

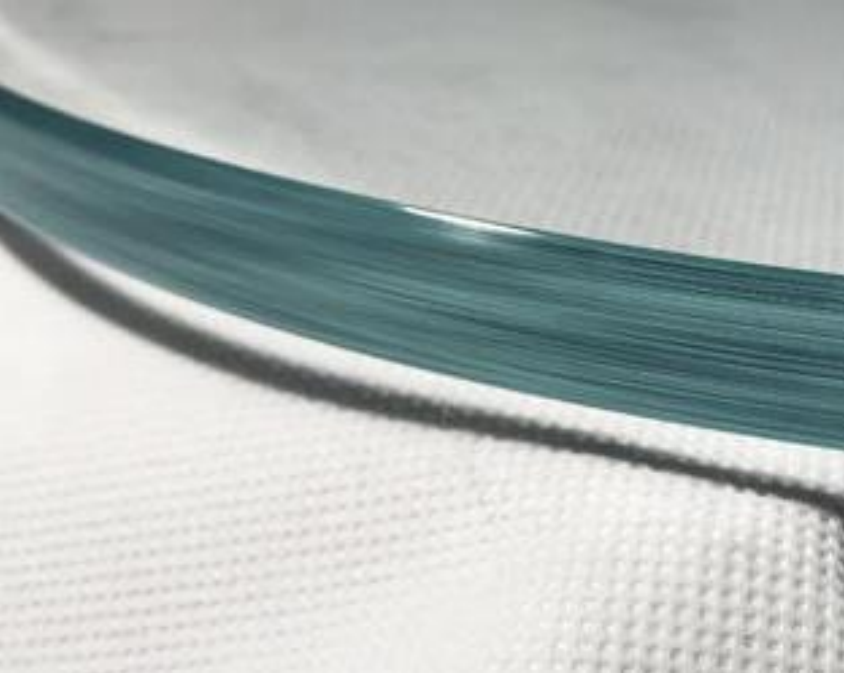
ESI 5247 SPRING 2023 ENGINEERING EXPERIMENTS TEAM PROJECT

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PROJECT TITLE:

STUDYING THE FACTORS AFFECTING THE TOLERANCE OF
GLASS AND RUBBER USING A 2-LEVEL FULL FACTORIAL
DESIGN

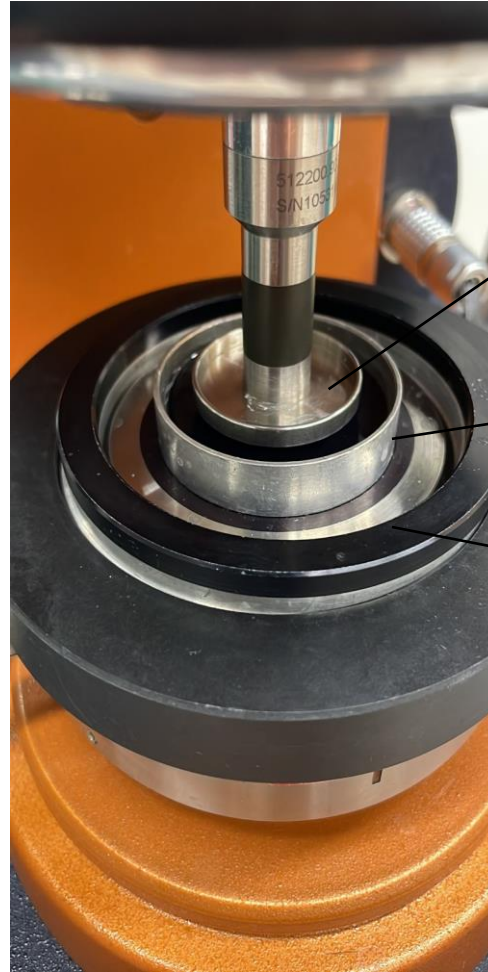
CONTENTS:

- Apparatus
- Factors
- Type of design used and methods
- Fractional Factorial Design Experiments
- Response Surface Analysis and Optimization Results
- Summary and Conclusion

Rheometer — Measures the way in which viscous fluid flows response to applied forces



Sample Loading



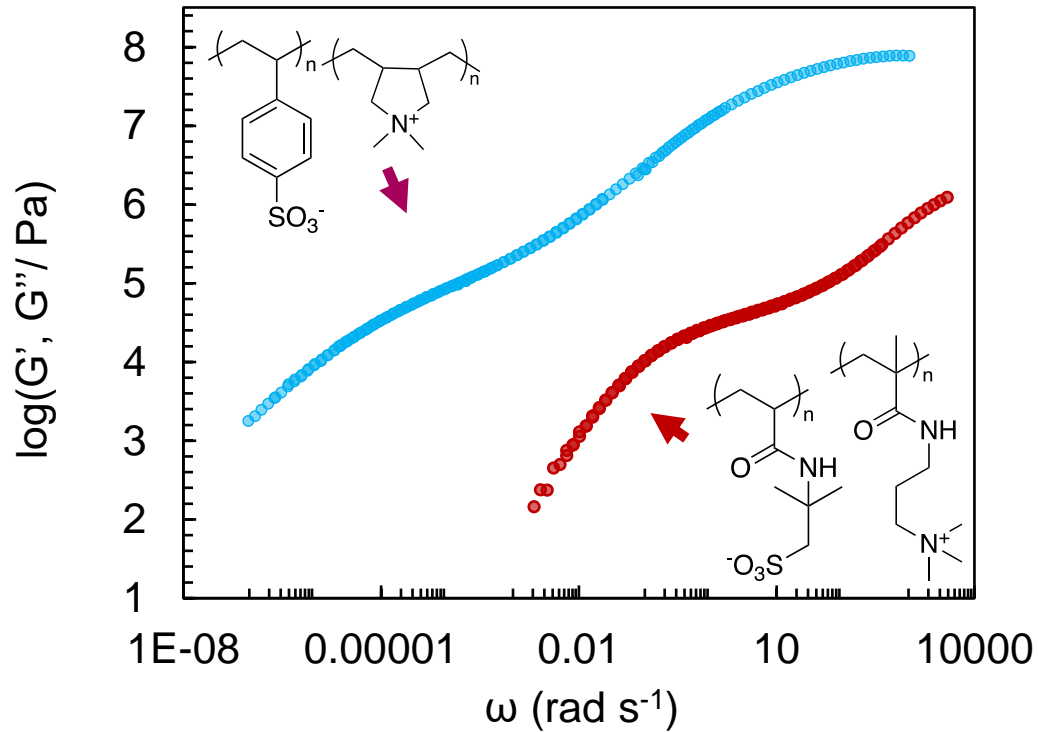
Upper plate - Oscillates at different frequencies causing an oscillatory shear force

Material

Lower Plate — Release the plat

The polymers that we used are charged and therefore retain water. Samples are immersed between an upper and a lower plate

Considering the data values



Temperature = 30 C
Salt Concentration = 0.05 M

The graph represents the relation between storage modulus and frequency.

Blue: it is the hard material it is called PSS/PDADMA. PSS is a negatively charged polymer and PDADMA is positively charged.

Red: it is a rubbery or elastic material. It is called PMAPTA/PAMPS where PMAPTA is the positive polymer and PAMPS is the negative polymer.

Here we have taken data at 10 rad s^{-1} of the polymer materials.

FACTORS

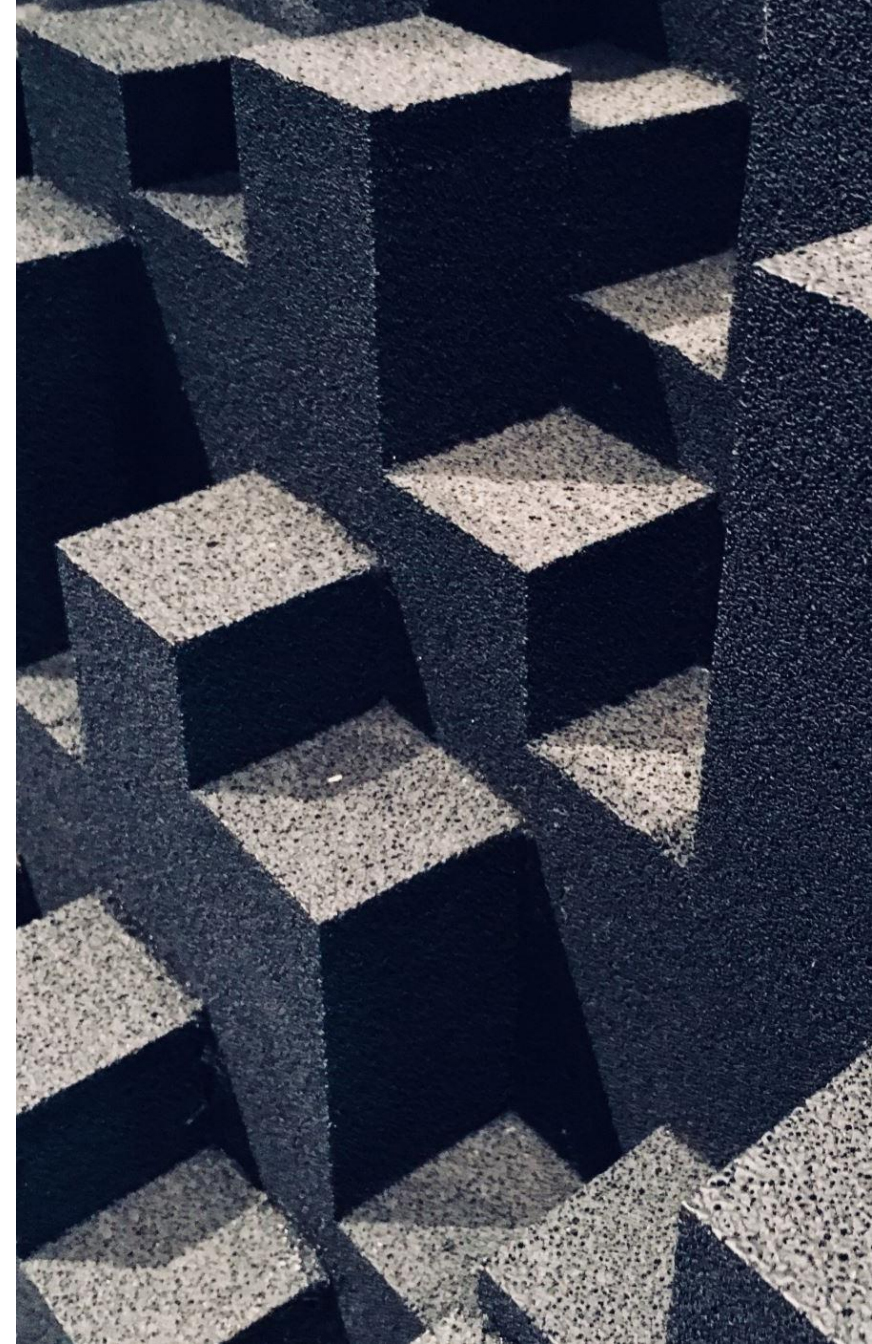
There are four factors effecting the design model

Temperature	30	70
Sample size	1000	1800
Salt concentration	0.05	0.025
Type	Rubber	Glass

- Temperature in measured in Fahrenheit
- Sample size is measured in millimeter
- Salt concentration is measured in grams of salt per liter /kilogram of water
- Type of material used is Polyelectrolyte.

TYPE OF DESIGN AND METHODS

- The type of design is 2^4 full factorial design, each factor has two level
- Methods used are factorial design and response surface design.



FACTOR EFFECT ESTIMATES AND SUM OF SQUARES FOR THE 2^4

Coded Coefficients for Transformed Response

Term	Effect	Coef	SE Coef	95% CI	T-Value
Constant		11.33	*	(*, *)	*
Tempreture	-2.313	-1.157	*	(*, *)	*
salt concentration	-1.4603	-0.7302	*	(*, *)	*
size	0.5345	0.2672	*	(*, *)	*
Type	4.548	2.274	*	(*, *)	*
Tempreture*salt concentration	-0.07637	-0.03819	*	(*, *)	*
Tempreture*size	-0.15788	-0.07894	*	(*, *)	*
Tempreture*Type	-1.3585	-0.6793	*	(*, *)	*
salt concentration*size	0.18451	0.09226	*	(*, *)	*
salt concentration*Type	0.6593	0.3296	*	(*, *)	*
size*Type	0.09141	0.04571	*	(*, *)	*
Tempreture*salt concentration*size	-0.15822	-0.07911	*	(*, *)	*
Tempreture*salt concentration*Type	0.15442	0.07721	*	(*, *)	*
Tempreture*size*Type	-0.3470	-0.1735	*	(*, *)	*
salt concentration*size*Type	0.12854	0.06427	*	(*, *)	*
Tempreture*salt concentration*size*Type	-0.18470	-0.09235	*	(*, *)	*

FULL MODEL ANOVA: NO DEGREE FREEDOM FOR ERROR, CANNOT TEST SIGNIFICANCE OF EFFECTS

Analysis of Variance for Transformed Response

Source	DF	Seq SS	Contribution	Adj SS	Adj MS
Model	15	124.098	100.00%	124.098	8.2732
Linear	4	113.805	91.71%	113.805	28.4511
Tempreture	1	21.409	17.25%	21.409	21.4087
salt concentration	1	8.530	6.87%	8.530	8.5304
size	1	1.143	0.92%	1.143	1.1426
Type	1	82.723	66.66%	82.723	82.7230
2-Way Interactions	6	9.413	7.59%	9.413	1.5689
Tempreture*salt concentration	1	0.023	0.02%	0.023	0.0233
Tempreture*size	1	0.100	0.08%	0.100	0.0997
Tempreture*Type	1	7.382	5.95%	7.382	7.3821
salt concentration*size	1	0.136	0.11%	0.136	0.1362
salt concentration*Type	1	1.739	1.40%	1.739	1.7385
size*Type	1	0.033	0.03%	0.033	0.0334
3-Way Interactions	4	0.743	0.60%	0.743	0.1858
Tempreture*salt concentration*size	1	0.100	0.08%	0.100	0.1001
Tempreture*salt concentration*Type	1	0.095	0.08%	0.095	0.0954
Tempreture*size*Type	1	0.482	0.39%	0.482	0.4817
salt concentration*size*Type	1	0.066	0.05%	0.066	0.0661
4-Way Interactions	1	0.136	0.11%	0.136	0.1365
Tempreture*salt concentration*size*Type	1	0.136	0.11%	0.136	0.1365
Error	0	*	*	*	*
Total	15	124.098	100.00%		

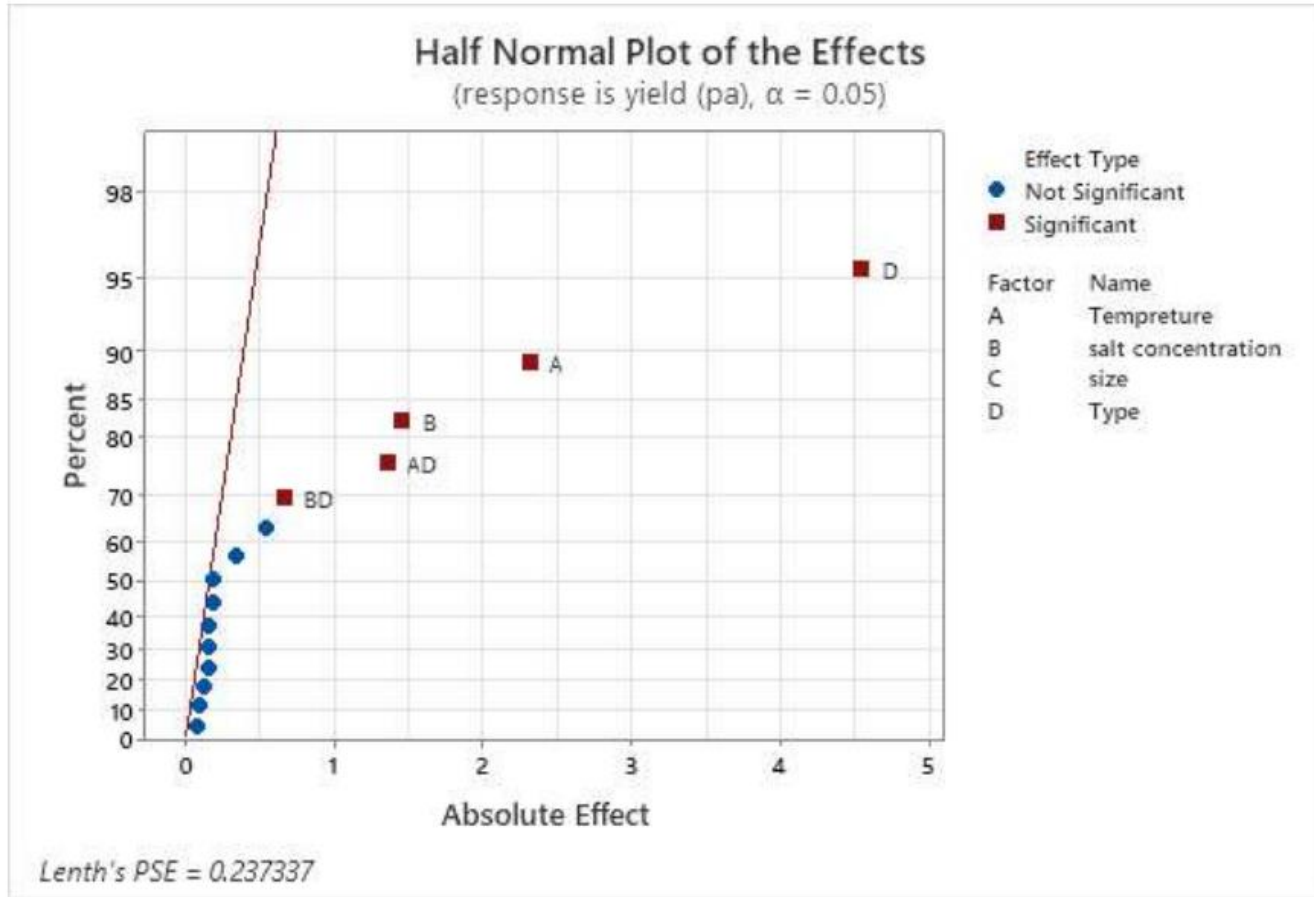
MODEL SUMMARY FOR THE UNREPLICATED

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
*	100.00%	*	*	*	*	*

- The model summary for unreplicated states that we cannot measure the error as the degree of freedom is zero we cannot do the f-test.
- As we can observe from the above, the value of R-sq is 100% so we can consider this as not acceptable.

HALF NORMAL PROBABILITY PLOT OF THE EFFECTS FOR THE 2^4 FACTORIAL



From the NPP of effect estimates of the Full Model following effects appear to be non-negligible

A, B, D, AD, BD

- To form the SSE, drop the insignificant terms
- Fit a model that contains only the non-negligible terms: Refined Model
10 terms are dropped, thus for the refined model, $df_E = 10$.

REFINED MODEL ANOVA

Analysis of Variance for Transformed Response

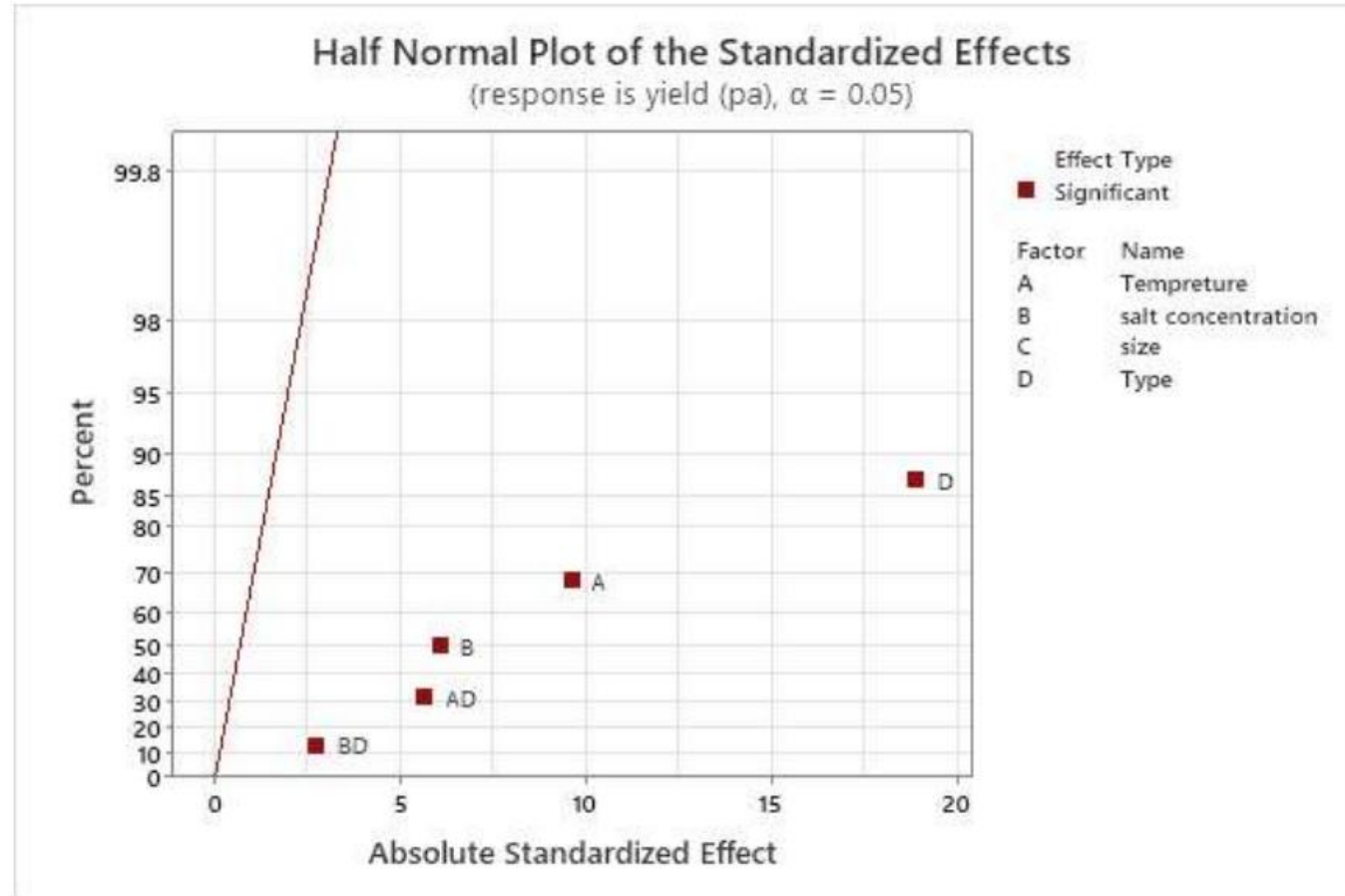
Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Model	5	121.783	98.13%	121.783	24.3565	105.21	0.000
Linear	3	112.662	90.78%	112.662	37.5540	162.22	0.000
Tempreture	1	21.409	17.25%	21.409	21.4087	92.48	0.000
salt concentration	1	8.530	6.87%	8.530	8.5304	36.85	0.000
Type	1	82.723	66.66%	82.723	82.7230	357.33	0.000
2-Way Interactions	2	9.121	7.35%	9.121	4.5603	19.70	0.000
Tempreture*Type	1	7.382	5.95%	7.382	7.3821	31.89	0.000
salt concentration*Type	1	1.739	1.40%	1.739	1.7385	7.51	0.021
Error	10	2.315	1.87%	2.315	0.2315		
Total	15	124.098	100.00%				

MODEL SUMMARY FOR THE UNREPLICATED REFINED

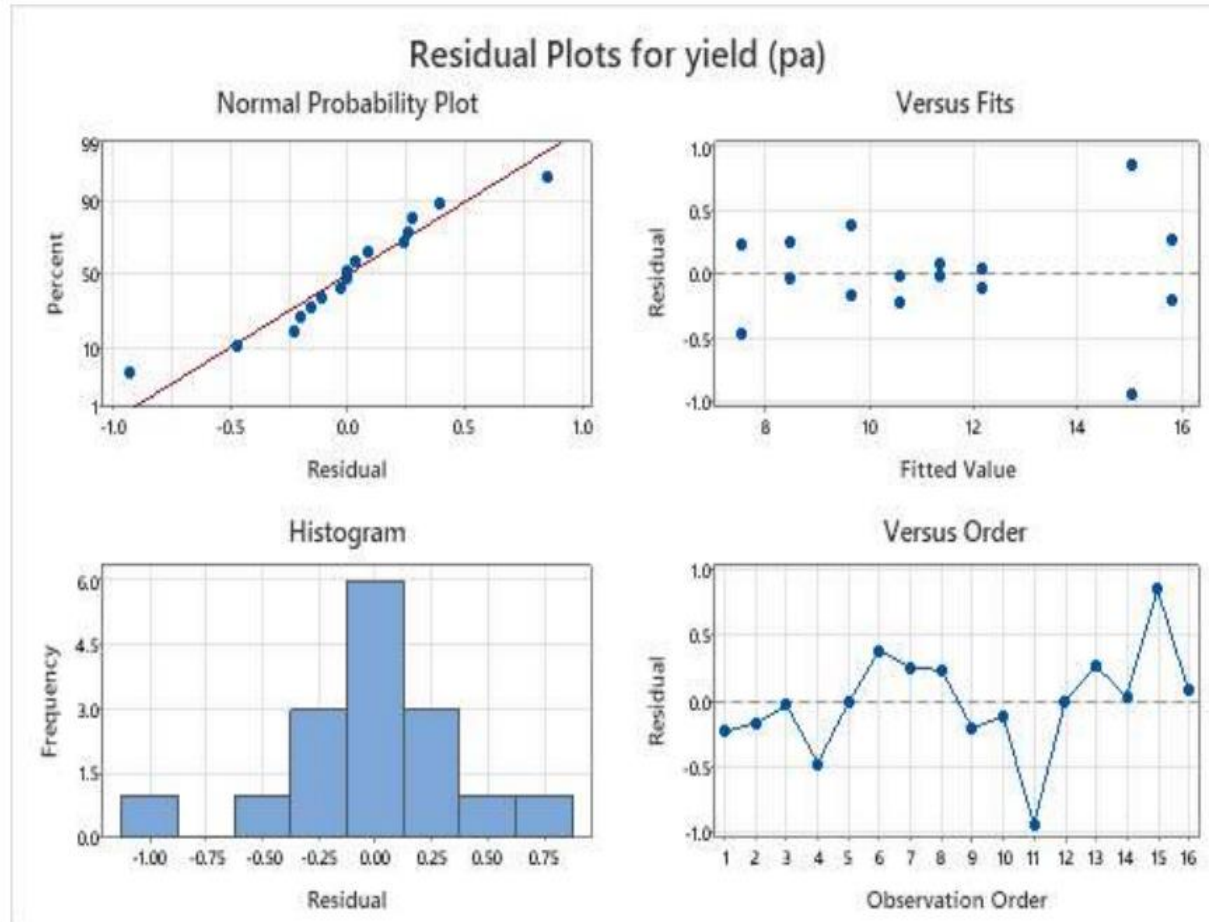
Model Summary for Transformed Response

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
0.481150	98.13%	97.20%	5.92653	95.22%	42.48	33.88

HALF NORMAL PROBABILITY PLOT OF THE EFFECTS FOR THE 2^4 FACTORIAL (REFINED MODEL)



RESIDUAL PLOTS



- NPP – Stability of the regression model for the data and help to identify the validity of statical inference.
- Versus Fits - The versus fit plot is useful for detecting patterns in the data that may suggest the model is mis specified.

ADDING THE CENTER POINTS

- Use 2^k design with center points to test for curvature.
- To check the possibility of curvature, add six center points to this experiment.
- Consider the number of center points $n_c = 6$
- Adding six center points to the 2^4 design
- Center points allow an independent estimate of error (Pure error) in unreplicated factorials.

EXPERIMENTAL RESULTS IN MINITAB BY ADDING THE CENTER POINTS

Factorial Regression: yield (pa) versus Temp, salt concentration, sample size, ...

Method

Box-Cox transformation

Rounded λ -0.133429

Estimated λ -0.133429

95% CI for λ (-0.244929, -0.0219290)

Coded Coefficients for Transformed Response

Term	Effect	Coef	SE Coef	95% CI
Constant		-0.23534	0.00343	(-0.24415, -0.22652)
Temp	-0.06005	-0.03003	0.00343	(-0.03884, -0.02122)
salt concentration	-0.05180	-0.02590	0.00343	(-0.03471, -0.01709)
sample size	0.01518	0.00759	0.00343	(-0.00122, 0.01640)
type	0.12674	0.06337	0.00292	(0.05585, 0.07088)
Temp*salt concentration	-0.00846	-0.00423	0.00343	(-0.01304, 0.00458)
Temp*sample size	0.00035	0.00018	0.00343	(-0.00864, 0.00899)
Temp*type	-0.02002	-0.01001	0.00343	(-0.01882, -0.00120)
salt concentration*sample size	0.00544	0.00272	0.00343	(-0.00609, 0.01153)
salt concentration*type	0.03384	0.01692	0.00343	(0.00811, 0.02573)
sample size*type	-0.00354	-0.00177	0.00343	(-0.01058, 0.00704)
Temp*salt concentration*sample size	-0.00188	-0.00094	0.00343	(-0.00975, 0.00787)
Temp*salt concentration*type	0.00636	0.00318	0.00343	(-0.00563, 0.01199)
Temp*sample size*type	-0.00867	-0.00433	0.00343	(-0.01314, 0.00448)
salt concentration*sample size*type	0.00040	0.00020	0.00343	(-0.00861, 0.00901)
Temp*salt concentration*sample size*type	-0.00462	-0.00231	0.00343	(-0.01112, 0.00650)
Ct Pt		0.00257	0.00656	(-0.01431, 0.01944)

EXPERIMENTAL
RESULTS IN
MINITAB BY
ADDING THE
CENTER
POINTS

Term	T-Value	P-Value	VIF
Constant	-68.66	0.000	
Temp	-8.76	0.000	1.00
salt concentration	-7.56	0.001	1.00
sample size	2.21	0.078	1.00
type	21.68	0.000	1.00
Temp*salt concentration	-1.23	0.272	1.00
Temp*sample size	0.05	0.961	1.00
Temp*type	-2.92	0.033	1.00
salt concentration*sample size	0.79	0.464	1.00
salt concentration*type	4.94	0.004	1.00
sample size*type	-0.52	0.627	1.00
Temp*salt concentration*sample size	-0.27	0.794	1.00
Temp*salt concentration*type	0.93	0.396	1.00
Temp*sample size*type	-1.26	0.262	1.00
salt concentration*sample size*type	0.06	0.956	1.00
Temp*salt concentration*sample size*type	-0.67	0.530	1.00
Ct Pt	0.39	0.712	1.00

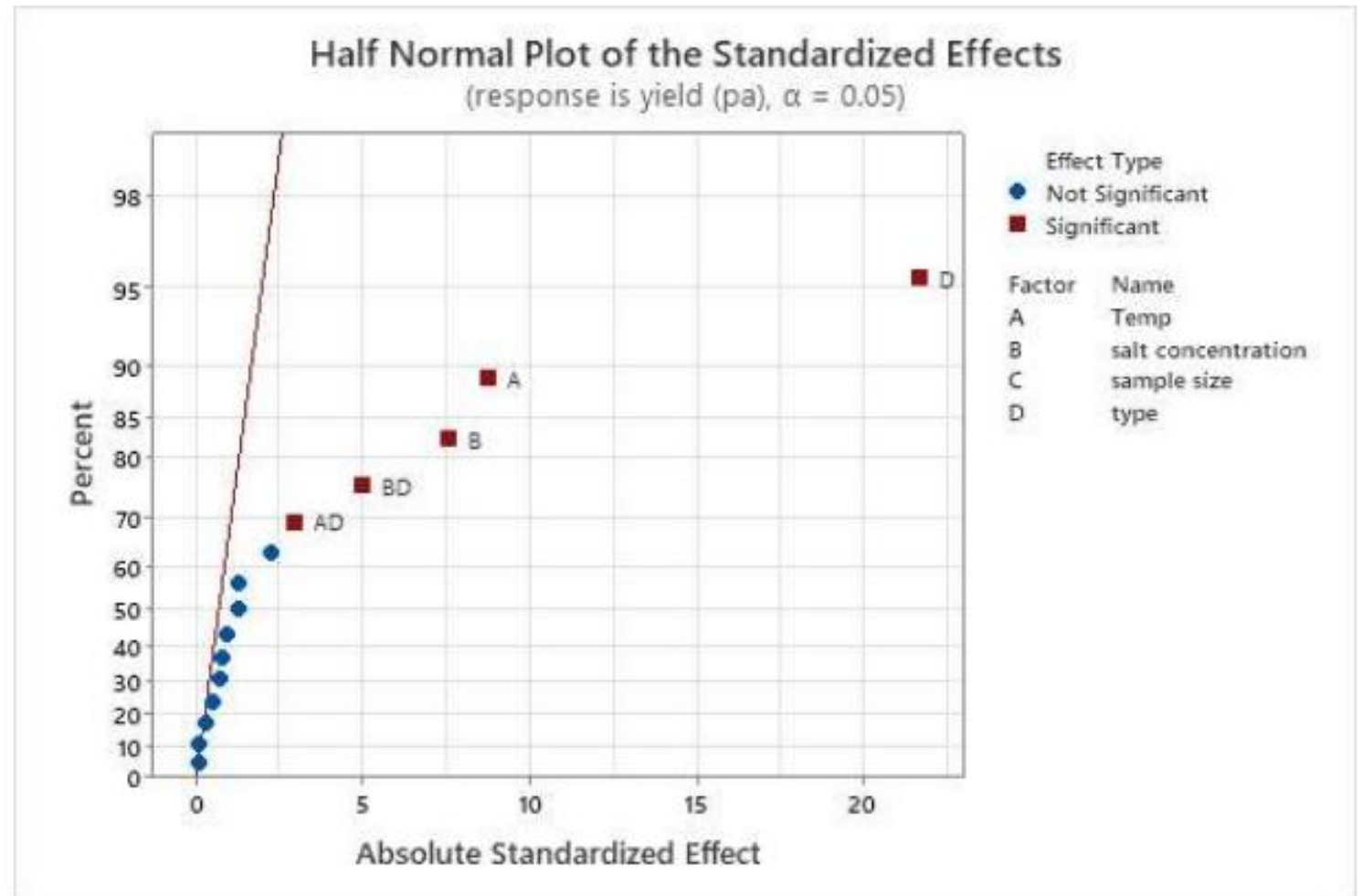
EXPERIMENTAL RESULTS IN MINITAB BY ADDING THE CENTER POINTS

Analysis of Variance for Transformed Response

Source	DF	Seq SS	Contribution	Adj SS	Adj MS
Model	16	0.121655	99.23%	0.121655	0.007603
Linear	4	0.114423	93.33%	0.114423	0.028606
Temp	1	0.014426	11.77%	0.014426	0.014426
salt concentration	1	0.010735	8.76%	0.010735	0.010735
sample size	1	0.000922	0.75%	0.000922	0.000922
type	1	0.088341	72.06%	0.088341	0.088341
2-Way Interactions	6	0.006641	5.42%	0.006641	0.001107
Temp*salt concentration	1	0.000286	0.23%	0.000286	0.000286
Temp*sample size	1	0.000000	0.00%	0.000000	0.000000
Temp*type	1	0.001603	1.31%	0.001603	0.001603
salt concentration*sample size	1	0.000118	0.10%	0.000118	0.000118
salt concentration*type	1	0.004582	3.74%	0.004582	0.004582
sample size*type	1	0.000050	0.04%	0.000050	0.000050
3-Way Interactions	4	0.000477	0.39%	0.000477	0.000119
Temp*salt concentration*sample size	1	0.000014	0.01%	0.000014	0.000014
Temp*salt concentration*type	1	0.000162	0.13%	0.000162	0.000162
Temp*sample size*type	1	0.000300	0.25%	0.000300	0.000300
salt concentration*sample size*type	1	0.000001	0.00%	0.000001	0.000001
4-Way Interactions	1	0.000085	0.07%	0.000085	0.000085
Temp*salt concentration*sample size*type	1	0.000085	0.07%	0.000085	0.000085
Curvature	1	0.000029	0.02%	0.000029	0.000029
Error	5	0.000940	0.77%	0.000940	0.000188
Lack-of-Fit	1	0.000840	0.69%	0.000840	0.000840
Pure Error	4	0.000100	0.08%	0.000100	0.000025
Total	21	0.122595	100.00%		

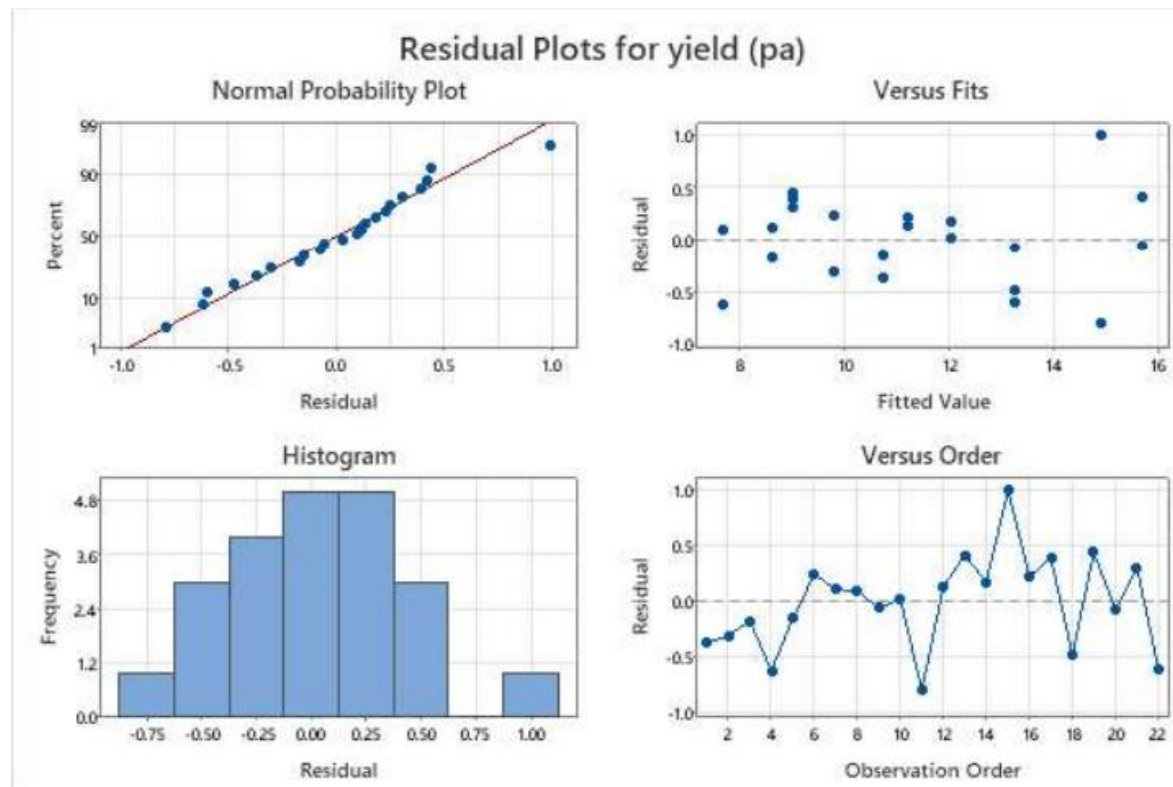
Source	F-Value	P-Value
Model	40.45	0.000
Linear	152.17	0.000
Temp	76.74	0.000
salt concentration	57.11	0.001
sample size	4.90	0.078
type	469.94	0.000
2-Way Interactions	5.89	0.035
Temp*salt concentration	1.52	0.272
Temp*sample size	0.00	0.961
Temp*type	8.53	0.033
salt concentration*sample size	0.63	0.464
salt concentration*type	24.37	0.004
sample size*type	0.27	0.627
3-Way Interactions	0.63	0.660
Temp*salt concentration*sample size	0.08	0.794
Temp*salt concentration*type	0.86	0.396
Temp*sample size*type	1.60	0.262
salt concentration*sample size*type	0.00	0.956
4-Way Interactions	0.45	0.530
Temp*salt concentration*sample size*type	0.45	0.530
Curvature	0.15	0.712
Error		
Lack-of-Fit	33.57	0.004
Pure Error		
Total		

EXPERIMENTAL RESULTS HALF NORMAL PLOT OF THE STANDARDIZED EFFECTS



From the half-normal plot above, we can observe that some factors are not significant, and we run again our design with refined data to achieve a significant effect type.

EXPERIMENTAL RESULTS FOR RESIDUAL PLOTS FOR YIELD(PA)



- The above design is not significant, the residual plots for the refined model show random and scattered points around the horizontal line at zero. This indicates that the residuals are normally distributed and that the model is a good fit for the data.
- We run the design one more time to achieve a refined model by not considering the non-significant factors.

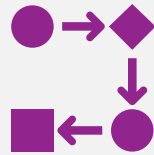
MODEL SUMMARY FOR THE CENTER POINTS IN THE DESIGN

Model Summary for Transformed Response

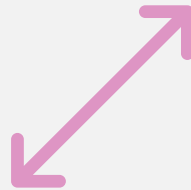
S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
0.0137106	99.23%	96.78%	0.789476	0.00%	105.10	-103.26

- The model summary for center points indicates a higher R-sq touching 99.23% and higher R-sq(adj) at 96.78%.

REFINED MODEL WITH CENTER POINTS



From the half normal plot from the previous slide, we can observe that some factors are not significant, and we run again our design with refined data.



After running the design, we tend to observe that the error has significantly decreased, and we achieve our desired factorial design.

EXPERIMENTAL RESULTS IN MINITAB FOR THE REFINED MODEL HAVING CENTER POINTS

Factorial Regression: yield (pa) versus Temp, salt concentration, type, Center...

Method

Box-Cox transformation

Rounded λ 0
Estimated λ -0.0856129
95% CI for λ (-0.172113, 0.000887110)

Coded Coefficients for Transformed Response

Term	Effect	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant		11.331	0.124	(11.067, 11.595)	91.45	0.000	
Temp	-2.313	-1.157	0.124	(-1.421, -0.893)	-9.34	0.000	1.00
salt concentration	-1.460	-0.730	0.124	(-0.994, -0.466)	-5.89	0.000	1.00
type	4.261	2.130	0.106	(1.905, 2.356)	20.16	0.000	1.00
Temp*type	-1.359	-0.679	0.124	(-0.943, -0.415)	-5.48	0.000	1.00
salt concentration*type	0.659	0.330	0.124	(0.066, 0.594)	2.66	0.018	1.00
Ct Pt		-0.203	0.237	(-0.709, 0.303)	-0.86	0.406	1.00

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
0.495610	97.42%	96.39%	8.09020	94.33%	50.20	47.85

EXPERIMENTAL RESULTS FOR REFINED MODEL

Analysis of Variance for Transformed Response

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Model	6	139.088	97.42%	139.088	23.1813	94.38	0.000
Linear	3	129.788	90.91%	129.788	43.2625	176.13	0.000
Temp	1	21.409	14.99%	21.409	21.4087	87.16	0.000
salt concentration	1	8.530	5.97%	8.530	8.5304	34.73	0.000
type	1	99.849	69.94%	99.849	99.8486	406.50	0.000
2-Way Interactions	2	9.121	6.39%	9.121	4.5603	18.57	0.000
Temp*type	1	7.382	5.17%	7.382	7.3821	30.05	0.000
salt concentration*type	1	1.739	1.22%	1.739	1.7385	7.08	0.018
Curvature	1	0.180	0.13%	0.180	0.1797	0.73	0.406
Error	15	3.684	2.58%	3.684	0.2456		
Lack-of-Fit	11	3.522	2.47%	3.522	0.3201	7.87	0.031
Pure Error	4	0.163	0.11%	0.163	0.0407		
Total	21	142.772	100.00%				

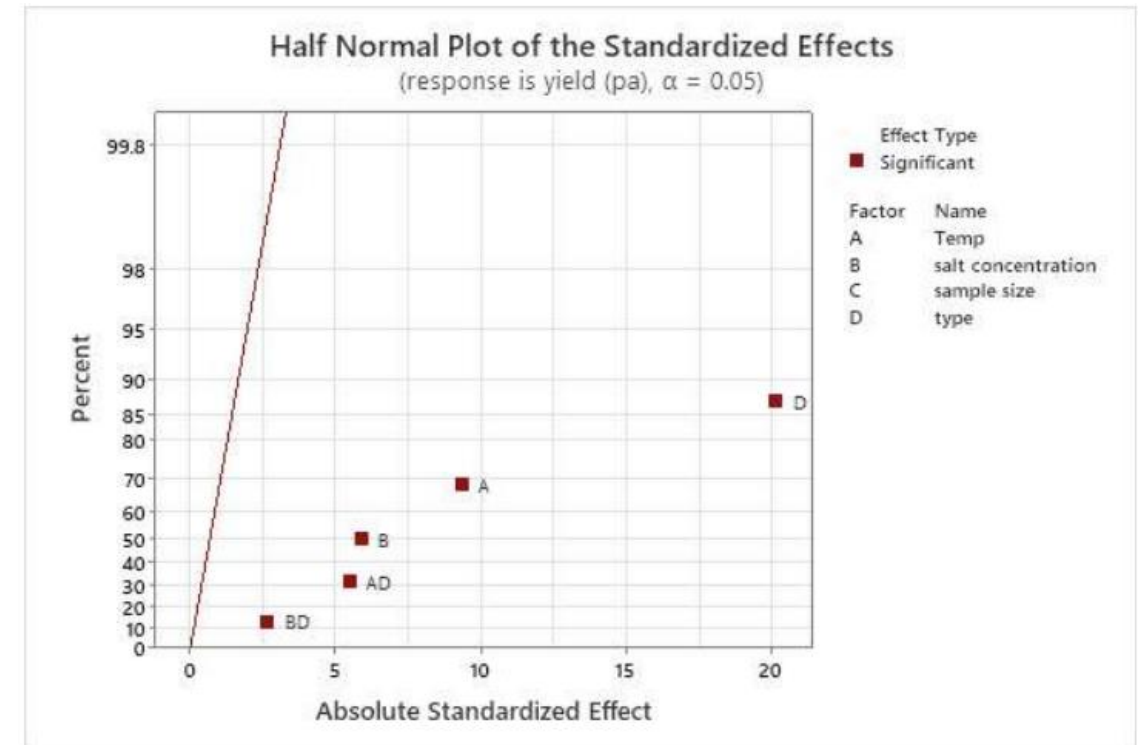
Regression Equation in Uncoded Units

$$\ln(\text{yield (pa)}) = 15.318 - 0.05784 \text{ Temp} - 7.30 \text{ salt concentration} + 3.334 \text{ type} \\ - 0.03396 \text{ Temp*type} + 3.30 \text{ salt concentration*type} - 0.203 \text{ Ct Pt}$$

EXPERIMENTAL RESULTS FOR REFINED MODEL

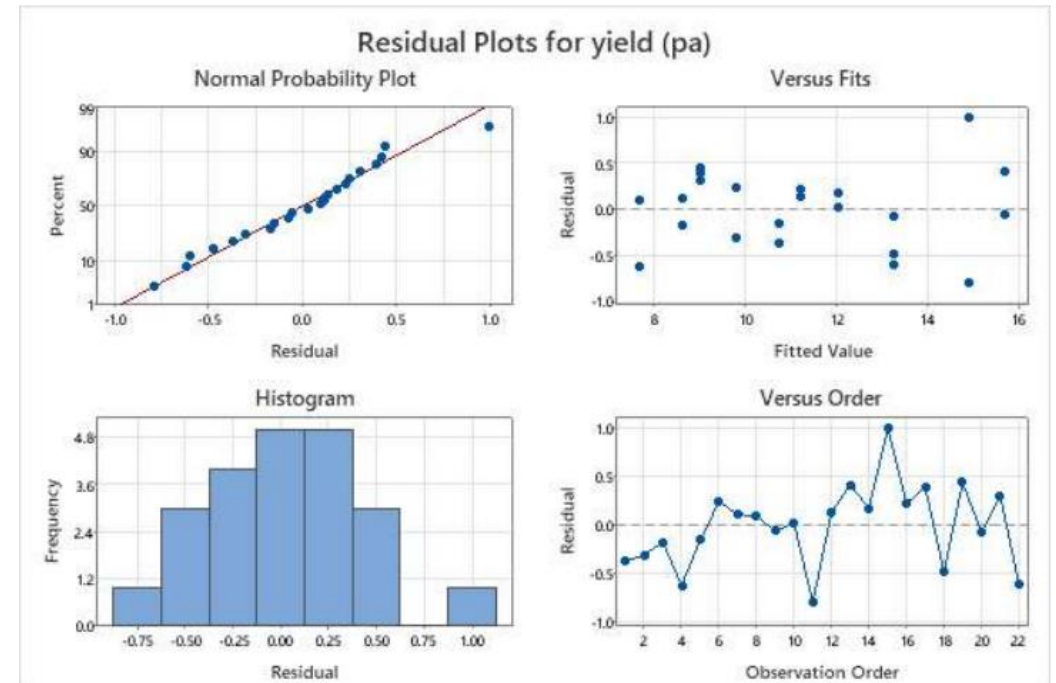
The half-normal plot of the standardized effects:

- From the half-normal plot above, we can observe that all factors are significant indicated in red on the plot.
- Now as all the factors A, B, C, and D are significant we have achieved the desired design model.



EXPERIMENTAL RESULTS FOR REFINED MODEL

- In a 2^4 factorial design, a refined model is a regression model that includes only the significant main effects and interaction terms identified from the full factorial model.
- This provides confidence in the conclusions drawn from the analysis and the ability to make accurate predictions about the response variable based on the factors included in the refined model.



MODEL SUMMARY FOR THE REFINED MODEL

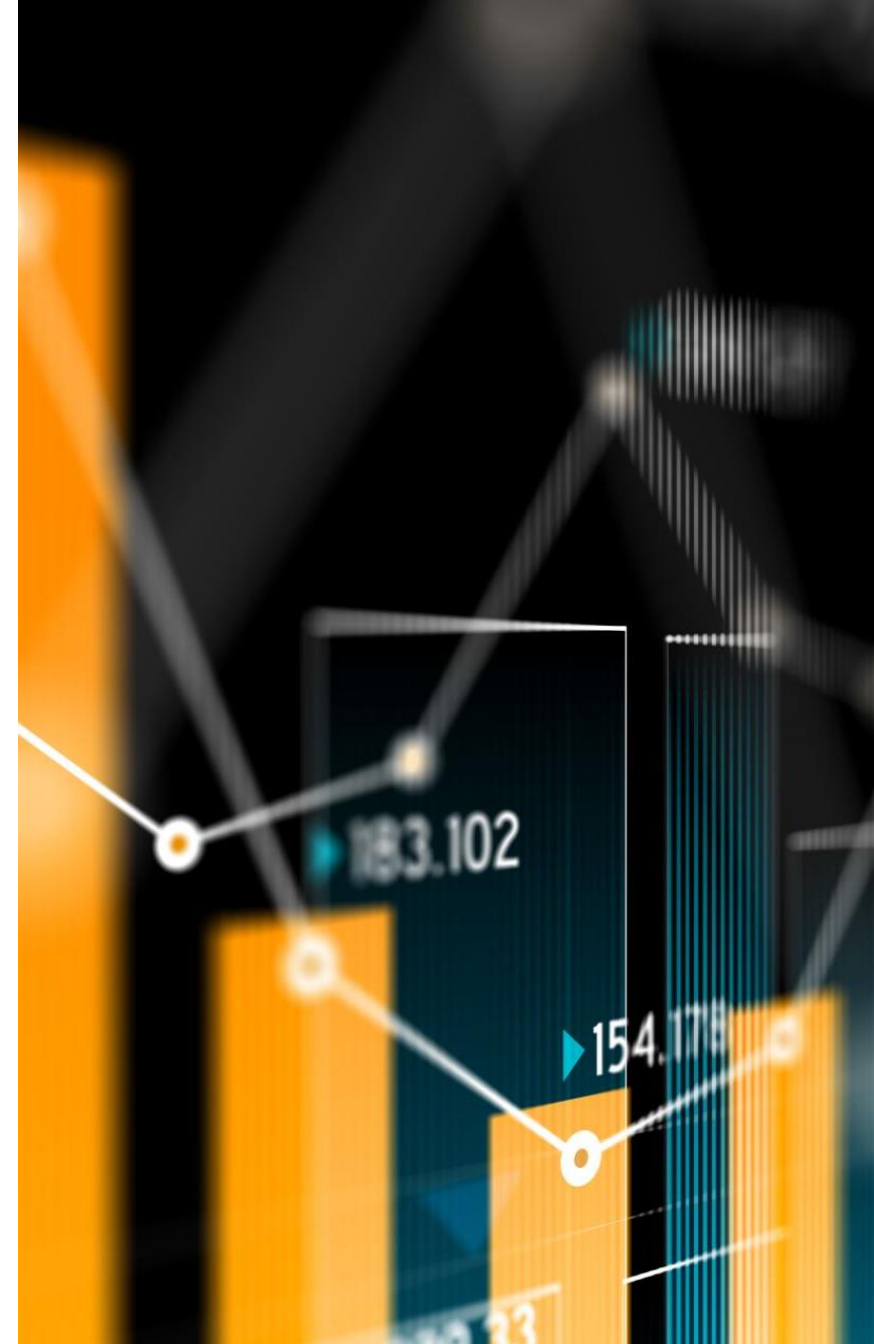
Model Summary for Transformed Response

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
0.495610	97.42%	96.39%	8.09020	94.33%	50.20	47.85

- The model summary for center points indicates a higher R-sq touching 97.42% and higher R-sq(adj) at 96.39%.

RESPONSE SURFACE METHODOLOGY

- The objective of RSM is process optimization.
- Process optimization in RSM includes predicting response and exploring response space by varying the important factors such as temperature, sample size, salt concentration, and type of material.
- In RSM the focus is mainly on problems with quantitative factors.
- The primary focus of two-level factorial designs is process characterization by screening and estimation of the effects of factors. The main objective is to focus on treatment comparison where factors are modeled as qualitative.



RESPONSE SURFACE

Coded Coefficients for Transformed Response

Term	Coef	SE Coef	95% CI	T-Value	P-Value
Constant	-0.26256	0.00519	(-0.27413, -0.25099)	-50.55	0.000
Temp	-0.03143	0.00318	(-0.03851, -0.02434)	-9.88	0.000
salt concentration	-0.02644	0.00318	(-0.03353, -0.01936)	-8.31	0.000
sample size	0.00786	0.00318	(0.00077, 0.01495)	2.47	0.033
type					
rubber	-0.06569	0.00271	(-0.07173, -0.05965)	-24.22	0.000
Temp*Temp	-0.00194	0.00609	(-0.01551, 0.01162)	-0.32	0.756
Temp*salt concentration	-0.00411	0.00318	(-0.01119, 0.00298)	-1.29	0.226
Temp*sample size	-0.00002	0.00318	(-0.00711, 0.00706)	-0.01	0.994
Temp*type					
rubber	0.01123	0.00318	(0.00415, 0.01832)	3.53	0.005
salt concentration*sample size	0.00279	0.00318	(-0.00429, 0.00988)	0.88	0.401
salt concentration*type					
rubber	-0.01690	0.00318	(-0.02399, -0.00982)	-5.32	0.000
sample size*type					
rubber	0.00158	0.00318	(-0.00551, 0.00866)	0.50	0.631

Analysis of Variance for Transformed Response

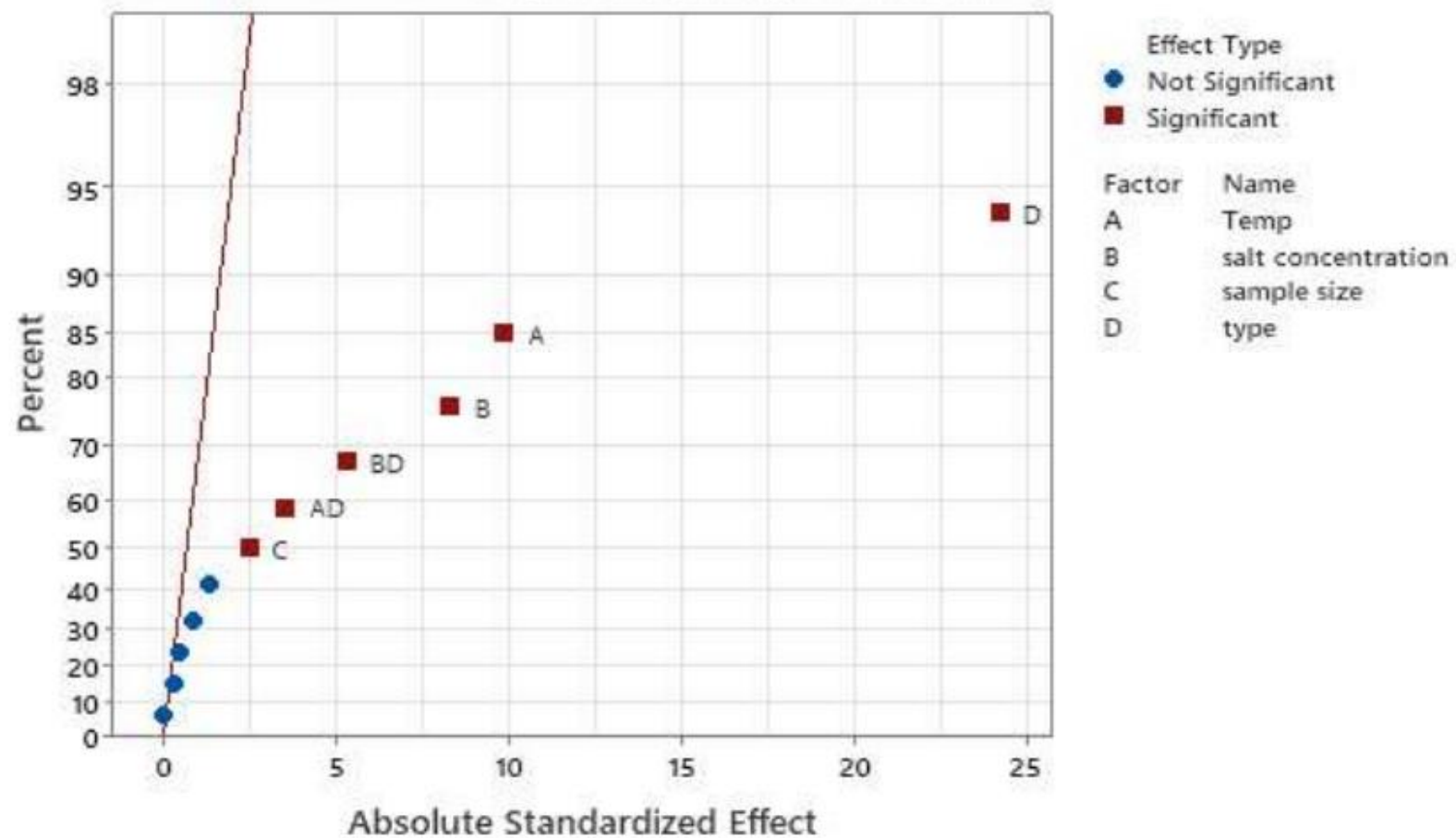
Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value
Model	11	0.129950	98.77%	0.129950	0.011814	73.00
Linear	4	0.122908	93.42%	0.122908	0.030727	189.86
Temp	1	0.015801	12.01%	0.015801	0.015801	97.64
salt concentration	1	0.011187	8.50%	0.011187	0.011187	69.13
sample size	1	0.000988	0.75%	0.000988	0.000988	6.11
type	1	0.094931	72.15%	0.094931	0.094931	586.58
Square	1	0.000017	0.01%	0.000017	0.000017	0.10
Temp*Temp	1	0.000017	0.01%	0.000017	0.000017	0.10
2-Way Interaction	6	0.007026	5.34%	0.007026	0.001171	7.24
Temp*salt concentration	1	0.000270	0.21%	0.000270	0.000270	1.67
Temp*sample size	1	0.000000	0.00%	0.000000	0.000000	0.00
Temp*type	1	0.002019	1.53%	0.002019	0.002019	12.48
salt concentration*sample size	1	0.000125	0.09%	0.000125	0.000125	0.77
salt concentration*type	1	0.004572	3.48%	0.004572	0.004572	28.25
sample size*type	1	0.000040	0.03%	0.000040	0.000040	0.25
Error	10	0.001618	1.23%	0.001618	0.000162	
Lack-of-Fit	6	0.001507	1.15%	0.001507	0.000251	9.05
Pure Error	4	0.000111	0.08%	0.000111	0.000028	
Total	21	0.131569	100.00%			

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
0.0127215	98.77%	97.42%	0.0090512	93.12%	-75.45	-106.77

Half Normal Plot of the Standardized Effects

(response is yield (pa), $\alpha = 0.05$)



REFINED RESPONSE SURFACE

Coded Coefficients for Transformed Response

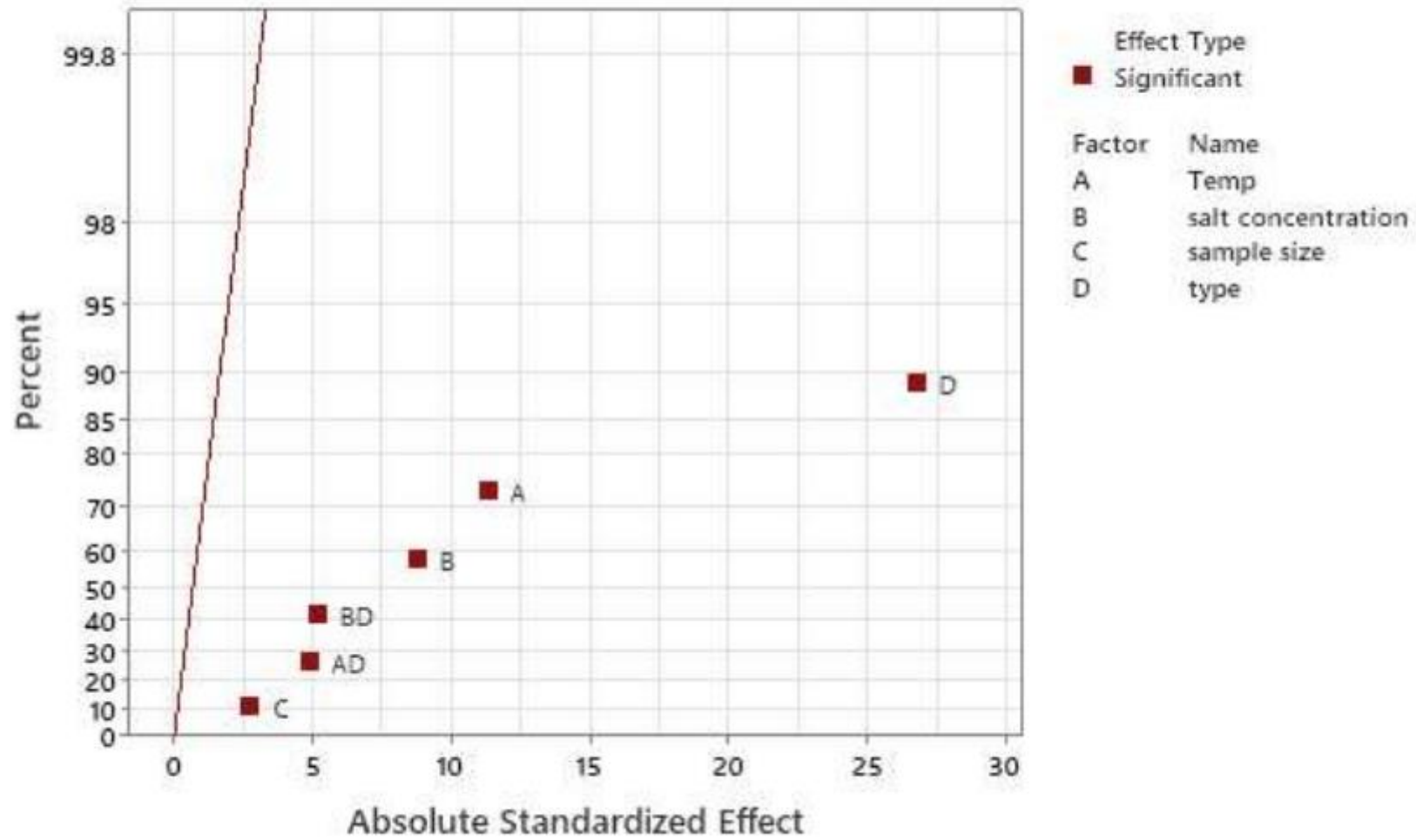
Term	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant	-0.38912	0.00258	(-0.39462, -0.38362)	-150.77	0.000	
Temp	-0.03421	0.00303	(-0.04066, -0.02776)	-11.30	0.000	1.00
salt concentration	-0.02649	0.00303	(-0.03294, -0.02003)	-8.75	0.000	1.00
sample size	0.00829	0.00303	(0.00184, 0.01474)	2.74	0.015	1.00
type						
rubber	-0.06913	0.00258	(-0.07463, -0.06363)	-26.79	0.000	1.00
Temp*type						
rubber	0.01480	0.00303	(0.00835, 0.02125)	4.89	0.000	1.00
salt concentration*type						
rubber	-0.01563	0.00303	(-0.02208, -0.00918)	-5.16	0.000	1.00

Analysis of Variance for Transformed Response

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Model	6	0.143607	98.49%	0.143607	0.023935	163.33	0.000
Linear	4	0.136194	93.41%	0.136194	0.034048	232.34	0.000
Temp	1	0.018724	12.84%	0.018724	0.018724	127.77	0.000
salt concentration	1	0.011223	7.70%	0.011223	0.011223	76.59	0.000
sample size	1	0.001100	0.75%	0.001100	0.001100	7.51	0.015
type	1	0.105145	72.11%	0.105145	0.105145	717.50	0.000
2-Way Interaction	2	0.007414	5.08%	0.007414	0.003707	25.30	0.000
Temp*type	1	0.003505	2.40%	0.003505	0.003505	23.92	0.000
salt concentration*type	1	0.003909	2.68%	0.003909	0.003909	26.68	0.000
Error	15	0.002198	1.51%	0.002198	0.000147		
Lack-of-Fit	11	0.002062	1.41%	0.002062	0.000187	5.50	0.057
Pure Error	4	0.000136	0.09%	0.000136	0.000034		
Total	21	0.145806	100.00%				

Half Normal Plot of the Standardized Effects

(response is yield (pa), $\alpha = 0.05$)



MODEL SUMMARY FOR THE REFINED RESPONSE SURFACE

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
0.0127215	98.77%	97.42%	0.0090512	93.12%	-75.45	-106.77

- The model summary for center points indicates a higher R-sq touching 98.77% and a higher R-sq(adj) at 97.42%.

STEEPEST ASCENT

- We have 2 different type so will assume rubber is type 1 =0 and glass is type 2 = 1
- From the first-order regression model we can get the coeff

Type 1 = 0

- $Y = 11.331 - 1.157 \cdot \text{Temp} - 0.73 \cdot \text{salt}$

Type 2 =1

$$Y = 11.331 - (1.157 + 0.679 \cdot \text{type}) \cdot \text{Temp} - (0.730 - 0.330 \cdot \text{Type}) \cdot \text{salt} + 2.130 \cdot \text{Type}$$

Coded Coefficients for Transformed Response

Term	Effect	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant		11.331	0.124	(11.067, 11.595)	91.45	0.000	
Temp	-2.313	-1.157	0.124	(-1.421, -0.893)	-9.34	0.000	1.00
salt concentration	-1.460	-0.730	0.124	(-0.994, -0.466)	-5.89	0.000	1.00
type	4.261	2.130	0.106	(1.905, 2.356)	20.16	0.000	1.00
Temp*type	-1.359	-0.679	0.124	(-0.943, -0.415)	-5.48	0.000	1.00
salt concentration*type	0.659	0.330	0.124	(0.066, 0.594)	2.66	0.018	1.00
Ct Pt		-0.203	0.237	(-0.709, 0.303)	-0.86	0.406	1.00

STEEPEST ASCENT OF RUBBER

$$Y = 11.331 - (1.157 + 0.679 \cdot \text{type}) \cdot \text{Temp} - (0.730 - 0.330 \cdot \text{Type}) \cdot \text{salt} + 2.130 \cdot \text{Type}$$

R1	20
C1	50
R2	0.1
C2	0.15
B2/b1	0.2178

$$X2 = (b2/b1) \cdot x1$$

step	x1(temp)	x2(salt)	v1(temp)	v2(salt)	yield (pa)
origin	0	0	50	0.15	
delta	1	0.2178649	20	0.0217865	
origin+delta	1	0.2178649	70	0.1717865	11.537854
origin+2delta	2	0.4357298	90	0.193573	9.6147081
origin+3delta	3	0.6535948	110	0.2153595	7.6915621
origin+4delta	4	0.8714597	130	0.237146	5.7684161
origin+5delta	5	1.0893246	150	0.2589325	3.8452702
origin+6delta	6	1.3071895	170	0.280719	1.9221242
origin+7delta	7	1.5250545	190	0.3025054	-0.001022
origin+delta	8	1.7429194	210	0.3242919	-1.924168
origin+delta	9	1.9607843	230	0.3460784	-3.847314

STEEPEST ASCENT OF GLASS

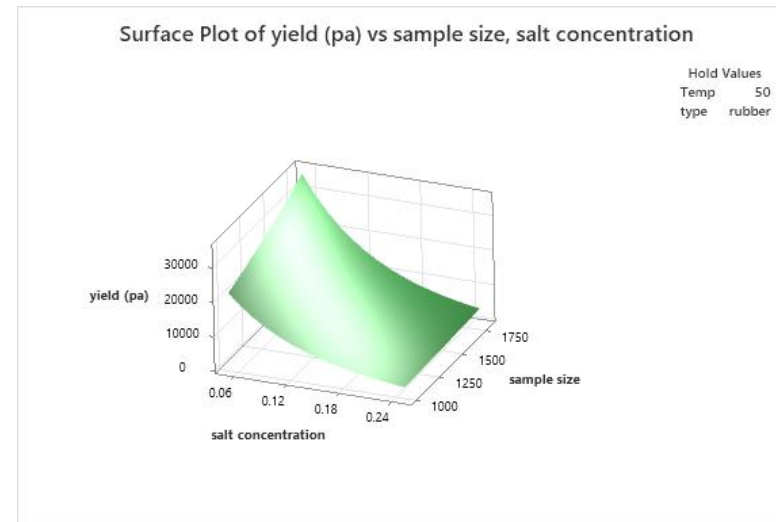
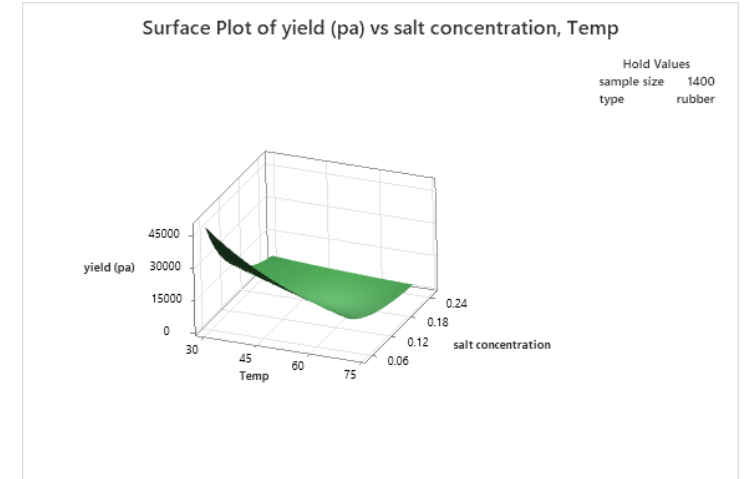
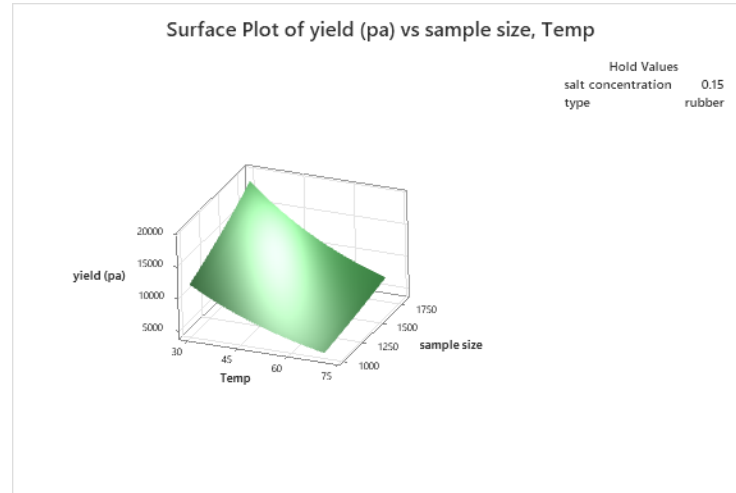
$$Y = 11.331 - 1.157 * \text{Temp} - 0.73 * \text{salt}$$

R1	20
C1	50
R2	0.1
C2	0.15
B2/b1	0.6309

$$X2 = (b2/b1) * x1$$

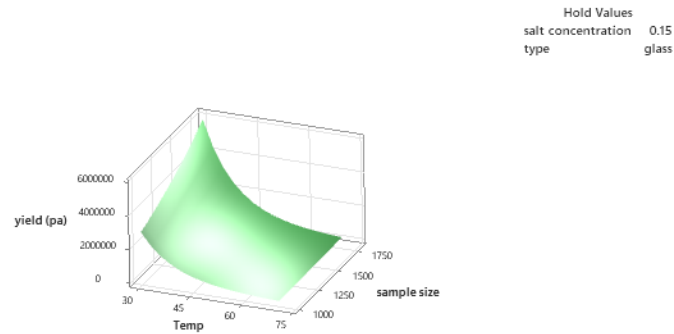
step	x1(temp)	x2(salt)	v1(temp)	v2(salt)	yield (pa)
origin	0	0	50	0.15	
delta	1	0.6309421	20	0.0630942	
origin+delta	1	0.6309421	70	0.2130942	9.7134123
origin+2delta	2	1.2618842	90	0.2761884	8.0958245
origin+3delta	3	1.8928263	110	0.3392826	6.4782368
origin+4delta	4	2.5237684	130	0.4023768	4.8606491
origin+5delta	5	3.1547105	150	0.465471	3.2430614
origin+6delta	6	3.7856525	170	0.5285653	1.6254736
origin+7delta	7	4.4165946	190	0.5916595	0.0078859
origin+delta	8	5.0475367	210	0.6547537	-1.609702
origin+delta	9	5.6784788	230	0.7178479	-3.22729

SURFACE PLOT OF RUBBER

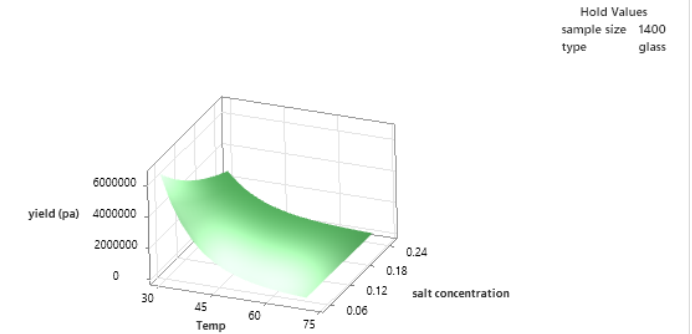


SURFACE PLOT OF GLASS

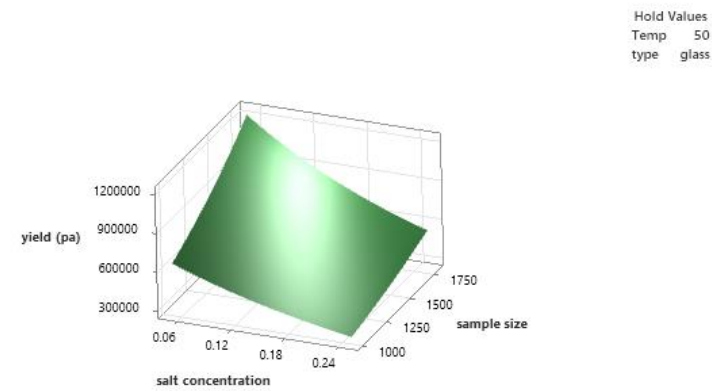
Surface Plot of yield (pa) vs sample size, Temp



Surface Plot of yield (pa) vs salt concentration, Temp



Surface Plot of yield (pa) vs sample size, salt concentration



EXPERIMENTS WITH RANDOM FACTORS

Variance Components, using Adjusted SS

Source	Variance	% of Total	StDev	% of Total
Temp	1.32122	11.51%	1.14944	33.93%
type	8.96939	78.16%	2.99489	88.41%
Error	1.18530	10.33%	1.08872	32.14%
Total	11.4759		3.38761	

Model Summary for Transformed Response

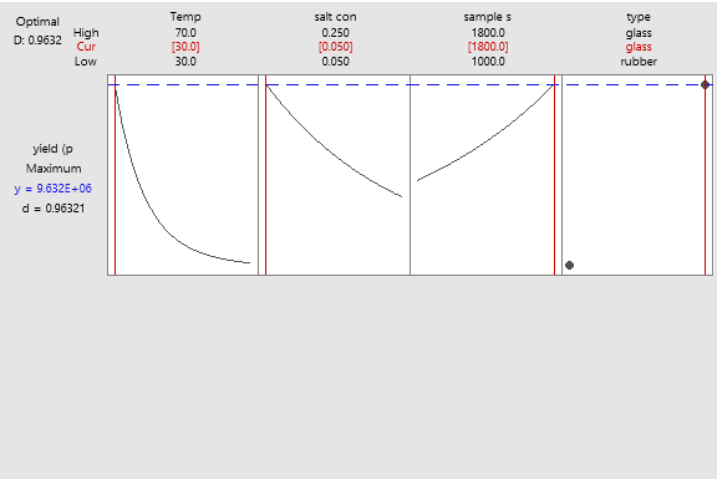
S	R-sq	R-sq(adj)	R-sq(pred)
1.08872	85.06%	82.57%	78.17%

RESPONSE
OPTIMIZATION
OF REFINED
MODEL OF
RSM

Multiple Response Prediction

Variable	Setting
Temp	30
salt concentration	0.05
sample size	1800
type	glass

Response	Fit	95% CI	95% PI
yield (pa)	9632171	(4617788, 21110928)	(2532542, 43557179)



Solution

Solution	salt				yield (pa)	Composite
	Temp	concentration	sample size	type	Fit	Desirability
1	30	0.05	1800	glass	9632171	0.963213

CONCLUSION

We conclude from the all model design that the sample size is insignificant factor and the most important factor are the type of polymer (chemical structure of it)

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
* 100.00%		*	*	*	*	*

original

full two factorial model

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
0.0137106	99.23%	96.78%	0.789476	0.00%	105.10	-103.26

original

full two factorial model with center point

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
0.0127215	98.77%	97.42%	0.0090512	93.12%	-75.45	-106.77

original

Response Surface Design

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
0.481150	98.13%	97.20%	5.92653	95.22%	42.48	33.88

Refined

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
0.495610	97.42%	96.39%	8.09020	94.33%	50.20	47.85

Refined

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
0.0121055	98.49%	97.89%	0.0047478	96.74%	-113.14	-115.48

Refined