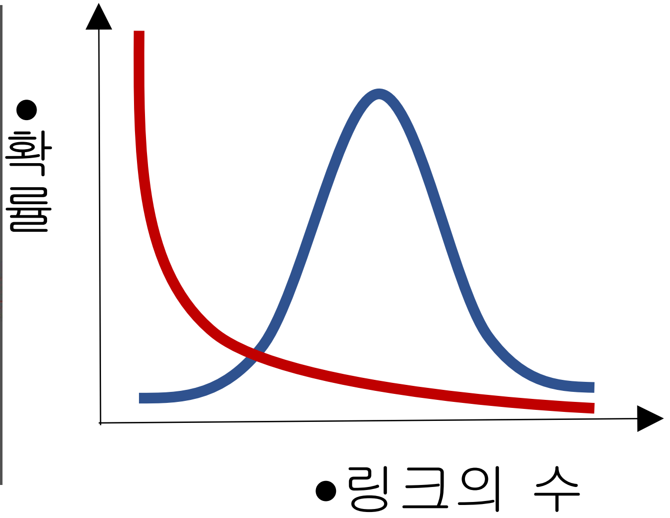
(Regular Network)

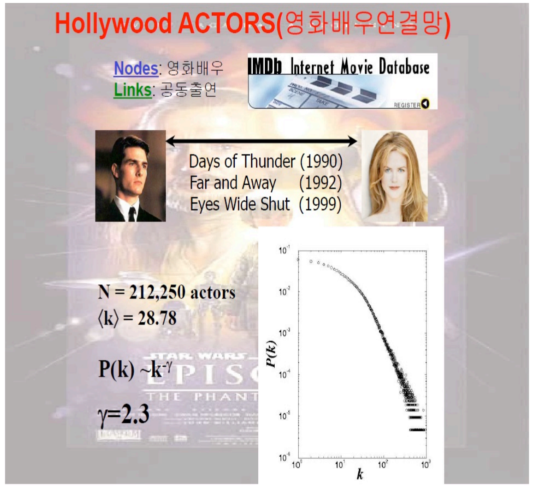


Small World Network

Network : Scale free network/ Power law





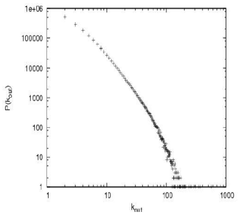


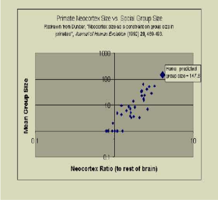
CyWorld

Nodes: 싸이 미니홈피

Links: 일촌(평)

(sampled over 100,000명
out of ~10,000,000명
via snowball sampling)
7 degrees of separation!



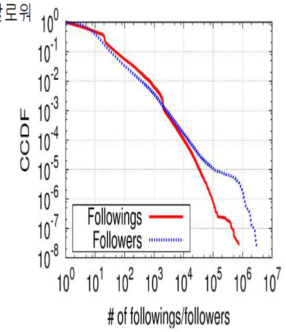


Twitter : Follower, following

Nodes: 트위터 사용자

Links: 팔로잉,팔로워

On June 2009,
41.7M users,
1.47B links
are sampled
(via BFS from
Perez Hilton)



H. Kwak et al (2009)

Measurement: Centralities

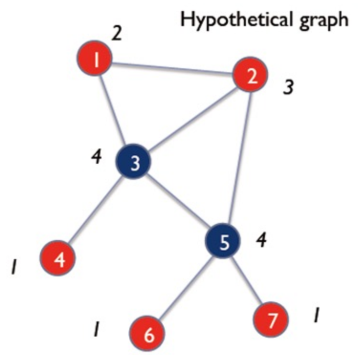
- Degree
- Closeness Centrality

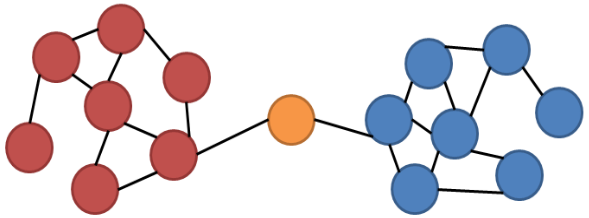
$$CC_i = \frac{N-1}{\sum_{j \in G_{j \neq i}} d_{ij}}$$

- Betweenness Centrality

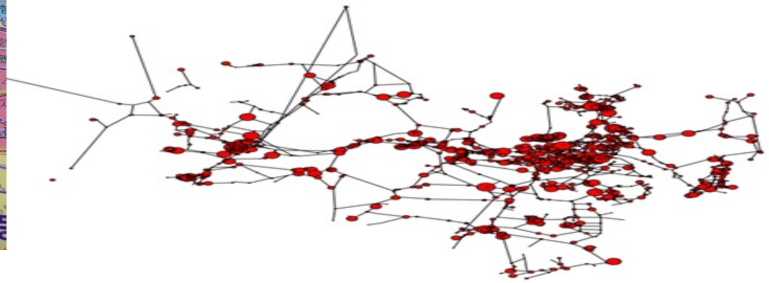
$$BC_i = \frac{1}{(N-1)(N-2)} \sum_{j,k \in G_{j \neq i, j \neq k}} n_{jk}(i) / n_{jk}$$

- Eigenvector Centrality





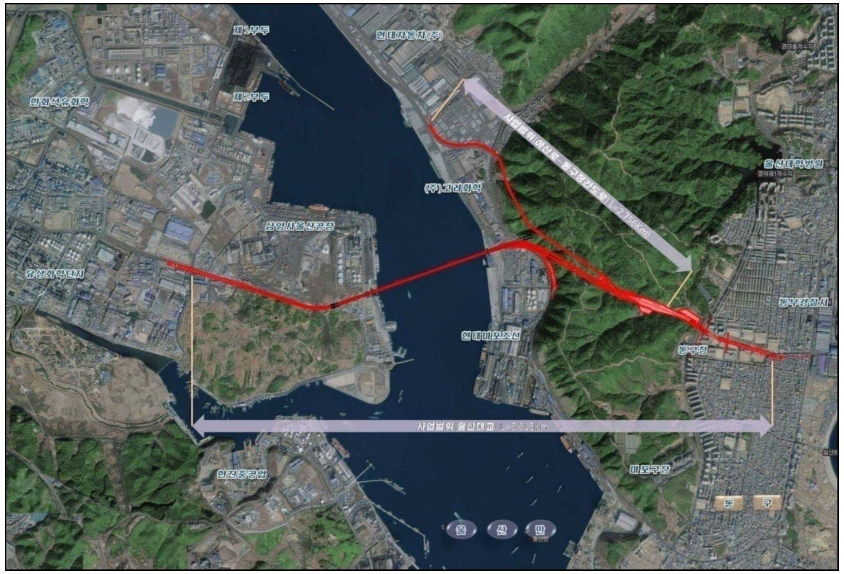
Transportation Network



국가 표준노드링크(ITS):
http://nodelink.its.go.kr/Default.aspx

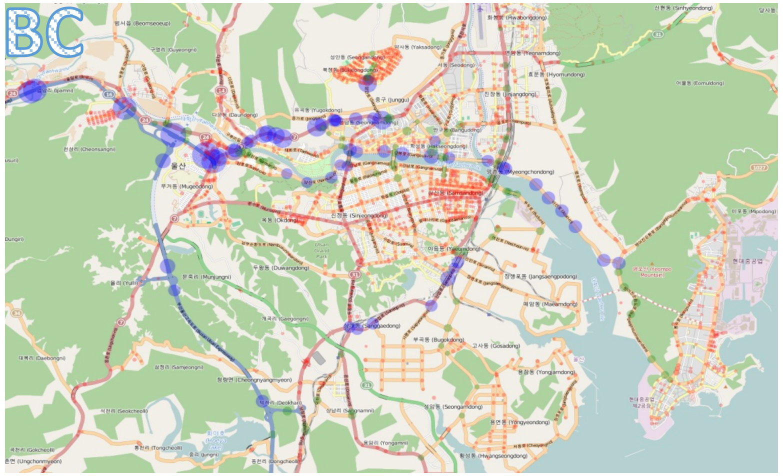
노드링크를 구분하는 점(링크 시작점,링크 종료점): 도로교차점,도로 시종점, 교통통제점, 도로 구조 변환점, 행정구역 변환점, 도로 운영 변환점, 교통 진출입점. 그 외에 ITS사업 주체가 필요에 따라 정하는 지점

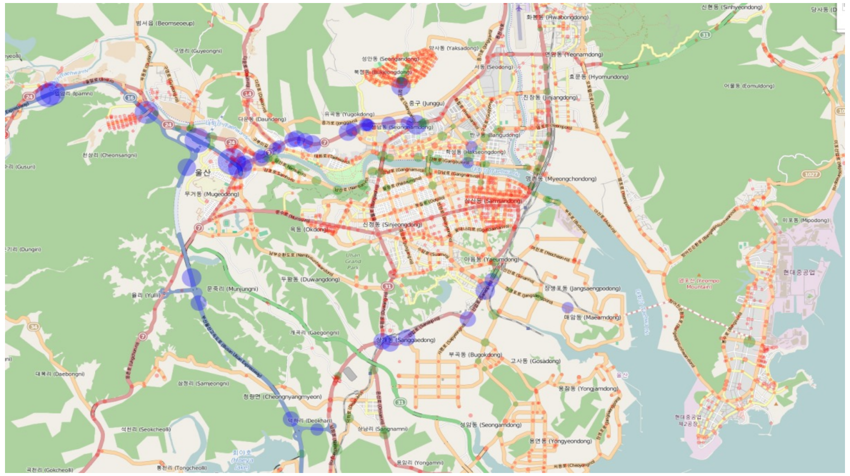
Ulsan Bridge

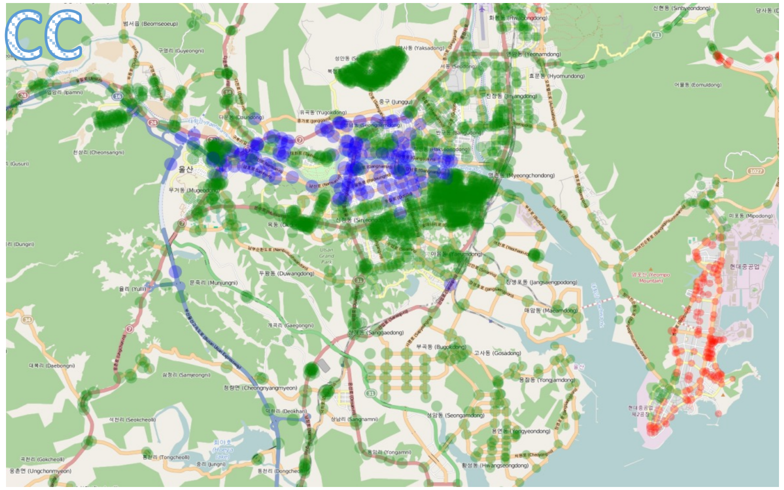


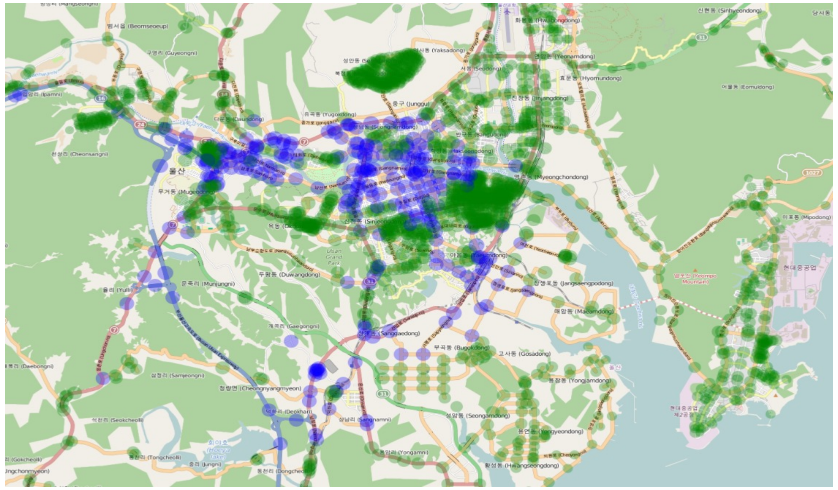
기간: 2010 - 2015년 5월 완공 예정
단경간 현수교(주탑과 주탑 사이의 거리인 경간이 하나로 연결된 현수교)
북구 염포동과 동구 화정동을 연결하는 다리
예산: 5398 억원/ 8.38 km(교량: 1.15km)

Centrality Analysis Before/After





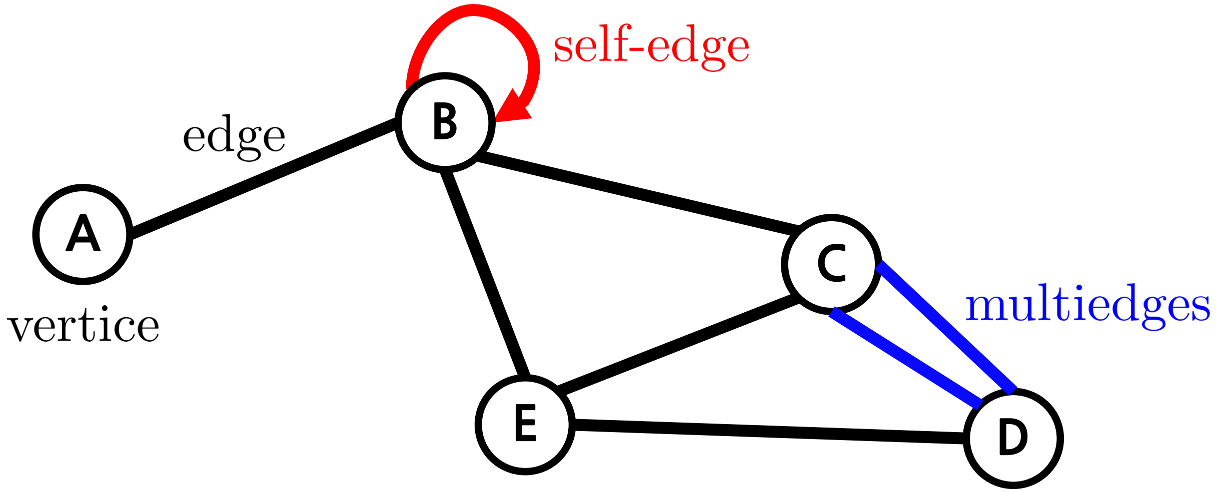




4.3 Networks And Their Representation

A **Network** or **Graph** is a collection of vertices joined by edges.

$*$ Math : verties, edges Computer : Nodes, Links
Physics : sites, bunds Sociology : actors, ties



A network that has neither self-edges nor multideges is called a **simple network** or **simple graph**.

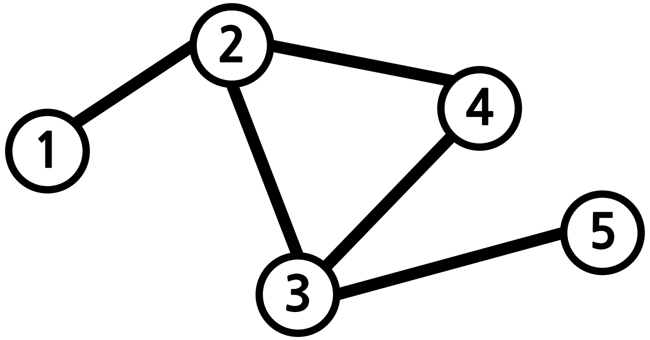
4.3 The Adjacency Matrix

Definition 2.3.1 (Adjacency matrix)

The **adjacency matrix** A has an element A_{ij},

$$A_{ij} = \begin{cases} 1 & \text{if there is an edge between } (i,j) \text{ vertices} \\ 0 & \text{otherwise} \end{cases}$$

Example (Simple Network)



$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

adjacency matrix.

symmetric matrix

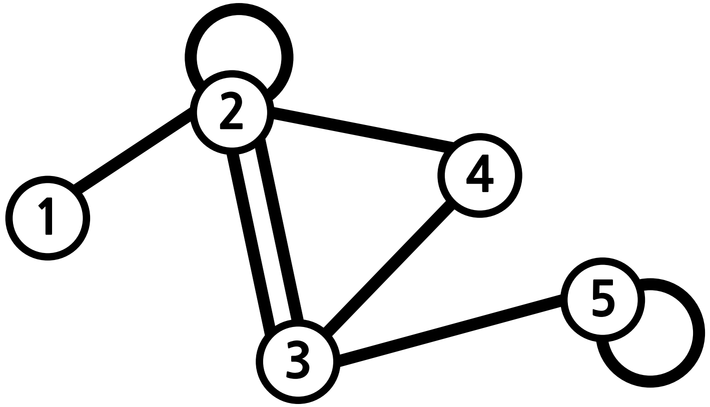
4.3 The Adjacency Matrix

Definition 4.3.2 (Multiple and self-edges)

$\star$ For nth multiple deges between i and j, $A_{ij} = n$.

$\star$ self-edge i, $A_{ii} = 2$

Example (Multiple and self-edges)



$\Rightarrow$

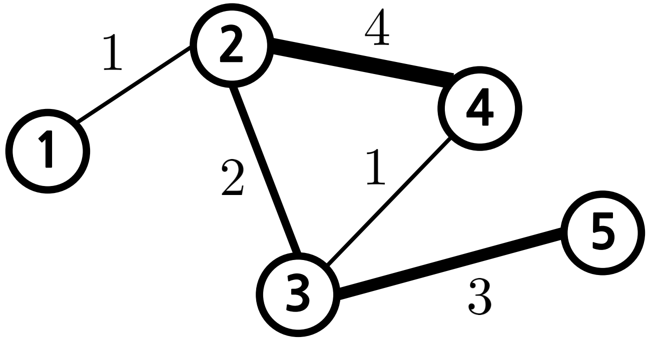
$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 2 & 1 & 0 \\ 0 & 2 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 \end{bmatrix} \end{array}$$

4.3 The Adjacency Matrix

Definition 4.3.3 (Weighted Network)

$$A_{ij} = \begin{cases} \alpha & \text{if } (i,j) \text{ connected} \\ 0 & \text{otherwise} \end{cases}$$

Example (Weighted Network)



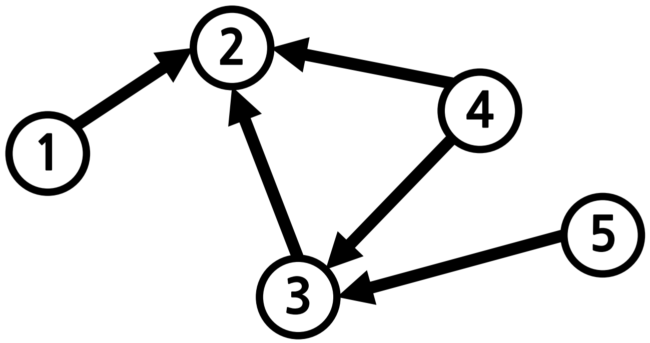
$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 4 & 0 \\ 0 & 2 & 0 & 1 & 3 \\ 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \end{bmatrix} \end{array}$$

4.3 The Adjacency Matrix

Definition 4.3.4 (Directed Network)

$$A_{ij} = \begin{cases} 1 & \text{if there is an edge from } j \text{ to } i \\ 0 & \text{otherwise} \end{cases}$$

Example (Directed Network)



$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

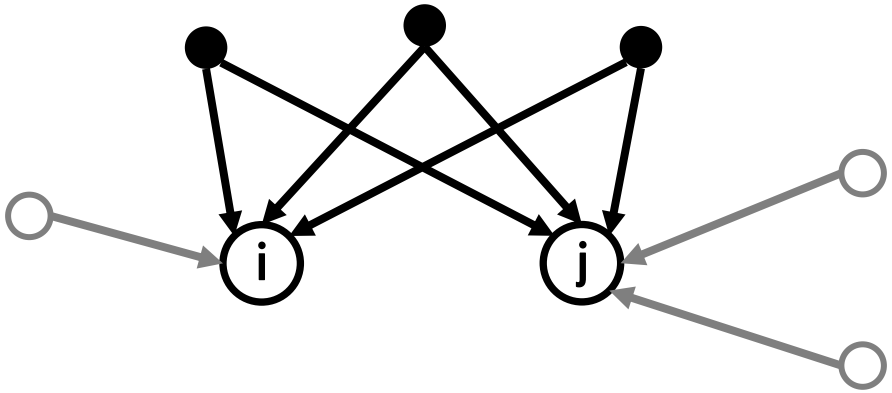
Not symmetric adjacent matrix

4.3 The Adjacency Matrix: Directed Networks

Definition 4.3.4. (Cocitation)

The **cocitation** of i and j vertices in a directed network is number of vertices that have outgoing edges pointing to both i and j

Example (Cocitation)



Vertices i and j are cited by three common papers, so their cocitation is 3.

4.3 The Adjacency Matrix: Directed Networks

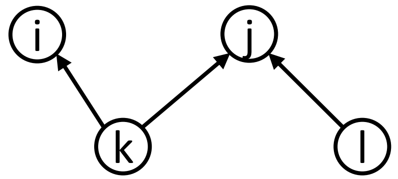
Note

How to find cocitation C_{ij}?

$$A_{ij} = \begin{cases} 1 & j \to i \\ 0 & \text{otherwise} \end{cases},$$

if i and j are both cited by k and zero otherwize, then $A_{ik} = 1$ and $A_{jk} = 1$.

$$\therefore A_{ik}A_{jk} = \begin{cases} 1 & (k \to i) \text{ and } (k \to j) \\ 0 & \text{otherwise} \end{cases}$$



$$C_{ij} = \sum_{k=1}^{n} A_{ik}A_{jk} = \sum_{k=1}^{n} A_{ik}A_{kj}^{\top}$$

$$\left(Recall\ :\ (AB)_{ij} = \sum_{k=1}^{n} A_{ik}B_{kj} \right)$$

Definition 4.3.4 (Cocitation Matrix)

The **cocitation matrix** C is $n \times n$ matrix with elemetns C_{ij} which is thus given by

$C = AA^{\top}$ where A is adjacency matrix of a directed network

Moreover $C^{\top} = (AA^{\top})^{\top} = AA^{\top} = C$ (symmetric)

4.3 The Adjacency Matrix: Directed Networks

Definition 4.3.5. (Cocitation Network)

A **Cocitation network** in which there is an edge between i and j if $C_{ij} > 0$, for $i \neq j$, i.e, an edge between any two vertices that are cocited in the original directed network.

Note

Better still, we can make the cocitation network a weighted network with positive integer weights on the edges equal to the corresponding elements C_{ij}. Then vertex pairs cited by more common neighbors have a stronger connection than those cited by fewer. Since the cocitation matrix is symmetric, the cocitation network is undirected, making it easier to deal with in many respects than the original directed network from which it was constructed.

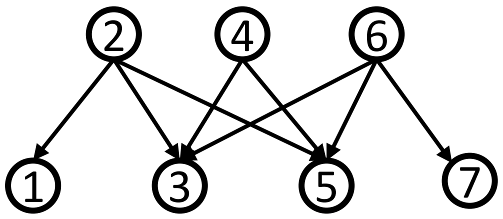
The cocitation matrix thus plays a role similar to an adjacency matrix for the cocitation network. There is however one aspect in which the cocitation matrix differs from an adjacency matrix: its diagonal elements. The diagonal elements of the cocitation matrix are given by

$$C_{ii} = \sum_{k=1}^{n} A_{ik} A_{ik} = \sum_{k=1}^{n} A_{ik}$$

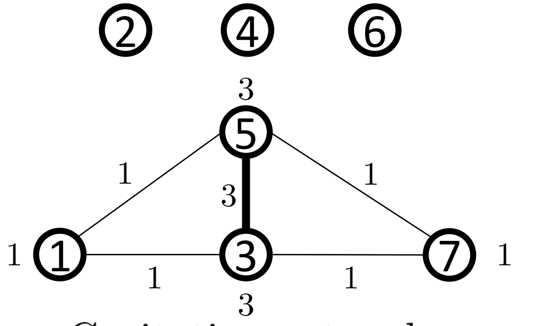
directed network $\{0, 1\}$

4.3 The Adjacency Matrix: Directed Networks

Example (Cocitation Graph)



A



Cocitation network

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(Adjacent matrix of directed network A)

$$\hat{A} = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(Adjacent matrix of Cocitation network)

$$C = AA^\top = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 3 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 3 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(Cocitation matrix)

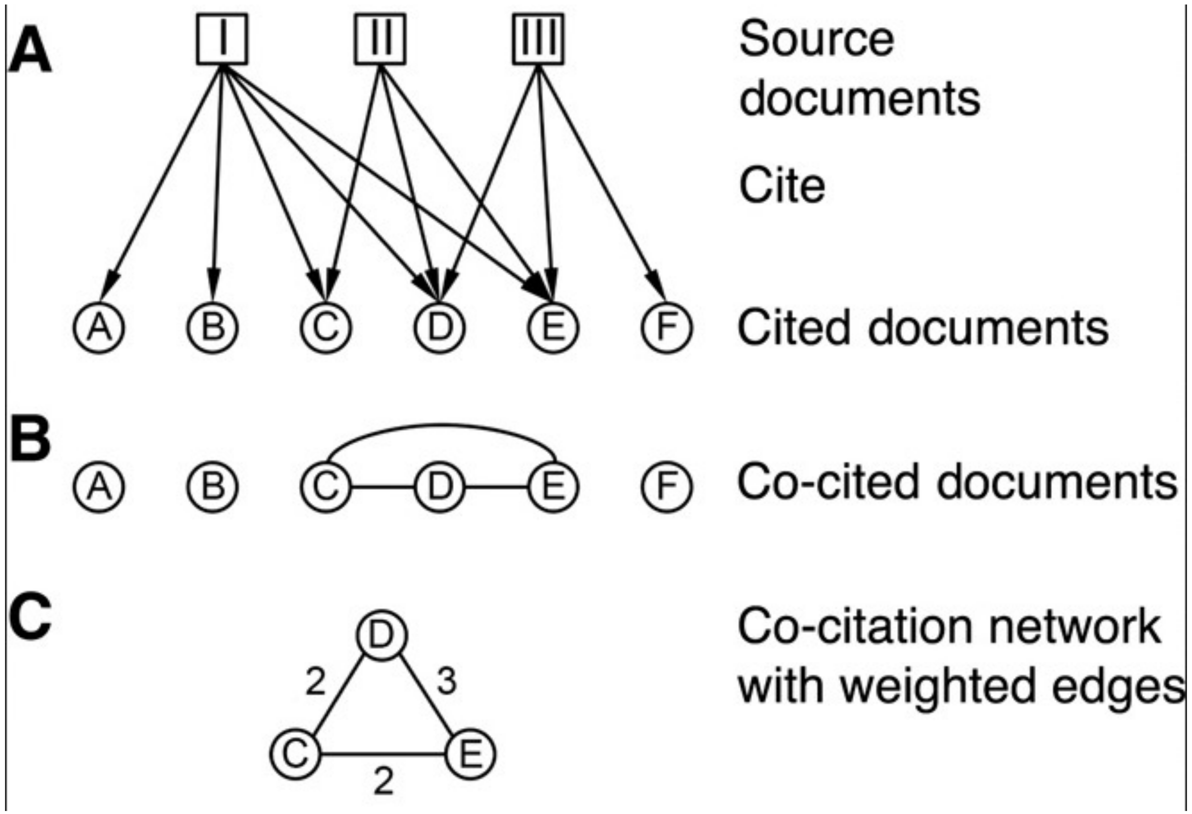
The cocitation matrix thus plays a role similar to an adjacency matrix for the cocitation network. There is however one aspect in which the cocitation matrix differs from an adjacency matrix: its diagonal elements. The diagonal elements of the cocitation matrix are given by

$$C_{ii} = \sum_{k=1}^{n} A_{ik} A_{ik} = \sum_{k=1}^{n} A_{ik}$$

where we have assumed that the directed network is a simple graph, with no multiedges, so that all elements $\hat{A}_{ik}$ of the adjacency matrix of cocitation network are zero or one.

4.3 The Adjacency Matrix: Directed Networks

Example (Cocitation Graph)



https://www.science.org/doi/10.1126/sciadv.1701130

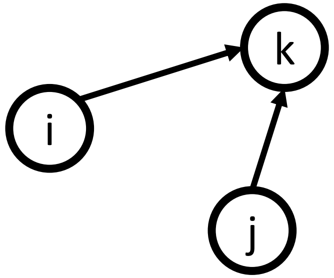
4.3 The Adjacency Matrix: Directed Networks

Definition 4.3.6. (Bibliographic coupling)

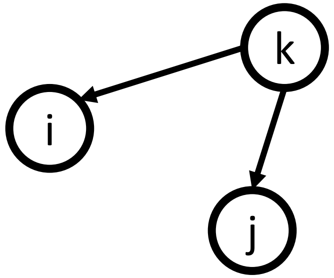
The **bibliographic coupling** of two vertices in a directed network is the number of other vertices to which both point.

In a citation network, A (Directed network)

$$A_{ki}A_{kj} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ both cite } k \\ 0 & \text{otherewise} \end{cases}$$



Bibliopraphic coupling



Cocitation

A bibliographic coupling of i and j

$$B_{ij} = \sum_{k=1}^{n} A_{ki}A_{kj} = \sum_{k=1}^{n} A^{\top}_{ik}A_{kj} \;\Rightarrow\; B = A^{\top}A$$

Definition 4.3.7. (Bibliographic coupling matrix)

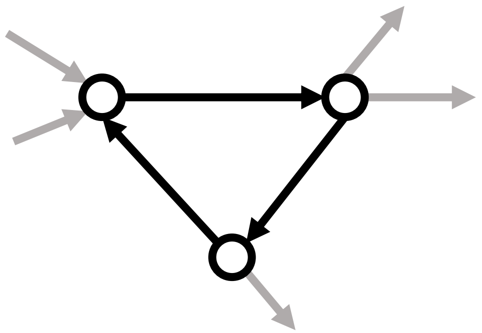
The **bibliographic coupling matrix** B to be the $n \times n$ matrix with elements B_{ij} so that

$$B = A^{\top}A$$

4.3 The Adjacency Matrix: Acyclic Directed Network

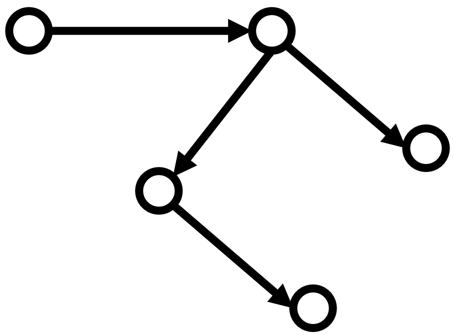
Definition 4.3.8 (Cycle)

A **cycle** in a directed network is a closed loop of edges with the arrows on each of the edges pointg the same way around the loop.



Definition 4.3.9 (Acyclic network)

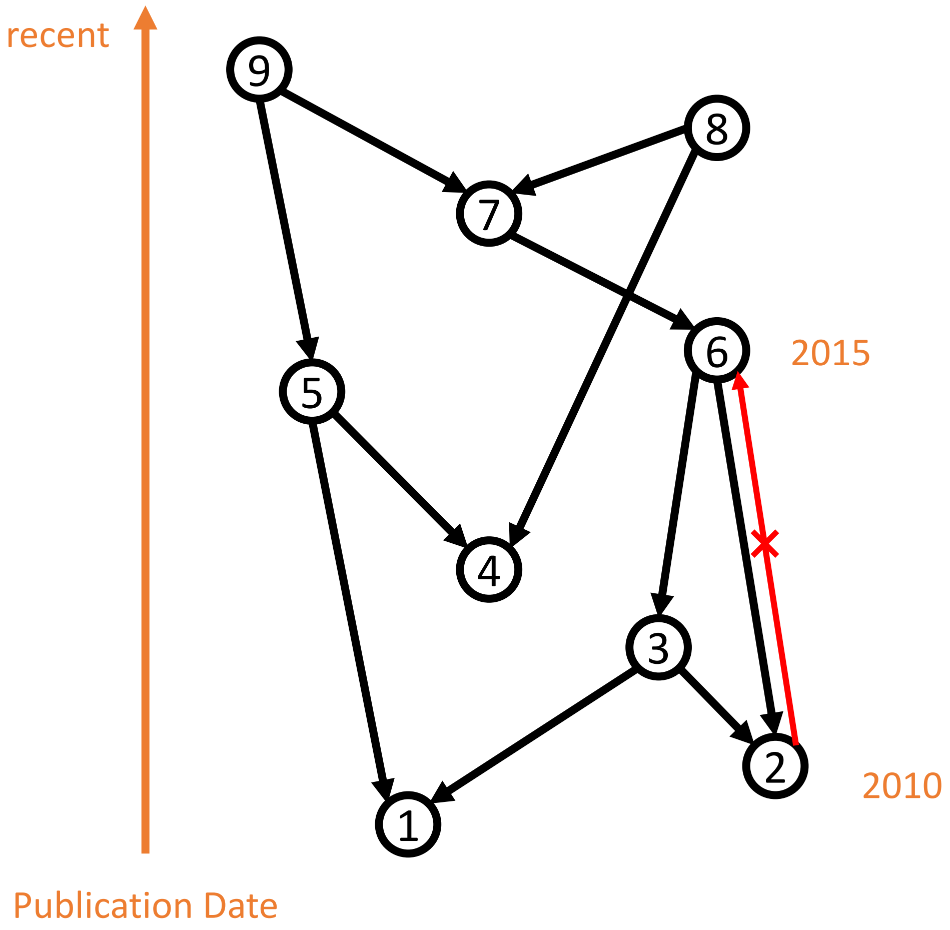
Some directed networks however have no cycles and there are called **acyclic networks**.



4.3 The Adjacency Matrix: Acyclic Directed Network

Example (Acyclic)

Citation Network



Acyclic

4.3 The Adjacency Matrix: Acyclic Directed Network

Note

Simple Algorithm for determining wherher a network is acyclic is:

Step 1 : Find a vertex with no outgoing edges

Step 2 : If no such vertex, exists, $\Rightarrow$ a cycle
otherwise, if such vertex exists, remove it and all its ingoing edges.

Step 3 : if all vertices have been removed, $\Rightarrow$ Acyclic
otherwise, go back to step 1

Adjacency matrix A of acyclic network is a strict upper triangular matrix

If a graph is acyclic, then it must have at least one node with no outgoing edge

4.3 The Adjacency Matrix: Acyclic Directed Network

Properties 4.3.1

The adjacency matrix have the propety that all of its eigenvalues are zero
$\Leftrightarrow$ a networks is a cycle

Proof) Recall eigenvalues $\lambda:\ A\mathbf{x} = \lambda\mathbf{x} \Rightarrow (A - \lambda I)\mathbf{x} = 0$

$|A - \lambda I| = 0$ is a characteristic equation.

$\Leftarrow$) Suppose a network is acycle.

A is strictly upper triangular matrix then, eigenvalues of A are all zeros.

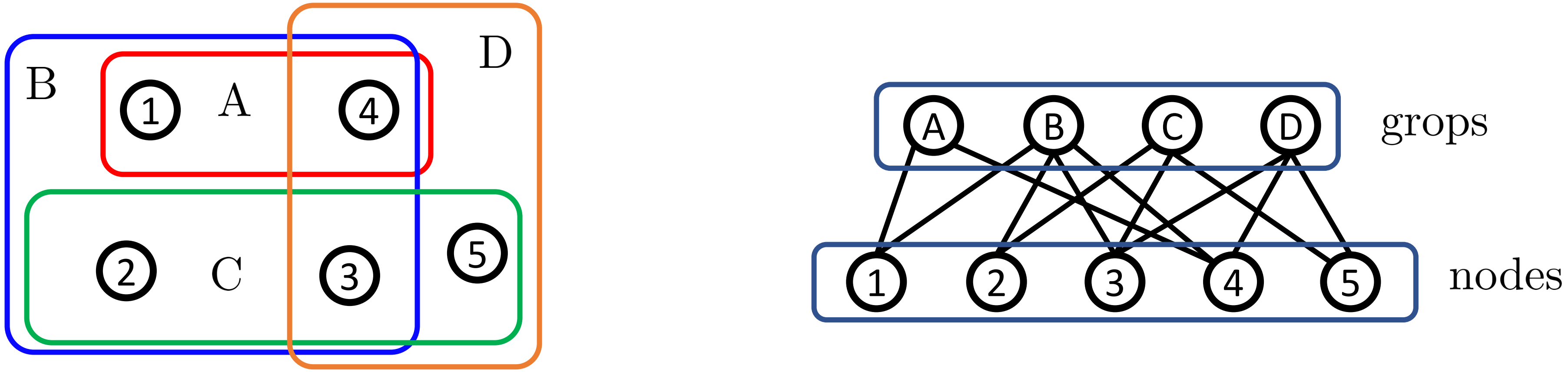
$\Rightarrow$) If all eigenvalues are zero then, acyclic

$$\Leftrightarrow \begin{cases} \text{If a network is cycle,} \\ \text{there exists } \lambda \neq 0 \text{ such that } A\mathbf{x} = \lambda\mathbf{x}. \end{cases}$$

$\square$

4.3 The Adjacency Matrix: Hypergraphs and Bipartite Graph

Definition 4.3.10. (Hypergraphs and Bipartite Graph)

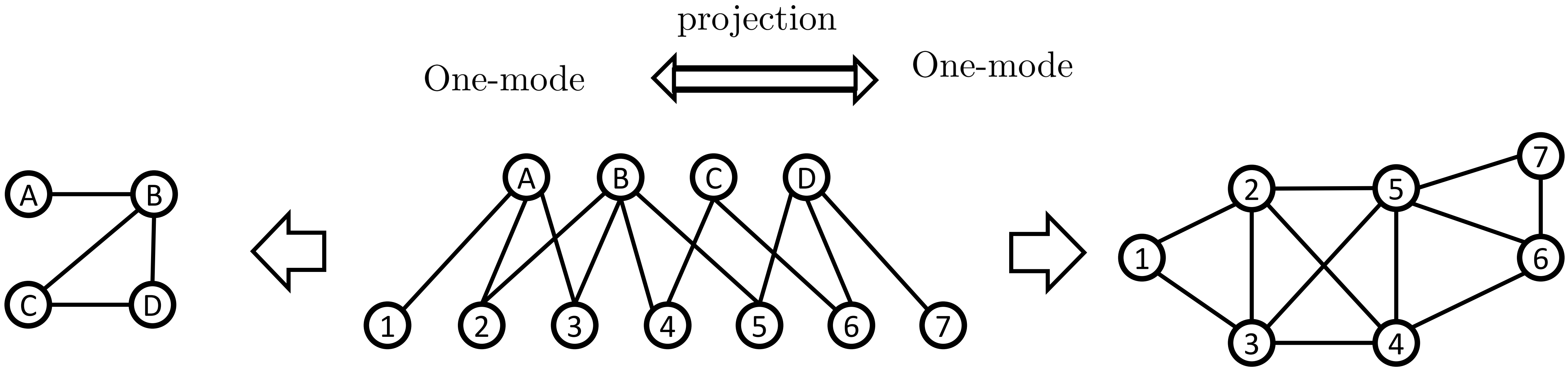


Hypergraph and corresponding bipartite graph

$$B_{ij} = \begin{cases} 1 & \text{if a vertex } j \text{ belongs to group } i \\ 0 & \text{otherwise} \end{cases} \quad \Rightarrow \quad B: \text{ an incidence matrix}$$

$$B = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} \mathrm{A} \\ \mathrm{B} \\ \mathrm{C} \\ \mathrm{D} \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array} : \text{ Rectangular Matrix}$$

4.3 The Adjacency Matrix: Hypergraphs and Bipartite Graph



The two one-mode projections of a bipartite network,

P_{ij} : the total nubmer of groups to whcih both i and j belong

$$P_{ij} = \sum_{k=1}^{g} B_{ki} B_{kj} = \sum_{k=1}^{g} B_{ik}^{\top} B_{kj},\ k \text{ is group index.}$$

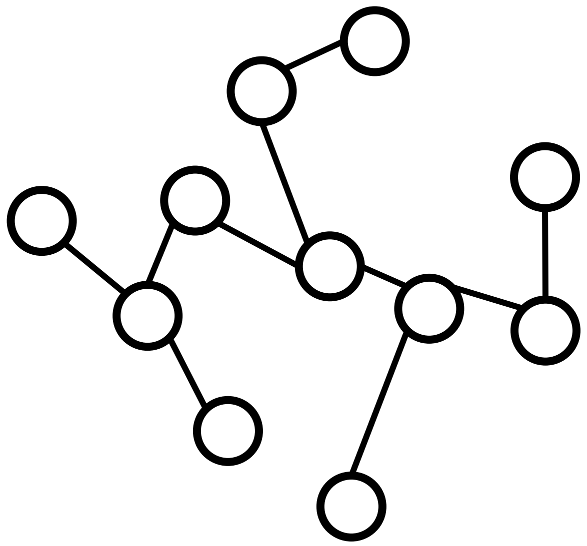
$$\therefore P = B^{\top} B$$

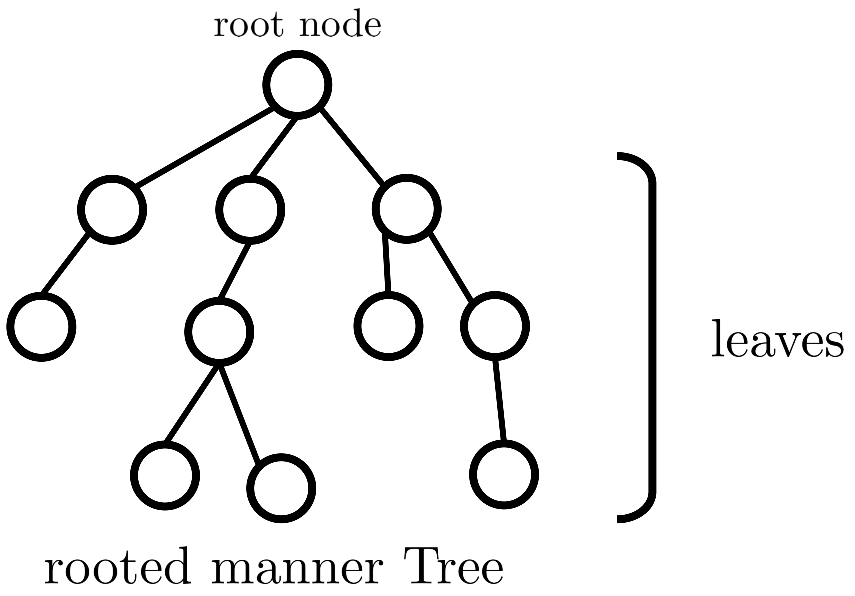
4.3 The Adjacency Matrix: Tree

Definition 4.3.11. (Tree)

A **tree** is a connected, undirected network that contains no closed loops.

Example



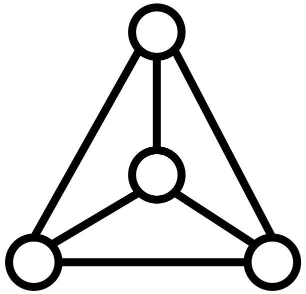


4.3 The Adjacency Matrix: Planar Network

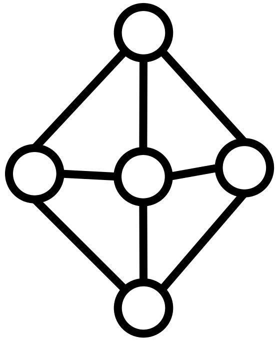
Definition 4.3.12 (Planar network)

A **planar network** is a network that can be drawn on a plane without having any edges cross.

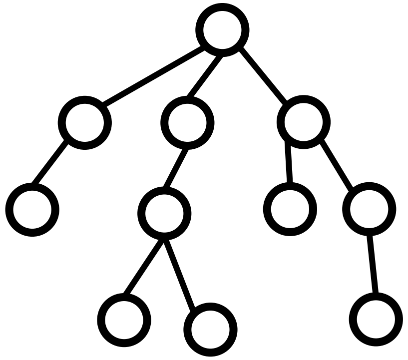
Example



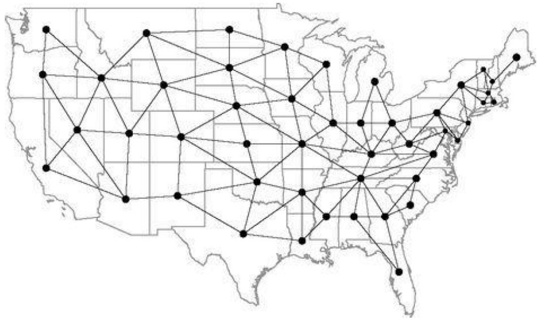
Planar



No Planar



Tree $\Rightarrow$ Planar

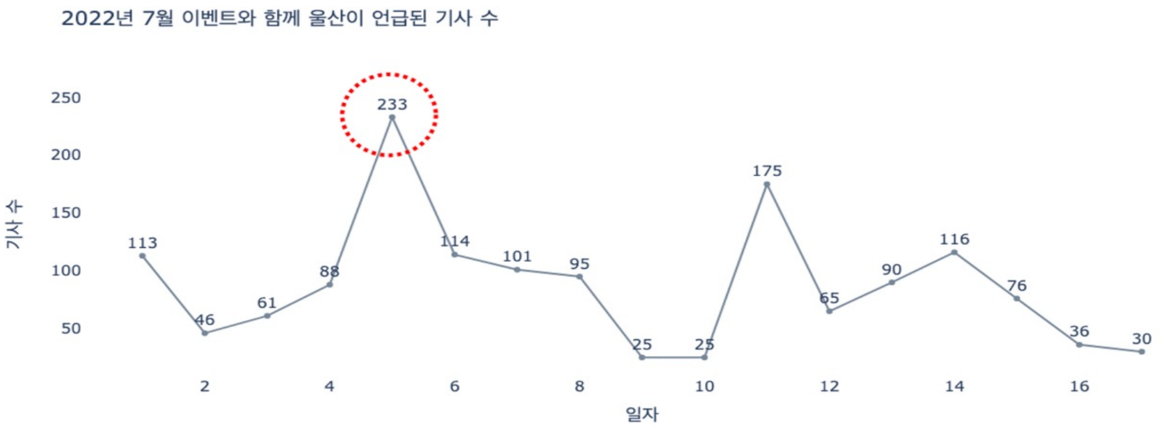


4.4 Experiments

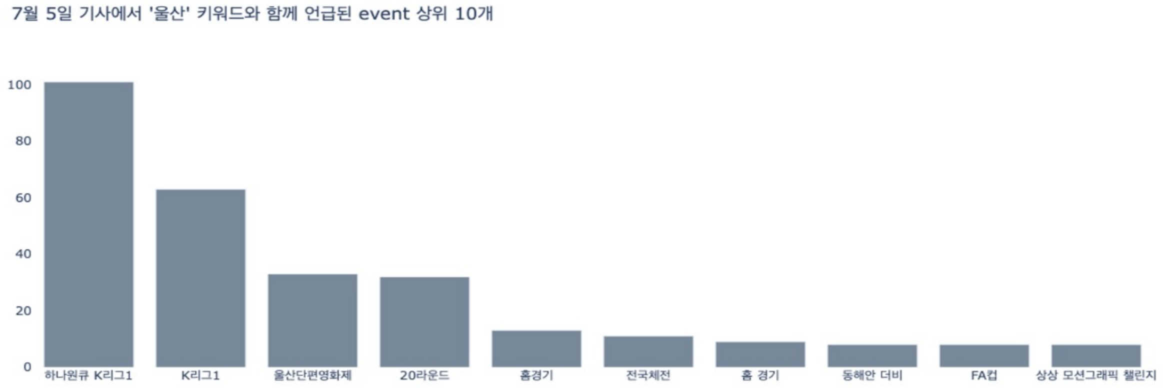
- **[데이터셋] 울산 지역 뉴스**

2022.07 뉴스기사 키워드 탐색 - "울산" | 코어닷투데이

https://v5.core.today/story/xM342O4AAO



	n	title	press	date	ner	ner_processed
0	0	"아련함에 가려졌다"..빅톤, 이제 섹시할 시간(종합)	더팩트	2022-06-01 00:00:00	[[('31일', 'DATE'), ('7집', 'QUANTITY'), ('Chaos...	[[(31일, DATE), (7집, QUANTITY), (Chaos, ARTIFAC...
1	1	국민의힘, '경기'에 화력 집중..계양·제주에선 '이재명' 직격	데일리안	2022-06-01 00:00:00	[[('안철수', 'PERSON'), ('국민의힘', 'ORGANIZATION'),...	[[(안철수, PERSON), (국민의힘, ORGANIZATION), (성남, LO...
2	2	'세월호 분향소서 성관계' 주장 60대 유튜버 1심 유죄	더팩트	2022-06-01 00:00:00	[[('벌금', 'CIVILIZATION'), ('150만원', 'QUANTITY'...	[[(벌금, CIVILIZATION), (150만원, QUANTITY), (세월호,...



4.4 Experiments

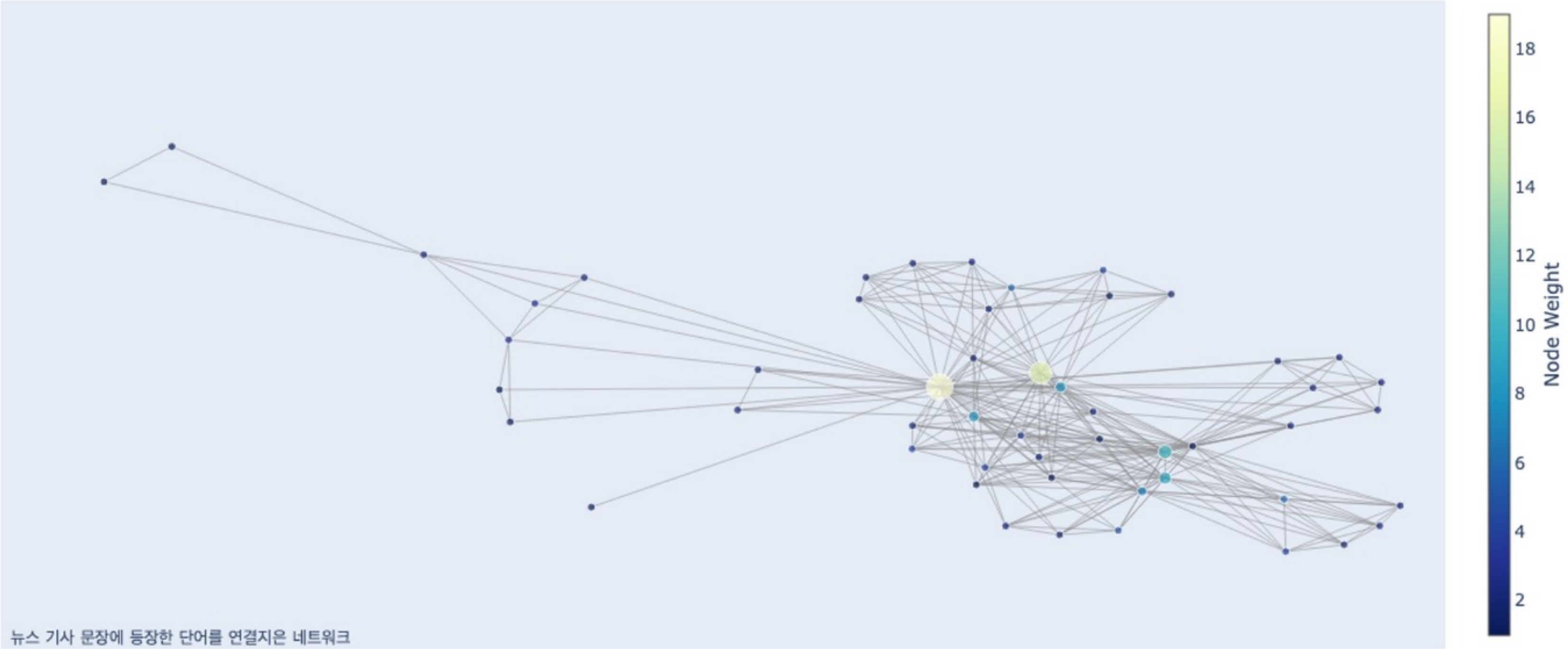
- **[노트북] 울산 지역 뉴스를 네트워크로 만들기**

코어닷투데이 (Core.Today) | 코어닷투데이

https://v5.core.today/notebook/K7949Iwy0#networkx_notebook_t...

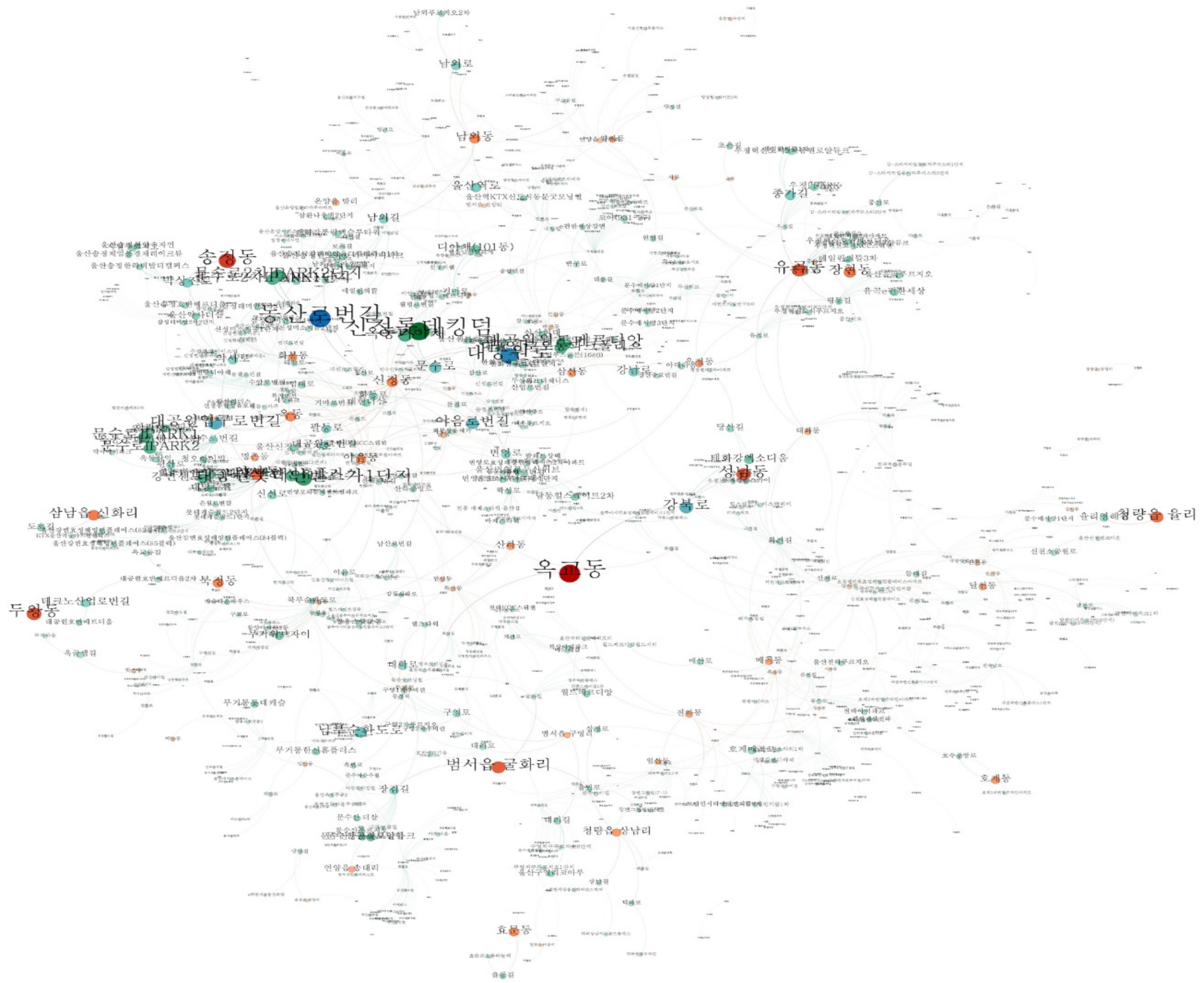
Core.Today

1월 뉴스에 대한 네트워크 매버릭 삭제



4.4 Experiments

- **[노트북] 울산 지역 법정동, 도로명, 아파트 네트워크 만들기**



4.4 Experiments

- **[노트북] 뉴스 NER 데이터셋을 이용한 네트워크 생성 튜토리얼**

Networkx_tutorial 및 Plotly 시각화 | 코어닷투데이

Networkx 튜토리얼을 진행하고, 뉴스에 등장한 단어 그래프를 그려봅니다. 단어 그래프의 데이터로는 20220601 뉴스 NER를 사용해서 Network를 생성했습니다.

https://v5.core.today/notebook/3qi4fF454#networkx_notebook_to...

- **[노트북] 네트워크의 시각화 배우기**

In [1]:
```
from pyvis.network import Network
import networkx as nx
```

In [3]:
```
g=Network(height=800, width=800, notebook=True)
g.toggle_hide_edges_on_drag(False)
g.barnes_hut() # use this particular physics solver
g.from_nx(nx.davis_southern_women_graph()) # just show a networkx graph
g.show("ex.html")
```

Out[3]: