式 e ::=定数 プリミティブ演算 $op(e_1,\ldots,e_n)$ if e_1 then e_2 else e_3 条件分岐 $\mathtt{let}\ x = e_1\ \mathtt{in}\ e_2$ 変数定義 変数の読み出し let rec $x y_1 \ldots y_n = e_1$ in e_2 再帰関数定義 関数呼び出し $e e_1 \ldots e_n$ 組の作成 (e_1,\ldots,e_n) $\mathtt{let}\;(x_1,\ldots,x_n)=e_1\;\mathtt{in}\;e_2$ 組の読み出し Array.create e_1 e_2 配列の作成 配列の読み出し $e_1.(e_2)$ $e_1.(e_2) \leftarrow e_3$ 配列への書き込み

図 1: MinCaml の抽象構文(型は省略)

au :::= 型 au au au au au au au 月数型 $au_1 arrow \dots arrow au_n arrow au$ 組型 au au array au 配列型 au au 型変数(型推論用)

図 2: MinCaml の型

```
\Gamma \vdash e_1 : \pi_1 \quad \dots \quad \Gamma \vdash e_n : \pi_n
                                   op は \pi_1,\,\ldots,\,\pi_n 型の値を受け取って \pi 型の値を返すプリミティブ演算
c は\pi型の定数
    \Gamma \vdash c : \pi
                                                                            \Gamma \vdash op(e_1, \ldots, e_n) : \pi
 \Gamma, x: \tau_1 \to \ldots \to \tau_n \to \tau, y_1: \tau_1, \ldots, y_n: \tau_n \vdash e_1: \tau \Gamma \vdash e: \tau_1 \to \ldots \to \tau_n \to \tau
            \frac{\Gamma \vdash e_1 : \tau_1 \quad \dots \quad \Gamma \vdash e_n : \tau_n}{\Gamma \vdash (e_1, \dots, e_n) : \tau_1 \times \dots \times \tau_n} \qquad \frac{\Gamma \vdash e_1 : \tau_1 \times \dots \times \tau_n \quad \Gamma, x_1 : \tau_1, \dots, x_n : \tau_n \vdash e_2 : \tau}{\Gamma \vdash \mathsf{let} \ (x_1, \dots, x_n) = e_1 \ \mathsf{in} \ e_2 : \tau}
  \overline{\Gamma \vdash (e_1, \ldots, e_n) : \tau_1 \times \ldots \times \tau_n}
                                                        \Gamma \vdash e_1 : \mathtt{int} \quad \Gamma \vdash e_2 : \tau
                                                \overline{\Gamma \vdash \texttt{Array.create} \ e_1 \ e_2 : \tau} array
          \Gamma \vdash e_1 : \tau \text{ array } \Gamma \vdash e_2 : \text{int}
                                                                        \Gamma \vdash e_1 : \tau \text{ array } \Gamma \vdash e_2 : \text{int } \Gamma \vdash e_3 : \tau
                                                                                       \Gamma \vdash e_1.(e_2) \leftarrow e_3 : \mathtt{unit}
                       \Gamma \vdash e_1.(e_2) : \tau
                                                    図 3: MinCaml の型つけ規則
```

e ::= c $op(x_1,\ldots,x_n)$ if x=y then e_1 else e_2 if $x\leq y$ then e_1 else e_2 let $x=e_1$ in e_2 xlet $\operatorname{rec} x\ y_1\ \ldots\ y_n=e_1$ in e_2 $x\ y_1\ \ldots\ y_n$ (x_1,\ldots,x_n) let $(x_1,\ldots,x_n)=y$ in e x.(y) $x.(y)\leftarrow z$

```
\mathcal{K}: \mathtt{Syntax.t} 	o \mathtt{KNormal.t}
  \mathcal{K}(c)
  \mathcal{K}(\mathtt{not}(e))
                                                                = \mathcal{K}(\text{if } e \text{ then false else true})
  \mathcal{K}(e_1 = e_2)
                                                                = \mathcal{K}(\text{if }e_1=e_2 \text{ then true else false})
  \mathcal{K}(e_1 \leq e_2)
                                                                = \mathcal{K}(\text{if } e_1 \leq e_2 \text{ then true else false})
                                                                = let x_1 = \mathcal{K}(e_1) in ... let x_n = \mathcal{K}(e_n) in op(x_1, \ldots, x_n)
  \mathcal{K}(op(e_1,\ldots,e_n))
                                                                                                                     op が論理演算・比較以外の場合
  \mathcal{K}(\texttt{if not } e_1 \texttt{ then } e_2 \texttt{ else } e_3)
                                                                = \mathcal{K}(\text{if } e_1 \text{ then } e_3 \text{ else } e_2)
  \mathcal{K}(\text{if } e_1 = e_2 \text{ then } e_3 \text{ else } e_4)
                                                                = let x = \mathcal{K}(e_1) in let y = \mathcal{K}(e_2) in
                                                                       if x = y then \mathcal{K}(e_3) else \mathcal{K}(e_4)
                                                                = let x = \mathcal{K}(e_1) in let y = \mathcal{K}(e_2) in
  \mathcal{K}(\text{if } e_1 \leq e_2 \text{ then } e_3 \text{ else } e_4)
                                                                       if x \leq y then \mathcal{K}(e_3) else \mathcal{K}(e_4)
  \mathcal{K}(\texttt{if }e_1 \texttt{ then }e_2 \texttt{ else }e_3)
                                                                = \mathcal{K}(\text{if }e_1=\text{false then }e_3\text{ else }e_2)
                                                                                                                      e<sub>1</sub> が論理演算・比較以外の場合
  \mathcal{K}(\text{let } x = e_1 \text{ in } e_2)
                                                                = let x = \mathcal{K}(e_1) in \mathcal{K}(e_2)
  \mathcal{K}(x)
  \mathcal{K}(\text{let rec }x\;y_1\;\ldots\;y_n=e_1\;\text{in}\;e_2) \;\;=\;\; \text{let rec }x\;y_1\;\ldots\;y_n=\mathcal{K}(e_1)\;\text{in}\;\mathcal{K}(e_2)
                                                               = let x=\mathcal{K}(e) in let y_1=\mathcal{K}(e_1) in \dots let y_n=\mathcal{K}(e_n) in
  \mathcal{K}(e \ e_1 \ \dots \ e_n)
                                                                       x y_1 \ldots y_n
  \mathcal{K}(e_1,\ldots,e_n)
                                                               = let x_1 = \mathcal{K}(e_1) in ... let x_n = \mathcal{K}(e_n) in (x_1, \ldots, x_n)
                                                               = let y = \mathcal{K}(e_1) in let (x_1, \ldots, x_n) = y in \mathcal{K}(e_2)
  \mathcal{K}(\mathsf{let}\ (x_1,\ldots,x_n)=e_1\ \mathsf{in}\ e_2)
                                                               = let x = \mathcal{K}(e_1) in let y = \mathcal{K}(e_2) in create_array x \ y
  \mathcal{K}(\texttt{Array.create}\;e_1\;e_2)
                                                                = let x = \mathcal{K}(e_1) in let y = \mathcal{K}(e_2) in x.(y)
  \mathcal{K}(e_1.(e_2))
                                                                = let x=\mathcal{K}(e_1) in let y=\mathcal{K}(e_2) in let z=\mathcal{K}(e_3) in
  \mathcal{K}(e_1.(e_2) \leftarrow e_3)
                                                                       x.(y) \leftarrow z
```

図 5: K 正規化 (論理値の整数化と、insert_let による最適化は省略)。右辺に出現していて左辺に出現していない変数は、すべて新しい (fresh) とする。

```
\alpha: \mathtt{Id.t} \ \mathtt{M.t} \to \mathtt{KNormal.t} \to \mathtt{KNormal.t}
         \alpha_{\varepsilon}(c)
         \alpha_{\varepsilon}(op(x_1,\ldots,x_n))
                                                                                              = op(\varepsilon(x_1), \ldots, \varepsilon(x_n))
         \alpha_{\varepsilon}(\text{if } x = y \text{ then } e_1 \text{ else } e_2)
                                                                                              = if \varepsilon(x) = \varepsilon(y) then \alpha_{\varepsilon}(e_1) else \alpha_{\varepsilon}(e_2)
         \alpha_{\varepsilon}(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)
                                                                                             = if \varepsilon(x) \leq \varepsilon(y) then \alpha_{\varepsilon}(e_1) else \alpha_{\varepsilon}(e_2)
                                                                                              = let x'=lpha_{arepsilon}(e_1) in lpha_{arepsilon,x\mapsto x'}(e_2)
         \alpha_{\varepsilon}(\text{let } x = e_1 \text{ in } e_2)
                                                                                              = \varepsilon(x)
        \alpha_{\varepsilon}(x)
         \alpha_{\varepsilon}(\text{let rec }x\;y_1\;\ldots\;y_n=e_1\;\text{in }e_2) \;\;=\;\; \text{let rec }x'\;y_1'\;\ldots\;y_n'=\alpha_{\varepsilon,x\mapsto x',y_1\mapsto y_1',\ldots,y_n\mapsto y_n'}(e_1)\;\text{in }
                                                                                                      \alpha_{\varepsilon,x\mapsto x'}(e_2)
                                                                                              = \varepsilon(x) \varepsilon(y_1) \ldots \varepsilon(y_n)
         \alpha_{\varepsilon}(x \ y_1 \ \dots \ y_n)
         \alpha_{\varepsilon}((x_1,\ldots,x_n))
                                                                                           = (\varepsilon(x_1), \ldots, \varepsilon(x_n))
         \alpha_{arepsilon}(	exttt{let}\ (x_1,\ldots,x_n)=y\ 	exttt{in}\ e)
                                                                                         = let (x'_1,\ldots,x'_n)=\varepsilon(y) in \alpha_{\varepsilon,x_1\mapsto x'_1,\ldots,x_n\mapsto x'_n}(e)
                                                                                              = \varepsilon(x).(\varepsilon(y))
         \alpha_{\varepsilon}(x.(y))
         \alpha_{\varepsilon}(x.(y) \leftarrow z)
                                                                                              = \varepsilon(x).(\varepsilon(y)) \leftarrow \varepsilon(z)
```

図 6: α 変換。 ε は α 変換前の変数を受け取って、 α 変換後の変数を返す写像。右辺に出現していて左辺に出現していない変数(x' など)は、すべて fresh とする。

```
\beta: \mathtt{Id.t} \ \mathtt{M.t} \rightarrow \mathtt{KNormal.t} \rightarrow \mathtt{KNormal.t}
       \beta_{\varepsilon}(c)
       \beta_{\varepsilon}(op(x_1,\ldots,x_n))
                                                                                      = op(\varepsilon(x_1), \ldots, \varepsilon(x_n))
       \beta_{\varepsilon}(\text{if } x=y \text{ then } e_1 \text{ else } e_2)
                                                                                   = if \varepsilon(x)=arepsilon(y) then eta_{arepsilon}(e_1) else eta_{arepsilon}(e_2)
       \beta_{\varepsilon}(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)
                                                                                     = if \varepsilon(x) \leq \varepsilon(y) then \beta_{\varepsilon}(e_1) else \beta_{\varepsilon}(e_2)
       \beta_{\varepsilon}(\text{let } x = e_1 \text{ in } e_2)
                                                                                      = \beta_{\varepsilon,x\mapsto y}(e_2)
                                                                                                                                                              \beta_{\varepsilon}(e_1) が変数 y の場合
                                                                                      = let x=eta_{arepsilon}(e_1) in eta_{arepsilon}(e_2) eta_{arepsilon}(e_1) が変数でない場合
       \beta_{\varepsilon}(let x=e_1 in e_2)
                                                                                      = \varepsilon(x)
       \beta_{\varepsilon}(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \ \text{in} \ e_2) = \text{let rec } x \ y_1 \ \dots \ y_n = \beta_{\varepsilon}(e_1) \ \text{in} \ \beta_{\varepsilon}(e_2)
       \beta_{\varepsilon}(x \ y_1 \ \dots \ y_n)
                                                                                    = \varepsilon(x) \varepsilon(y_1) \ldots \varepsilon(y_n)
       \beta_{\varepsilon}((x_1,\ldots,x_n))
                                                                                      = (\varepsilon(x_1), \ldots, \varepsilon(x_n))
       \beta_{\varepsilon}(let (x_1,\ldots,x_n)=y in e)
                                                                                   = let (x_1,\ldots,x_n)=\varepsilon(y) in \beta_{\varepsilon}(e)
       \beta_{\varepsilon}(x.(y))
                                                                                      = \varepsilon(x).(\varepsilon(y))
                                                                                      = \varepsilon(x).(\varepsilon(y)) \leftarrow \varepsilon(z)
       \beta_{\varepsilon}(x.(y) \leftarrow z)
```

図 7: β 簡約。 ε は β 簡約前の変数を受け取って、 β 簡約後の変数を返す写像。 $\varepsilon(x)$ が定義されていない場合は、 $\varepsilon(x)=x$ とみなす。

```
\mathcal{A}: 	exttt{KNormal.t} 
ightarrow 	exttt{KNormal.t}
                 \mathcal{A}(c)
                                                                                = c
                 \mathcal{A}(op(x_1,\ldots,x_n))
                                                                                = op(x_1,\ldots,x_n)
                 \mathcal{A}(\texttt{if } x = y \texttt{ then } e_1 \texttt{ else } e_2)
                                                                                = if x=y then \mathcal{A}(e_1) else \mathcal{A}(e_2)
                                                                                = if x \leq y then \mathcal{A}(e_1) else \mathcal{A}(e_2)
                 \mathcal{A}(\texttt{if } x \leq y \texttt{ then } e_1 \texttt{ else } e_2)
                 \mathcal{A}(\texttt{let}\ x = e_1\ \texttt{in}\ e_2)
                                                                                = \ \ \texttt{let} \ \ldots \ \texttt{in let} \ x = e_1' \ \texttt{in} \ \mathcal{A}(e_2)
                                                                                              \mathcal{A}(e_1) = \mathtt{let} \ \ldots \ \mathtt{in} \ e_1' という形で
                                                                                              (let ... in は 0 個以上の let の列)
                                                                                               e_1' は let でない
                 \mathcal{A}(x)
                                                                                = x
                 \mathcal{A}(\text{let rec }x\;y_1\;\ldots\;y_n=e_1\;\text{in }e_2) \;\;=\;\; \text{let rec }x\;y_1\;\ldots\;y_n=\mathcal{A}(e_1)\;\text{in }\mathcal{A}(e_2)
                 \mathcal{A}(x \ y_1 \ \dots \ y_n)
                                                                                = x y_1 \dots y_n
                 \mathcal{A}((x_1,\ldots,x_n))
                                                                                = (x_1,\ldots,x_n)
                                                                               = let (x_1,\ldots,x_n)=y in \mathcal{A}(e)
                 \mathcal{A}(\mathsf{let}\ (x_1,\ldots,x_n)=y\ \mathsf{in}\ e)
                 \mathcal{A}(x.(y))
                                                                                = x.(y)
                 \mathcal{A}(x.(y) \leftarrow z)
                                                                                = x.(y) \leftarrow z
                                                                図 8: ネストした let の簡約
```

```
\mathcal{I}: (\mathtt{Id.t\ list} \times \mathtt{KNormal.t})\ \mathtt{M.t} \rightarrow \mathtt{KNormal.t} \rightarrow \mathtt{KNormal.t}
      \mathcal{I}_{\varepsilon}(c)
      \mathcal{I}_{\varepsilon}(op(x_1,\ldots,x_n))
                                                                           = op(x_1,\ldots,x_n)
      \mathcal{I}_{\varepsilon}(\text{if } x=y \text{ then } e_1 \text{ else } e_2)
                                                                           = if x=y then \mathcal{I}_{arepsilon}(e_1) else \mathcal{I}_{arepsilon}(e_2)
      \mathcal{I}_{\varepsilon}(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)
                                                                        = if x \leq y then \mathcal{I}_{arepsilon}(e_1) else \mathcal{I}_{arepsilon}(e_2)
      \mathcal{I}_{\varepsilon}(let x=e_1 in e_2)
                                                                           = let x = \mathcal{I}_{\varepsilon}(e_1) in \mathcal{I}_{\varepsilon}(e_2)
      \mathcal{I}_{\varepsilon}(x)
      \mathcal{I}_{\varepsilon}(\text{let rec }x\;y_1\;\ldots\;y_n=e_1\;\text{in }e_2) = \varepsilon'=\varepsilon, x\mapsto ((y_1,\ldots,y_n),e_1)\; \succeq \mathsf{LT}
                                                                                  let rec x \ y_1 \ \dots \ y_n = \mathcal{I}_{\varepsilon'}(e_1) in \mathcal{I}_{\varepsilon'}(e_2)
                                                                                                                                            size(e_1) \leq th の場合
      \mathcal{I}_{\varepsilon}(\texttt{let rec}\ x\ y_1\ \dots\ y_n = e_1\ \texttt{in}\ e_2) \quad = \quad \texttt{let rec}\ x\ y_1\ \dots\ y_n = \mathcal{I}_{\varepsilon}(e_1)\ \texttt{in}\ \mathcal{I}_{\varepsilon}(e_2)
                                                                                                                                            size(e_1) > th の場合
                                                                           = \alpha_{y_1\mapsto z_1,\dots,y_n\mapsto z_n}(e) \varepsilon(x)=((z_1,\dots,z_n),e) の場合
      \mathcal{I}_{\varepsilon}(x \ y_1 \ \dots \ y_n)
      \mathcal{I}_{\varepsilon}(x \ y_1 \ \dots \ y_n)
                                                                           = x y_1 \dots y_n
                                                                                                                           \varepsilon(x) が定義されていない場合
      \mathcal{I}_{\varepsilon}((x_1,\ldots,x_n))
                                                                           = (x_1,\ldots,x_n)
      \mathcal{I}_{\varepsilon}(let (x_1,\ldots,x_n)=y in e)
                                                                          = let (x_1,\ldots,x_n)=y in \mathcal{I}_{\varepsilon}(e)
      \mathcal{I}_{\varepsilon}(x.(y))
                                                                           = x.(y)
      \mathcal{I}_{\varepsilon}(x.(y) \leftarrow z)
                                                                           = x.(y) \leftarrow z
       size(c)
       size(op(x_1,\ldots,x_n))
       size(if x = y then e_1 else e_2)
                                                                        = 1 + size(e_1) + size(e_2)
       size(if \ x \leq y \ then \ e_1 \ else \ e_2)
                                                                        = 1 + size(e_1) + size(e_2)
       size(let x = e_1 in e_2)
                                                                           = 1 + size(e_1) + size(e_2)
       size(x)
                                                                            = 1
       size(let rec x y_1 \dots y_n = e_1 \text{ in } e_2) = 1 + size(e_1) + size(e_2)
       size(x y_1 \ldots y_n)
       size((x_1,\ldots,x_n))
                                                                           = 1
       size(let (x_1, \ldots, x_n) = y in e)
                                                                        = 1 + size(e)
       size(x.(y))
                                                                            = 1
       size(x.(y) \leftarrow z)
```

図 9: インライン展開。 ε はサイズの小さい関数名を受け取って、仮引数と本体を返す写像。th はインライン展開する関数の最大サイズ (ユーザ指定)。

```
\mathcal{F}: \mathtt{KNormal.t} \ \mathtt{M.t} 
ightarrow \mathtt{KNormal.t} 
ightarrow \mathtt{KNormal.t}
      \mathcal{F}_{\varepsilon}(c)
      \mathcal{F}_{\varepsilon}(op(x_1,\ldots,x_n))
                                                                                                                                    op(\varepsilon(x_1),\ldots,\varepsilon(x_n))=c の場合
                                                                                 = c
      \mathcal{F}_{\varepsilon}(op(x_1,\ldots,x_n))
                                                                                                                                                                       それ以外の場合
                                                                                 = op(x_1,\ldots,x_n)
      \mathcal{F}_{\varepsilon}(\text{if } x=y \text{ then } e_1 \text{ else } e_2)
                                                                                 = \mathcal{F}_{\varepsilon}(e_1)
                                                                                                                                     \varepsilon(x) と \varepsilon(y) が等しい定数の場合
      \mathcal{F}_{\varepsilon}(\texttt{if } x = y \texttt{ then } e_1 \texttt{ else } e_2)
                                                                                = \mathcal{F}_{\varepsilon}(e_2)
                                                                                                                                     \varepsilon(x) と \varepsilon(y) が異なる定数の場合
      \mathcal{F}_{arepsilon}(	ext{if } x=y 	ext{ then } e_1 	ext{ else } e_2)
                                                                             = if x=y then \mathcal{F}_arepsilon(e_1) else \mathcal{F}_arepsilon(e_2)
                                                                                                                                                                      それ以外の場合
      \mathcal{F}_{arepsilon}(	ext{if } x \leq y 	ext{ then } e_1 	ext{ else } e_2)
                                                                                = \mathcal{F}_{\varepsilon}(e_1) \varepsilon(x) と \varepsilon(y) が定数で、\varepsilon(x) \leq \varepsilon(y) の場合
      \mathcal{F}_{\varepsilon}(\texttt{if } x \leq y \texttt{ then } e_1 \texttt{ else } e_2)
                                                                                = \mathcal{F}_{arepsilon}(e_2) arepsilon(x) と arepsilon(y) が定数で、arepsilon(x)>arepsilon(y) の場合
      \mathcal{F}_{\varepsilon}(if x \leq y then e_1 else e_2)
                                                                                = if x \leq y then \mathcal{F}_{arepsilon}(e_1) else \mathcal{F}_{arepsilon}(e_2)
                                                                                                                                                                     それ以外の場合
      \mathcal{F}_{\varepsilon}(let x=e_1 in e_2)
                                                                                 = e'_1 = \mathcal{F}_{\varepsilon}(e_1) \succeq \mathsf{UT}
                                                                                         let x = e'_1 in \mathcal{F}_{\varepsilon, x \mapsto e'_1}(e_2)
      \mathcal{F}_{\varepsilon}(x)
      \mathcal{F}_{\varepsilon}(\text{let rec }x\;y_1\;\ldots\;y_n=e_1\;\text{in }e_2) \;\;=\;\; \text{let rec }x\;y_1\;\ldots\;y_n=\mathcal{F}_{\varepsilon}(e_1)\;\text{in }\mathcal{F}_{\varepsilon}(e_2)
      \mathcal{F}_{\varepsilon}(x \ y_1 \ \dots \ y_n)
                                                                                 = x y_1 \dots y_n
      \mathcal{F}_{\varepsilon}((x_1,\ldots,x_n))
                                                                                 = (x_1,\ldots,x_n)
                                                                                 = \ \ \text{let} \ x_1 = y_1 \ \text{in} \ \dots \ \text{let} \ x_n = y_n \ \text{in} \ \mathcal{F}_{\varepsilon}(e)
      \mathcal{F}_{\varepsilon}(let (x_1,\ldots,x_n)=y in e)
                                                                                                                                                  \varepsilon(y)=(y_1,\ldots,y_n) の場合
      \mathcal{F}_{\varepsilon}(let (x_1,\ldots,x_n)=y in e)
                                                                                 = let (x_1,\ldots,x_n)=y in \mathcal{F}_{\varepsilon}(e)
      \mathcal{F}_{\varepsilon}(x.(y))
                                                                                 = x.(y)
      \mathcal{F}_{\varepsilon}(x.(y) \leftarrow z)
                                                                                 = x.(y) \leftarrow z
```

図 10: 定数畳み込み。 ε は変数を受け取って、定数を返す写像。

7

```
\mathcal{E}: \mathtt{KNormal.t} 	o \mathtt{KNormal.t}
   \mathcal{E}(c)
                                                         = c
   \mathcal{E}(op(x_1,\ldots,x_n))
                                                         = op(x_1,\ldots,x_n)
   \mathcal{E}(\text{if } x = y \text{ then } e_1 \text{ else } e_2)
                                                         = if x=y then \mathcal{E}(e_1) else \mathcal{E}(e_2)
   \mathcal{E}(\texttt{if } x \leq y \texttt{ then } e_1 \texttt{ else } e_2)
                                                         = if x \leq y then \mathcal{E}(e_1) else \mathcal{E}(e_2)
                                                                            effect(\mathcal{E}(e_1)) = false かつ x \notin FV(\mathcal{E}(e_2)) の場合
   \mathcal{E}(\text{let } x = e_1 \text{ in } e_2)
                                                         = \mathcal{E}(e_2)
   \mathcal{E}(\text{let } x = e_1 \text{ in } e_2)
                                                         = let x = \mathcal{E}(e_1) in \mathcal{E}(e_2)
                                                                                                                              それ以外の場合
   \mathcal{E}(x)
   \mathcal{E}(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \ \text{in} \ e_2) = \mathcal{E}(e_2)
                                                                                                                    x \notin FV(\mathcal{E}(e_2)) の場合
   \mathcal{E}(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \ \text{in} \ e_2) \ = \ \text{let rec} \ x \ y_1 \ \dots \ y_n = \mathcal{E}(e_1) \ \text{in} \ \mathcal{E}(e_2)
                                                                                                                              それ以外の場合
   \mathcal{E}(x \ y_1 \ \dots \ y_n)
                                                         = x y_1 \dots y_n
   \mathcal{E}((x_1,\ldots,x_n))
                                                         = (x_1,\ldots,x_n)
                                                                                                \{x_1,\ldots,x_n\}\cap FV(\mathcal{E}(e))=\emptyset の場合
   \mathcal{E}(\text{let }(x_1,\ldots,x_n)=y \text{ in } e)
                                                       = \mathcal{E}(e)
                                                                                                                              それ以外の場合
   \mathcal{E}(\text{let }(x_1,\ldots,x_n)=y \text{ in } e)
                                                         = let (x_1,\ldots,x_n)=y in \mathcal{E}(e)
   \mathcal{E}(x.(y))
                                                         = x.(y)
   \mathcal{E}(x.(y) \leftarrow z)
                                                         = x.(y) \leftarrow z
effect: {	t KNormal.t} 	o {	t bool}
                           effect(c)
                                                                                       = false
                           effect(op(x_1,\ldots,x_n))
                                                                                        = false
                           effect(if x = y then e_1 else e_2)
                                                                                       = effect(e_1) \vee effect(e_2)
                           effect(if \ x \leq y \ then \ e_1 \ else \ e_2)
                                                                                      = effect(e_1) \vee effect(e_2)
                           effect(let x = e_1 in e_2)
                                                                                       = effect(e_1) \vee effect(e_2)
                           effect(x)
                                                                                       = false
                           effect(let rec x y_1 ... y_n = e_1 in e_2) = effect(e_2)
                           effect(x \ y_1 \ \ldots \ y_n)
                                                                                       = true
                           effect((x_1,\ldots,x_n))
                                                                                       = false
                           effect(let (x_1, \ldots, x_n) = y in e)
                                                                                       = effect(e)
                           effect(x.(y))
                                                                                       = false
                           effect(x.(y) \leftarrow z)
                                                                                        = true
                                                         図 11: 不要定義削除 (1/2)
```

```
FV: \mathtt{KNormal.t} 	o \mathtt{S.t}
           FV(c)
            FV(op(x_1,\ldots,x_n))
                                                             = \{x_1,\ldots,x_n\}
            FV(if x=y then e_1 else e_2)
                                                            = \{x, y\} \cup FV(e_1) \cup FV(e_2)
            FV(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)
                                                          = \{x,y\} \cup FV(e_1) \cup FV(e_2)
            FV(let x = e_1 \text{ in } e_2)
                                                             = FV(e_1) \cup (FV(e_2) \setminus \{x\})
            FV(x)
                                                             = \{x\}
            FV(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \ \text{in} \ e_2) = ((FV(e_1) \setminus \{y_1, \dots, y_n\}) \cup FV(e_2)) \setminus \{x\}
            FV(x y_1 \ldots y_n)
                                                            = \{x, y_1, \dots, y_n\}
            FV((x_1,\ldots,x_n))
                                                            = \{x_1,\ldots,x_n\}
            FV(let (x_1, \ldots, x_n) = y in e)
                                                          = \{y\} \cup (FV(e) \setminus \{x_1, \dots, x_n\})
            FV(x.(y))
                                                             = \{x, y\}
            FV(x.(y) \leftarrow z)
                                                             = \{x, y, z\}
                                                図 12: 不要定義削除 (2/2)
```

```
プログラム全体
P ::=
 (\{D_1,\ldots,D_n\},e)
                                       トップレベル関数定義の集合とメインルーチンの式
D ::=
                                       トップレベル関数定義
                                      関数のラベルと仮引数、自由変数、および本体
 L_x(y_1,\ldots,y_m)(z_1,\ldots,z_n)=e
e ::=
 op(x_1,\ldots,x_n)
 if x = y then e_1 else e_2
 if x \leq y then e_1 else e_2
 let x = e_1 in e_2
 make\_closure \ x = (L_x, (y_1, \ldots, y_n)) in e クロージャ生成
  apply\_closure(x, y_1, \dots, y_n)
                                      クロージャを用いた関数呼び出し
                                      クロージャを用いない関数呼び出し (known function call)
 apply\_direct(L_x, y_1, \dots, y_n)
 (x_1,\ldots,x_n)
 let (x_1,\ldots,x_n)=y in e
 x.(y)
  x.(y) \leftarrow z
                              図 13: クロージャ変換後の構文
```

```
\mathcal{C}: \mathtt{KNormal.t} \rightarrow \mathtt{Closure.t}
          \mathcal{C}(c)
          \mathcal{C}(op(x_1,\ldots,x_n))
                                                              = op(x_1,\ldots,x_n)
          C(\text{if } x = y \text{ then } e_1 \text{ else } e_2)
                                                             = if x = y then C(e_1) else C(e_2)
          C(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)
                                                             = if x \leq y then \mathcal{C}(e_1) else \mathcal{C}(e_2)
          C(let x = e_1 \text{ in } e_2)
                                                              = let x = \mathcal{C}(e_1) in \mathcal{C}(e_2)
          \mathcal{C}(x)
          \mathcal{C}(\mathsf{let}\;\mathsf{rec}\;x\;y_1\;\ldots\;y_n=e_1\;\mathsf{in}\;e_2) = \mathcal{D}に\mathtt{L}_x(y_1,\ldots,y_n)(z_1,\ldots,z_m)=e_1'を加え、
                                                                    make\_closure \ x = (\mathtt{L}_x, (z_1, \ldots, z_m)) in e_2'を返す
                                                                          ただし e'_1 = C(e_1), e'_2 = C(e_2),
                                                                          FV(e'_1) \setminus \{x, y_1, \dots, y_n\} = \{z_1, \dots, z_m\}
          C(x y_1 \ldots y_n)
                                                              = apply\_closure(x, y_1, \dots, y_n)
          \mathcal{C}((x_1,\ldots,x_n))
                                                              = (x_1,\ldots,x_n)
          C(\text{let }(x_1,\ldots,x_n)=y \text{ in } e)
                                                              = let (x_1,\ldots,x_n)=y in \mathcal{C}(e)
          \mathcal{C}(x.(y))
                                                              = x.(y)
                                                              = x.(y) \leftarrow z
          \mathcal{C}(x.(y) \leftarrow z)
FV: \mathtt{Closure.t} 	o \mathtt{S.t}
                FV(c)
                FV(op(x_1,\ldots,x_n))
                                                                                   = \{x_1,\ldots,x_n\}
                                                                                   = \{x,y\} \cup FV(e_1) \cup FV(e_2)
                FV(\text{if } x = y \text{ then } e_1 \text{ else } e_2)
                FV(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)
                                                                                  = \{x,y\} \cup FV(e_1) \cup FV(e_2)
                                                                                   = FV(e_1) \cup (FV(e_2) \setminus \{x\})
                FV(let x = e_1 \text{ in } e_2)
                FV(x)
                                                                                   = \{x\}
                FV(make\_closure \ x = (\mathtt{L}_x, (y_1, \dots, y_n)) \ \text{in} \ e) = \{y_1, \dots, y_n\} \cup (FV(e) \setminus \{x\})
                FV(apply\_closure(x, y_1, \dots, y_n))
                                                                                   = \{x, y_1, \dots, y_n\}
                                                                                   = \{y_1, \dots, y_n\}
                FV(apply\_direct(L_x, y_1, \ldots, y_n))
                FV((x_1,\ldots,x_n))
                                                                                   = \{x_1,\ldots,x_n\}
                FV(let (x_1, \ldots, x_n) = y in e)
                                                                                   = \{y\} \cup (FV(e) \setminus \{x_1, \dots, x_n\})
                FV(x.(y))
                                                                                   = \{x, y\}
                FV(x.(y) \leftarrow z)
                                                                                   = \{x, y, z\}
図 14: 賢くない Closure 変換 \mathcal{C}(e)。 \mathcal{D} はトップレベル関数定義の集合を記憶しておくためのグローバル
```

変数。

```
\mathcal{C}: \mathtt{S.t} 
ightarrow \mathtt{KNormal.t} 
ightarrow \mathtt{Closure.t}
       \mathcal{C}_s(	ext{let rec }x\;y_1\;\ldots\;y_n=e_1\;	ext{in }e_2) = \mathcal{D} に \mathtt{L}_x(y_1,\ldots,y_n)()=e_1' を加え、
                                                                 make\_closure \ x = (L_x, ())  in e_2'を返す
                                                                       ただし e'_1 = C_{s'}(e_1), e'_2 = C_{s'}(e_2), s' = s \cup \{x\},
                                                                       FV(e_1')\setminus\{y_1,\ldots,y_n\}=\emptyset の場合
       \mathcal{C}_s(	ext{let rec } x \ y_1 \ \dots \ y_n = e_1 \ 	ext{in} \ e_2) \ = \ \mathcal{D} \ 	ext{に} \ 	ext{L}_x(y_1, \dots, y_n)(z_1, \dots, z_m) = e_1' を加え、
                                                                 make\_closure \ x = (\mathtt{L}_x, (z_1, \ldots, z_m)) in e_2'を返す
                                                                       ただし e'_1 = C_s(e_1), e'_2 = C_s(e_2),
                                                                       FV(e'_1) \setminus \{y_1, \dots, y_n\} \neq \emptyset,
                                                                       FV(e_1')\setminus \{x,y_1,\ldots,y_n\}=\{z_1,\ldots,z_m\} の場合
       C_s(x y_1 \ldots y_n)
                                                           = apply\_closure(x, y_1, \dots, y_n)
                                                                                                                  x \not \in s の場合
       C_s(x y_1 \ldots y_n)
                                                           = apply\_direct(L_x, y_1, \dots, y_n)
                                                                                                                    x \in s の場合
         図 15: やや賢い Closure 変換 C_s(e)。 s は自由変数がないとわかっている関数の名前の集合。
```

```
\mathcal{C}: \mathtt{S.t} \rightarrow \mathtt{KNormal.t} \rightarrow \mathtt{Closure.t}
      \mathcal{C}_s(	ext{let rec } x \ y_1 \ \dots \ y_n = e_1 \ 	ext{in} \ e_2) \ = \ \mathcal{D} \ 	ext{に} \ 	ext{L}_x(y_1, \dots, y_n)() = e_1' を加え、
                                                                        make\_closure \ x = (L_x, ()) \ in \ e_2'を返す
                                                                               ただし e'_1 = \mathcal{C}_{s'}(e_1), e'_2 = \mathcal{C}_{s'}(e_2), s' = s \cup \{x\},
                                                                               FV(e_1')\setminus\{y_1,\ldots,y_n\}=\emptyset かつ x\in FV(e_2') の場合
      \mathcal{C}_s(	ext{let rec } x \ y_1 \ \dots \ y_n = e_1 \ 	ext{in} \ e_2) \ = \ \mathcal{D} \ 	ext{に} \ 	ext{L}_x(y_1, \dots, y_n)() = e_1' \ 	ext{を返す}
                                                                               ただし e'_1 = \mathcal{C}_{s'}(e_1), e'_2 = \mathcal{C}_{s'}(e_2), s' = s \cup \{x\},
                                                                               FV(e_1')\setminus\{y_1,\ldots,y_n\}=\emptyset かつ x\not\in FV(e_2') の場合
      \mathcal{C}_s(	ext{let rec } x \; y_1 \; \dots \; y_n = e_1 \; 	ext{in} \; e_2) \;\; = \;\; \mathcal{D} \; 	ext{に} \; \mathtt{L}_x(y_1, \dots, y_n)(z_1, \dots, z_m) = e_1' \;を加え、
                                                                        make\_closure \ x = (\mathtt{L}_x, (z_1, \ldots, z_m)) in e_2'を返す
                                                                               ただし e'_1 = C_s(e_1), e'_2 = C_s(e_2),
                                                                               FV(e_1') \setminus \{y_1, \dots, y_n\} \neq \emptyset,
                                                                               FV(e'_1) \setminus \{x, y_1, \dots, y_n\} = \{z_1, \dots, z_m\} の場合
      C_s(x y_1 \ldots y_n)
                                                                                                                                           x \notin s の場合
                                                                  = apply\_closure(x, y_1, \dots, y_n)
      C_s(x y_1 \ldots y_n)
                                                                  = apply\_direct(L_x, y_1, \dots, y_n)
                                                                                                                                           x \in s の場合
                                                     図 16: もっと賢い Closure 変換 C_s(e)
```

```
P ::=
 (\{D_1,\ldots,D_n\},E)
D ::=
 L_x(y_1,\ldots,y_n)=E
E ::=
                           命令の列
                           代入
 x \leftarrow e; E
                           返値
  e
                           式
                           即値
  c
                           ラベル
 L_x
                           算術演算
  op(x_1,\ldots,x_n)
  if x=y then E_1 else E_2 比較&分岐
  if x \leq y then E_1 else E_2 比較&分岐
                           mov 命令
  apply\_closure(x,y_1,\ldots,y_n) クロージャを用いた関数呼び出し
  apply\_direct(\mathbf{L}_x,y_1,\ldots,y_n) クロージャを用いない関数呼び出し
                           ロード
  x.(y)
                           ストア
  x.(y) \leftarrow z
                           変数 x の値をスタック位置 y に退避する
  save(x, y)
                           スタック位置 y から値を復元する
  \mathtt{restore}(y)
```

図 17: 仮想マシンコードの構文

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```
\mathcal{V}: \texttt{Closure.prog} \to \texttt{SparcAsm.prog}
                                                                                  = (\{\mathcal{V}(D_1), \dots, \mathcal{V}(D_n)\}, \mathcal{V}(e))
\mathcal{V}((\{D_1,\ldots,D_n\},e))
\mathcal{V}: \mathtt{Closure.fundef} 	o \mathtt{SparcAsm.fundef}
                                                                                  = L_x(y_1, \ldots, y_n) = z_1 \leftarrow R_0.(4); \ldots; z_n \leftarrow R_0.(4n); \mathcal{V}(e)
\mathcal{V}(\mathsf{L}_x(y_1,\ldots,y_n)(z_1,\ldots,z_n)=e)
\mathcal{V}: \mathtt{Closure.t} 	o \mathtt{SparcAsm.t}
\mathcal{V}(c)
                                                                                  = c
\mathcal{V}(op(x_1,\ldots,x_n))
                                                                                  = op(x_1,\ldots,x_n)
\mathcal{V}(\text{if } x = y \text{ then } e_1 \text{ else } e_2)
                                                                                  = if x = y then \mathcal{V}(e_1) else \mathcal{V}(e_2)
\mathcal{V}(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)
                                                                                  = if x \leq y then \mathcal{V}(e_1) else \mathcal{V}(e_2)
\mathcal{V}(\text{let } x = e_1 \text{ in } e_2)
                                                                                  = x \leftarrow \mathcal{V}(e_1); \mathcal{V}(e_2)
\mathcal{V}(x)
\mathcal{V}(\textit{make\_closure}\ x = (\mathtt{L}_x, (y_1, \ldots, y_n))\ \mathsf{in}\ e) \ = \ x \leftarrow \mathtt{R_{hp}}; \mathtt{R_{hp}} \leftarrow \mathtt{R_{hp}} + 4(n+1); z \leftarrow \mathtt{L}_x; x.(0) \leftarrow z;
                                                                                         x.(4) \leftarrow y_1; \dots; x.(4n) \leftarrow y_n; \mathcal{V}(e)
\mathcal{V}(apply\_closure(x, y_1, \dots, y_n))
                                                                                  = apply\_closure(x, y_1, \dots, y_n)
\mathcal{V}(apply\_direct(L_x, y_1, \dots, y_n))
                                                                                  = apply\_direct(L_x, y_1, \dots, y_n)
\mathcal{V}((x_1,\ldots,x_n))
                                                                                  = y \leftarrow R_{hp}; R_{hp} \leftarrow R_{hp} + 4n;
                                                                                         y.(0) \leftarrow x_1; \dots; y.(4(n-1)) \leftarrow x_n; y
\mathcal{V}(\text{let }(x_1,\ldots,x_n)=y \text{ in } e)
                                                                                  = \{x_1, \ldots, x_n\} \cap FV(e) = \{x_{i_1}, \ldots, x_{i_m}\} \succeq \bigcup \mathcal{T}
                                                                                         x_{i_1} \leftarrow y.(4(i_1-1)); \dots; x_{i_m} \leftarrow y.(4(i_m-1)); \mathcal{V}(e)
\mathcal{V}(x.(y))
                                                                                  = y' \leftarrow 4 \times y; x.(y')
\mathcal{V}(x.(y) \leftarrow z)
                                                                                  = y' \leftarrow 4 \times y; x.(y') \leftarrow z
```

図 18: 仮想マシンコード生成 $\mathcal{V}(P), \, \mathcal{V}(D)$ および $\mathcal{V}(e)$ 。右辺に出現して左辺に出現しない変数は fresh とする。 R_{hp} はヒープポインタ(専用レジスタ)。 $e_1; e_2$ はダミーの変数 x について $x \leftarrow e_1; e_2$ の略記。 $x \leftarrow E_1; E_2$ は、 $E_1 = (x_1 \leftarrow e_1; \dots; x_n \leftarrow e_n; e)$ として、 $x_1 \leftarrow e_1; \dots; x_n \leftarrow e_n; x \leftarrow e; E_2$ の略記。

```
FV: \mathtt{S.t} 	o \mathtt{SparcAsm.t} 	o \mathtt{S.t}
                                               = s' = FV_s(E) \setminus \{x\} \succeq \bigcup \mathsf{T} \ FV_{s'}(e)
FV_s(x \leftarrow e; E)
FV_s(e)
                                               = FV_s(e)
FV: \mathtt{S.t} 	o \mathtt{SparcAsm.exp} 	o \mathtt{S.t}
FV_s(c)
                                                = s
FV_s(L_x)
                                             = \{x_1, \dots, x_n\} \cup s
FV_s(op(x_1,\ldots,x_n))
FV_s(\text{if } x = y \text{ then } E_1 \text{ else } E_2) = \{x, y\} \cup FV_s(E_1) \cup FV_s(E_2)
FV_s(\text{if } x \leq y \text{ then } E_1 \text{ else } E_2) = \{x,y\} \cup FV_s(E_1) \cup FV_s(E_2)
                                             = \{x\} \cup s
FV_s(apply\_closure(x, y_1, \dots, y_n)) = \{x, y_1, \dots, y_n\} \cup s
FV_s(apply\_direct(L_x, y_1, \dots, y_n)) = \{y_1, \dots, y_n\} \cup s
FV_s(x.(y))
                                             = \{x, y\} \cup s
FV_s(x.(y) \leftarrow z)
                                              = \{x, y, z\} \cup s
FV_s(\mathtt{save}(x,y))
                                              = \{x\} \cup s
FV_s(\mathtt{restore}(y))
```

図 19: 命令の列 E および式 e において生きている変数の集合 $FV_s(E)$ および $FV_s(e)$ 。 s は E や e の後で使われる変数の集合。以後の FV(E) は $FV_\emptyset(E)$ の略記。

```
\mathcal{R}: \mathtt{SparcAsm.prog} \to \mathtt{SparcAsm.prog}
\mathcal{R}((\{D_1,\ldots,D_n\},E))
                                                               = (\{\mathcal{R}(D_1), \dots, \mathcal{R}(D_n)\}, \mathcal{R}_{\emptyset}(E, x, ()))
                                                                                                                                                         x はダミーの fresh な変数
\mathcal{R}: \mathtt{SparcAsm.fundef} \to \mathtt{SparcAsm.fundef}
\mathcal{R}(\mathsf{L}_x(y_1,\ldots,y_n)=E)
                                                               = L_x(R_1, \dots, R_n) = \mathcal{R}_{x \mapsto R_0, y_1 \mapsto R_1, \dots, y_n \mapsto R_n}(E, R_0, R_0)
\mathcal{R}: \mathtt{Id.t} \ \mathtt{M.t} \rightarrow \mathtt{SparcAsm.t} \times \mathtt{Id.t} \times \mathtt{SparcAsm.t} \rightarrow \mathtt{SparcAsm.t} \times \mathtt{Id.t} \ \mathtt{M.t}
\mathcal{R}_{\varepsilon}((x \leftarrow e; E), z_{\texttt{dest}}, E_{\texttt{cont}}) = E'_{\texttt{cont}} = (z_{\texttt{dest}} \leftarrow E; E_{\texttt{cont}}),
                                                                        \mathcal{R}_{\varepsilon}(e,x,E'_{\mathtt{cont}}) = (E',\varepsilon'),
                                                                        r \notin \{ \varepsilon'(y) \mid y \in FV(E'_{cont}) \},
                                                                        \mathcal{R}_{\varepsilon',x\mapsto r}(E,z_{\mathtt{dest}},E_{\mathtt{cont}})=(E'',\varepsilon'') 
 \succeq \mathsf{LT}
                                                                        ((r \leftarrow E'; E''), \varepsilon'')
                                                                                                                                                                  x がレジスタでない場合
\mathcal{R}_{\varepsilon}((r \leftarrow e; E), z_{\mathtt{dest}}, E_{\mathtt{cont}}) = E'_{\mathtt{cont}} = (z_{\mathtt{dest}} \leftarrow E; E_{\mathtt{cont}}),
                                                                        \mathcal{R}_{\varepsilon}(e, r, E'_{\mathtt{cont}}) = (E', \varepsilon'),
                                                                        \mathcal{R}_{\varepsilon'}(E,z_{\mathtt{dest}},E_{\mathtt{cont}}) = (E'',\varepsilon'') \; \textbf{LUT}
                                                                        ((r \leftarrow E'; E''), \varepsilon'')
                                                               = \mathcal{R}_{\varepsilon}(e, x, E_{\mathtt{cont}})
                                                                                                                                                                                             (次図参照)
\mathcal{R}_{\varepsilon}(e, x, E_{\mathtt{cont}})
```

図 20: 単純なレジスタ割り当て $\mathcal{R}(P)$, $\mathcal{R}(D)$ および $\mathcal{R}_{\varepsilon}(E,z_{\mathrm{dest}},E_{\mathrm{cont}})$ 。 ε は変数からレジスタへの写像、 z_{dest} は E の結果をセットする変数、 E_{cont} は E の後に実行される命令の列。 $\mathcal{R}_{\varepsilon}(E,x,E_{\mathrm{cont}})$ の返り値はレジスタ割り当てされた命令の列 E' と、E の後のレジスタ割り当てを表す写像 ε' の組。[ファイル regAlloc.notarget-nospill.ml 参照]

```
\mathcal{R}: \mathtt{Id.t} \ \mathtt{M.t} \rightarrow \mathtt{SparcAsm.exp} \times \mathtt{Id.t} \times \mathtt{SparcAsm.t} \rightarrow \mathtt{SparcAsm.t} \times \mathtt{Id.t} \ \mathtt{M.t}
  \mathcal{R}_{\varepsilon}(c, z_{\mathtt{dest}}, E_{\mathtt{cont}})
                                                                                                            = (c, \varepsilon)
  \mathcal{R}_{\varepsilon}(\mathsf{L}_x, z_{\mathtt{dest}}, E_{\mathtt{cont}})
                                                                                                            = (L_x, \varepsilon)
  \mathcal{R}_{\varepsilon}(op(x_1,\ldots,x_n),z_{\mathtt{dest}},E_{\mathtt{cont}})
                                                                                                           = (op(\varepsilon(x_1), \ldots, \varepsilon(x_n)), \varepsilon)
  \mathcal{R}_{\varepsilon}(\text{if }x=y \text{ then } E_1 \text{ else } E_2, z_{\texttt{dest}}, E_{\texttt{cont}}) \quad = \quad \mathcal{R}_{\varepsilon}(E_1, z_{\texttt{dest}}, E_{\texttt{cont}}) = (E_1', \varepsilon_1),
                                                                                                                     \mathcal{R}_{\varepsilon}(E_2, z_{\mathtt{dest}}, E_{\mathtt{cont}}) = (E'_2, \varepsilon_2),
                                                                                                                     \varepsilon' = \{ z \mapsto r \mid \varepsilon_1(z) = \varepsilon_2(z) = r \},
                                                                                                                     \{z_1,\ldots,z_n\}=
                                                                                                                                (\mathit{FV}(E_{\mathtt{cont}}) \setminus \{z_{\mathtt{dest}}\} \setminus \mathit{dom}(\varepsilon')) \cap \mathit{dom}(\varepsilon) \succeq \mathsf{LT}
                                                                                                                     ((\mathtt{save}(\varepsilon(z_1), z_1); \ldots; \mathtt{save}(\varepsilon(z_n), z_n);
                                                                                                                         if \varepsilon(x) \leq \varepsilon(y) then E_1' else E_2', \varepsilon')
  \mathcal{R}_{arepsilon}(	ext{if } x \leq y 	ext{ then } E_1 	ext{ else } E_2, z_{	ext{dest}}, E_{	ext{cont}}) = 同様
  \mathcal{R}_{\varepsilon}(x, z_{\mathtt{dest}}, E_{\mathtt{cont}})
                                                                                                            = (\varepsilon(x), \varepsilon)
  \mathcal{R}_{\varepsilon}(apply\_closure(x,y_1,\ldots,y_n),z_{\mathtt{dest}},E_{\mathtt{cont}}) \ = \ \{z_1,\ldots,z_n\} = (FV(E_{\mathtt{cont}}) \setminus \{z_{\mathtt{dest}}\}) \cap dom(\varepsilon) \ \succeq \ \mathsf{LT}
                                                                                                                     ((\mathtt{save}(\varepsilon(z_1), z_1); \dots; \mathtt{save}(\varepsilon(z_n), z_n);
                                                                                                                          apply\_closure(\varepsilon(x), \varepsilon(y_1), \ldots, \varepsilon(y_n))), \emptyset)
  \mathcal{R}_{\varepsilon}(apply\_direct(\mathtt{L}_x,y_1,\ldots,y_n),z_{\mathtt{dest}},E_{\mathtt{cont}})
                                                                                                           = 同様
  \mathcal{R}_{\varepsilon}(x.(y), z_{\mathtt{dest}}, E_{\mathtt{cont}})
                                                                                                            = (\varepsilon(x).(\varepsilon(y)), \varepsilon)
  \mathcal{R}_{\varepsilon}(x.(y) \leftarrow z, z_{\texttt{dest}}, E_{\texttt{cont}})
                                                                                                           = (\varepsilon(x).(\varepsilon(y)) \leftarrow \varepsilon(z), \varepsilon)
  \mathcal{R}_{\varepsilon}(\mathtt{save}(x,y),z_{\mathtt{dest}},E_{\mathtt{cont}})
                                                                                                            = (save(\varepsilon(x), y), \varepsilon)
  \mathcal{R}_{\varepsilon}(\mathtt{restore}(y), z_{\mathtt{dest}}, E_{\mathtt{cont}})
                                                                                                            = (restore(y), \varepsilon)
図 21: 単純なレジスタ割り当て \mathcal{R}_{\varepsilon}(e, z_{\mathtt{dest}}, E_{\mathtt{cont}})。 \mathcal{R}_{\varepsilon}(e) の右辺で変数 x のレジスタ \varepsilon(x) が定義されて
いない場合は、\mathcal{R}_{\varepsilon}(e) = \mathcal{R}_{\varepsilon}(x \leftarrow \mathtt{restore}(x); e) とする。ただしレジスタ r については \varepsilon(r) = r とする。
```

[ファイル regAlloc.notarget-nospill.ml 参照]

```
\mathcal{T}: \mathtt{Id.t} 	o \mathtt{SparcAsm.t} \times \mathtt{Id.t} 	o \mathtt{bool} \times \mathtt{S.t}
                                                                        = \mathcal{T}_x(e,y)=(c_1,s_1) として、もしc_1ならば(true,s_1)
\mathcal{T}_x((y \leftarrow e; E), z_{\texttt{dest}})
                                                                               そうでなければ T_x(E, z_{\texttt{dest}}) = (c_2, s_2) として (c_2, s_1 \cup s_2)
                                                                        = T_x(e, z_{\tt dest})
\mathcal{T}_x(e, z_{\mathtt{dest}})
\mathcal{T}: \mathtt{Id.t} \rightarrow \mathtt{SparcAsm.exp} \times \mathtt{Id.t} \rightarrow \mathtt{bool} \times \mathtt{S.t}
\mathcal{T}_x(x,z_{	exttt{dest}})
                                                                       = (false, \{z_{dest}\})
\mathcal{T}_x(\text{if }y=z \text{ then } E_1 \text{ else } E_2, z_{\texttt{dest}}) \quad = \quad \mathcal{T}_x(E_1, z_{\texttt{dest}}) = (c_1, s_1),
                                                                               \mathcal{T}_x(E_2, z_{\mathtt{dest}}) = (c_2, s_2) \ \mathsf{LLT}
                                                                               (c_1 \wedge c_2, s_1 \cup s_2)
                                                                       = 同上
T_x(\text{if } y \leq z \text{ then } E_1 \text{ else } E_2, z_{\texttt{dest}})
\mathcal{T}_x(apply\_closure(y_0, y_1, \dots, y_n), z_{\texttt{dest}}) = (true, \{R_i \mid x = y_i\})
T_x(apply\_direct(L_y, y_1, \dots, y_n), z_{\texttt{dest}}) = 同上
                                                                        = (false, \emptyset)
                                                                                                                                                          それ以外の場合
\mathcal{T}_x(e, z_{\mathtt{dest}})
```

図 22: 変数 x に割り当てるレジスタ r を選ぶときに使う $targeting \ T_x(E,z_{\tt dest})$ および $T_x(e,z_{\tt dest})$ 。E や e で関数呼び出しがあったかどうかを表す論理値 e と、e を割り当てると良いレジスタの集合 e の組を返す。前々図の「e がレジスタでない場合」において、e できれば e とする。e 「ファイル regAlloc.target-nospill.ml 参照]

```
\mathcal{R}: \mathtt{Id.t} \ \mathtt{M.t} 	o \mathtt{SparcAsm.t} 	imes \mathtt{Id.t} 	imes \mathtt{SparcAsm.t} 	o \mathtt{SparcAsm.t} 	imes \mathtt{Id.t} \ \mathtt{M.t} \mathcal{R}_{\varepsilon}((x \leftarrow e; E), z_{\mathtt{dest}}, E_{\mathtt{cont}}) \ = \ E'_{\mathtt{cont}} = (z_{\mathtt{dest}} \leftarrow E; E_{\mathtt{cont}}), \\ \mathcal{R}_{\varepsilon}(e, x, E'_{\mathtt{cont}}) = (E', \varepsilon'), \\ y \in FV(E'_{\mathtt{cont}}), \\ \mathcal{R}_{\varepsilon' \setminus \{y \mapsto \varepsilon'(y)\}, x \mapsto \varepsilon'(y)}(E, z_{\mathtt{dest}}, E_{\mathtt{cont}}) = (E'', \varepsilon'') \ \succeq \mathsf{UT} \\ \left\{ \ ((\mathtt{save}(\varepsilon(y), y); \varepsilon'(y) \leftarrow E'; E''), \varepsilon'') \quad y \in dom(\varepsilon) \ \mathfrak{O} \succeq \mathtt{SparcAsm.t} \times \mathtt{Id.t} \ \mathtt{M.t} \right\} \\ \left\{ \ ((\varepsilon'(y) \leftarrow E'; E''), \varepsilon'') \quad y \in \mathsf{Mom}(\varepsilon) \ \mathfrak{O} \succeq \mathtt{SparcAsm.t} \times \mathtt{Id.t} \ \mathtt{M.t} \times \mathtt{Id.t} \ \mathtt{M.t} \times \mathtt{Id.t} \ \mathtt{M.t} \times \mathtt{Id.t} \times \mathtt{M.t} \times \mathtt{M.t} \times \mathtt{M.t} \times \mathtt{Id.t} \times \mathtt{M.t} \times \mathtt{Id.t} \times \mathtt{M.t} \times \mathtt{Id.t} \times \mathtt{M.t} \times \mathtt{Id.t} \times \mathtt{M.t} \times \mathtt{M.
```

図 23: spilling をするレジスタ割り当て $\mathcal{R}_{\varepsilon}(E, z_{\texttt{dest}}, E_{\texttt{cont}})$ [ファイル regAlloc.target-latespill.ml 参照]

```
\mathcal{S}: \mathtt{SparcAsm.prog} \to \mathtt{string}
    \mathcal{S}((\{D_1,\dots,D_n\},E)) \quad = \quad \mathtt{.section} \ \mathtt{".text"}
                                                   \mathcal{S}(D_1)
                                                  . . .
                                                  S(D_n)
                                                   .global min_caml_start
                                                  min_caml_start:
                                                   save %sp, -112, %sp
                                                   \mathcal{S}(E, \%g0)
                                                  ret
                                                  restore
    \mathcal{S}: \texttt{SparcAsm.fundef} \to \texttt{string}
    \mathcal{S}(L_x(y_1,\ldots,y_n)=E) = x:
                                                  \mathcal{S}(E,\mathtt{R}_0)
                                                  retl
                                                  nop
    \mathcal{S}: \texttt{SparcAsm.t} \times \texttt{Id.t} \rightarrow \texttt{string}
    \mathcal{S}((x \leftarrow e; E), z_{\texttt{dest}})
                                            = \mathcal{S}(e, x)
                                                  \mathcal{S}(E, z_{	exttt{dest}})
    \mathcal{S}(e,z_{\texttt{dest}})
                                            = \mathcal{S}(e, z_{\mathtt{dest}})
図 24: 単純なアセンブリ生成 \mathcal{S}(P),\,\mathcal{S}(D) および \mathcal{S}(E,z_{	exttt{dest}})
```

```
\mathcal{S}: \mathtt{SparcAsm.exp} \times \mathtt{Id.t} \rightarrow \mathtt{string}
\mathcal{S}(c, z_{\mathtt{dest}})
                                                                                           \mathtt{set}\ c, z_{\mathtt{dest}}
\mathcal{S}(L_x, z_{\mathtt{dest}})
                                                                                            \operatorname{set} L_x, z_{\operatorname{dest}}
S(op(x_1,\ldots,x_n),z_{\tt dest})
                                                                                            op \ x_1, \ldots, x_n, z_{\texttt{dest}}
\mathcal{S}(	ext{if } x=y 	ext{ then } E_1 	ext{ else } E_2, z_{	ext{dest}})
                                                                                         cmp x, y
                                                                                            bne b_1
                                                                                            nop
                                                                                            S(E_1, z_{\texttt{dest}})
                                                                                            b b_2
                                                                                            nop
                                                                                            b_1:
                                                                                            \mathcal{S}(E_2,z_{\mathtt{dest}})
                                                                                            b_2:
S(\text{if } x \leq y \text{ then } E_1 \text{ else } E_2, z_{\texttt{dest}})
                                                                                         同様
\mathcal{S}(x, z_{\mathtt{dest}})
                                                                                            \mathtt{mov}\ x, z_{\mathtt{dest}}
\mathcal{S}(apply\_closure(x, y_1, \dots, y_n), z_{\texttt{dest}})
                                                                                         shuffle((x, y_1, \ldots, y_n), (R_0, R_1, \ldots, R_n))
                                                                                            \mathtt{st}\ \mathtt{R}_{\mathtt{ra}}, [\mathtt{R}_{\mathtt{st}} + 4\#\varepsilon]
                                                                                            ld[R_0], R_{n+1}
                                                                                            call R_{n+1}
                                                                                            add \mathbf{R}_{\mathrm{st}}, 4(\#\varepsilon+1), \mathbf{R}_{\mathrm{st}} ! delay\ slot
                                                                                            \operatorname{sub} R_{\operatorname{st}}, 4(\#\varepsilon+1), R_{\operatorname{st}}
                                                                                            ld [R_{st} + 4\#\varepsilon], R_{ra}
                                                                                            mov R_0, z_{dest}
S(apply\_direct(L_x, y_1, \dots, y_n), z_{dest})
                                                                                   = shuffle((y_1, \ldots, y_n), (R_1, \ldots, R_n))
                                                                                            \mathtt{st}\ \mathtt{R}_{\mathtt{ra}}, [\mathtt{R}_{\mathtt{st}} + 4\#\varepsilon]
                                                                                            add R_{\rm st}, 4(\#\varepsilon+1), R_{\rm st} ! delay\ slot
                                                                                            \operatorname{sub} R_{\operatorname{st}}, 4(\#\varepsilon+1), R_{\operatorname{st}}
                                                                                            ld [R_{st} + 4\#\varepsilon], R_{ra}
                                                                                            \mathtt{mov}\ \mathtt{R}_0, z_{\mathtt{dest}}
S(x.(y), z_{\text{dest}})
                                                                                   = 1d [x+y], z_{\text{dest}}
S(x.(y) \leftarrow z, z_{\texttt{dest}})
                                                                                   = st z, [x+y]
\mathcal{S}(\mathtt{save}(x,y),z_{\mathtt{dest}})
                                                                                   = もしy \not\in dom(\varepsilon)なら\varepsilonにy \mapsto 4\#\varepsilonを加えて
                                                                                            st x, [\mathbf{R}_{\mathsf{st}} + \varepsilon(y)]
\mathcal{S}(\mathtt{restore}(y), z_{\mathtt{dest}})
                                                                                   = \ \text{ld} \ [\mathtt{R}_{\mathtt{st}} + \varepsilon(y)], z_{\mathtt{dest}}
```

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図 25: 単純なアセンブリ生成 $\mathcal{S}(e,z_{\mathtt{dest}})$ 。 ε はスタック位置を記憶するグローバル変数。 $\#\varepsilon$ は ε の要素の

個数。 $\mathit{shuffle}((x_1,\ldots,x_n),(r_1,\ldots,r_n))$ は x_1,\ldots,x_n を r_1,\ldots,r_n に適切な順序で移動する命令。

```
\mathcal{S}: \mathtt{S.t} \rightarrow \mathtt{SparcAsm.t} \times \mathtt{Id.t} \rightarrow \mathtt{S.t} \times \mathtt{string}
S_s((x \leftarrow e; E), z_{\texttt{dest}})
                                                                    = \mathcal{S}_s(e, x) = (s', S),
                                                                           S_{s'}(E, z_{\texttt{dest}}) = (s'', S') \succeq \mathsf{UT}
                                                                           (s'', SS')
\mathcal{S}_s(e,z_{	exttt{dest}})
                                                                    = S_s(e, z_{\tt dest})
\mathcal{S}: \mathtt{S.t} \rightarrow \mathtt{SparcAsm.exp} \times \mathtt{Id.t} \rightarrow \mathtt{S.t} \times \mathtt{string}
S_s(\text{if } x = y \text{ then } E_1 \text{ else } E_2, z_{\text{dest}}) = S_s(E_1, z_{\text{dest}}) = (s_1, S_1),
                                                                           S_s(E_2, z_{\texttt{dest}}) = (s_2, S_2) \succeq \mathsf{UT}
                                                                           (s_1 \cap s_2,
                                                                            cmp \ x, y
                                                                            bne b_1
                                                                            nop
                                                                            S_1
                                                                            b b_2
                                                                            nop
                                                                            b_1:
                                                                            S_2
                                                                            b_2:)
S_s(if x \leq y then E_1 else E_2, z_{	exttt{dest}}) = 同樣
S_s(\mathtt{save}(x,y), z_{\mathtt{dest}})
                                                                    = (s, nop)
                                                                                                                                   y \in s の場合
S_s(\mathtt{save}(x,y),z_{\mathtt{dest}})
                                                                    = もしy \notin dom(\varepsilon)なら\varepsilonにy \mapsto 4\#\varepsilonを加えて
                                                                           (s \cup \{y\}, \mathtt{st}\ x, [\mathtt{R}_{\mathtt{st}} + \varepsilon(y)])
                                                                                                                                   y \notin s の場合
S_s(e, z_{\tt dest})
                                                                                                                             上述以外の場合
                                                                    = (s,以前と同様)
```

図 26: 無駄な save を省略するアセンブリ生成 $\mathcal{S}_s(E,z_{\mathtt{dest}})$ および $\mathcal{S}_s(e,z_{\mathtt{dest}})$ 。s はすでに save された変数の名前の集合。以前の $\mathcal{S}(E,z_{\mathtt{dest}})$ は $\mathcal{S}_{\emptyset}(E,z_{\mathtt{dest}})=(s,S)$ として S の略記とする。

```
\mathcal{S}: \texttt{SparcAsm.fundef} \to \texttt{string}
                                                                    = S_{\emptyset}(E, tail) = (s, S) \succeq U \mathsf{T}
\mathcal{S}(\mathsf{L}_x(y_1,\ldots,y_n)=E)
                                                                           x:
                                                                           S
\mathcal{S}: \mathtt{S.t} \rightarrow \mathtt{SparcAsm.exp} \times \mathtt{Id.t} \rightarrow \mathtt{S.t} \times \mathtt{string}
\mathcal{S}_s(\text{if } x=y \text{ then } E_1 \text{ else } E_2, \text{tail}) \ = \ \mathcal{S}_s(E_1, \text{tail}) = (s_1, S_1),
                                                                           S_s(E_2, 	exttt{tail}) = (s_2, S_2) 	exttt{ bl}
                                                                           (\emptyset,
                                                                            cmp \ x, y
                                                                            \mathtt{bne}\ b
                                                                            nop
                                                                            S_1
                                                                            b:
                                                                            S_2
S_s( 	ext{if } x \leq y 	ext{ then } E_1 	ext{ else } E_2, 	ext{tail} ) =
                                                                           同樣
S_s(apply\_closure(x, y_1, \dots, y_n), \texttt{tail}) =
                                                                          (\emptyset,
                                                                            shuffle((x, y_1, \ldots, y_n), (R_0, R_1, \ldots, R_n))
                                                                            ld[R_0], R_{n+1}
                                                                            {\tt jmp}\ {\tt R}_{n+1}
                                                                            nop)
S_s(apply\_direct(L_x, y_1, \dots, y_n), tail) = (\emptyset,
                                                                            shuffle((y_1,\ldots,y_n),(\mathtt{R}_1,\ldots,\mathtt{R}_n))
                                                                            \mathbf{b} \ x
                                                                            nop)
                                                                    = S_s(e, \mathbf{R}_0) = (s', S) \succeq \mathsf{UT}
\mathcal{S}_s(e, \mathtt{tail})
                                                                           (\emptyset,
                                                                            S
                                                                            retl
                                                                                                                   上述以外の場合
                                                                            nop)
```

図 27: 末尾呼び出し最適化をするアセンブリ生成 $\mathcal{S}_s(D)$ および $\mathcal{S}_s(e,z_{\mathtt{dest}})$ 。 $z_{\mathtt{dest}}=\mathtt{tail}$ の場合が末尾。