Lecture Notes for

Machine Learning in Python

Professor Eric Larson

Gradients of Convolutional Networks

Class logistics and Agenda

- Wide/Deep Lab due soon!
- Agenda:
  - Finish CNN Discussion
  - CNN Demo
  - History of CNNs
    - with Modern CNN Architectures
- Next Time:
  - More Advanced CNN Demo

Class Overview, by topic

Table Data Visualization

Numpy, Pandas, Seaborn
Overviews with some in-depth discussion

Dimension Reduction and Image Processing

Scikit-learn, Scikit Image,
Intuition only, Some mathematics

Linear and Logistic Regression

Numpy, Recreate API for Scikit-learn
Detailed mathematics for simple optimization
intuition for advanced optimization

Neural Networks and Back Prop.

Numpy
Detailed mathematics for NN operations

Wide and Deep Networks

Convolutional Networks

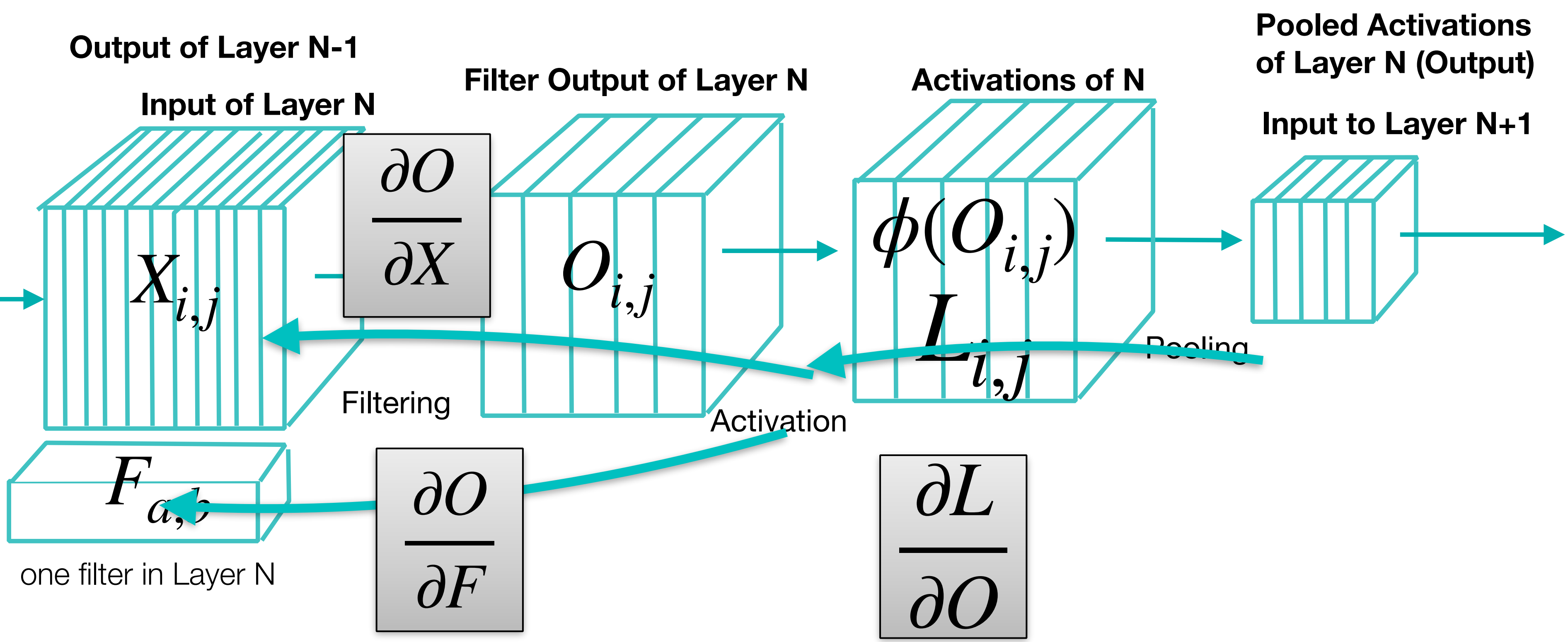
Recurrent Networks

Keras, Tensorflow
Intuition, Detailed implement.

Ethics in Language Models

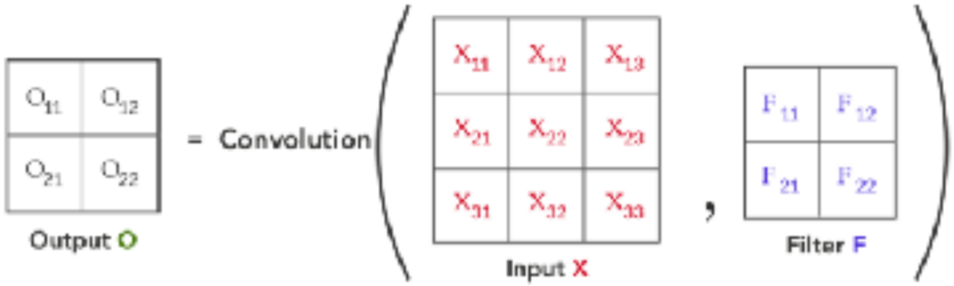
ConceptNet
Case studies

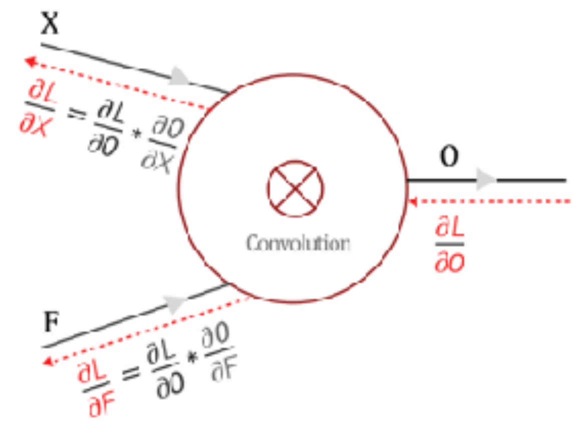
Last Time: CNNs, Putting it together

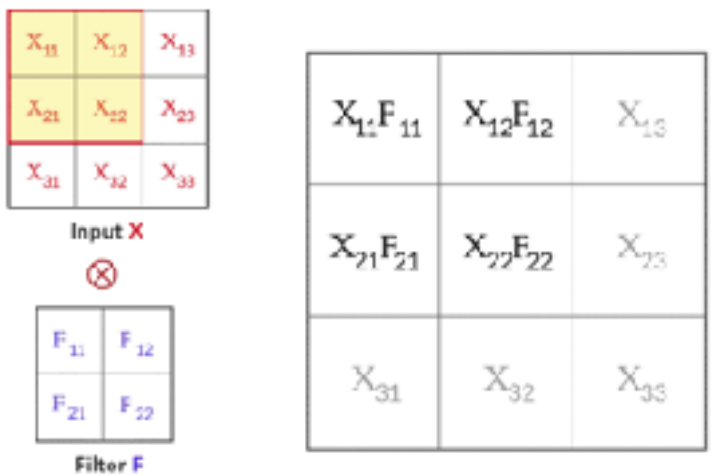


Structure of Each Tensor: Channels x Rows x Columns

Reminder: Convolution







$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

$$O_{12} = X_{12}F_{11} + X_{13}F_{12} + X_{22}F_{21} + X_{23}F_{22}$$

$$O_{21} = X_{21}F_{11} + X_{22}F_{12} + X_{31}F_{21} + X_{32}F_{22}$$

$$O_{22} = X_{22}F_{11} + X_{23}F_{12} + X_{32}F_{21} + X_{33}F_{22}$$

https://medium.com/@pavisj/convolutions-and-backpropagations-46026a8f5d2c

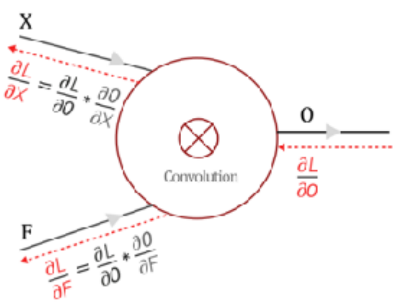
Gradient of Convolution

$$\frac{\partial L}{\partial X} = \frac{\partial L}{\partial O}\frac{\partial O}{\partial X}$$

for back propagation

$$\frac{\partial L}{\partial F} = \frac{\partial L}{\partial O}\frac{\partial O}{\partial F}$$

for weight updates



$$\underbrace{\begin{bmatrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{bmatrix}}_{\text{Output O}} = \text{Convolution}\left(\underbrace{\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}}_{\text{Input X}}, \underbrace{\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}}_{\text{Filter F}} \right)$$

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$
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$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

Finding derivatives with respect to F_{11}, F_{12}, F_{21} and F_{22}

$$\frac{\partial O_{11}}{\partial F_{11}} = X_{11} \quad \frac{\partial O_{11}}{\partial F_{12}} = X_{12} \quad \frac{\partial O_{11}}{\partial F_{21}} = X_{21} \quad \frac{\partial O_{11}}{\partial F_{22}} = X_{22}$$

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{11}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{11}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{11}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{11}}$$
$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{12}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{12}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{12}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{12}}$$
$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{21}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{21}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{21}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{21}}$$
$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{22}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{22}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{22}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{22}}$$

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial O_{11}} * X_{11} + \frac{\partial L}{\partial O_{12}} * X_{12} + \frac{\partial L}{\partial O_{21}} * X_{21} + \frac{\partial L}{\partial O_{22}} * X_{22}$$
$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial O_{11}} * X_{12} + \frac{\partial L}{\partial O_{12}} * X_{13} + \frac{\partial L}{\partial O_{21}} * X_{22} + \frac{\partial L}{\partial O_{22}} * X_{23}$$
$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial O_{11}} * X_{21} + \frac{\partial L}{\partial O_{12}} * X_{22} + \frac{\partial L}{\partial O_{21}} * X_{31} + \frac{\partial L}{\partial O_{22}} * X_{32}$$
$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial O_{11}} * X_{22} + \frac{\partial L}{\partial O_{12}} * X_{23} + \frac{\partial L}{\partial O_{21}} * X_{32} + \frac{\partial L}{\partial O_{22}} * X_{33}$$

$$\underbrace{\begin{bmatrix} \frac{\partial L}{\partial F_{11}} & \frac{\partial L}{\partial F_{12}} \\ \frac{\partial L}{\partial F_{21}} & \frac{\partial L}{\partial F_{22}} \end{bmatrix}}_{\text{Filter updates}} = \text{Convolution}\left(\underbrace{\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}}_{\text{Input}}, \underbrace{\begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix}}_{\text{Sensitivity from next layer}} \right)$$

https://medium.com/@pavisj/convolutions-and-backpropagations-46026a8f5d2c

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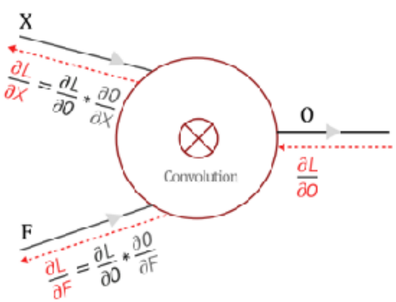

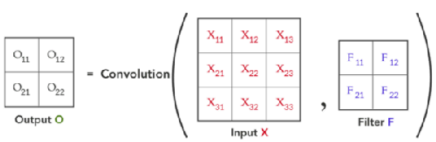

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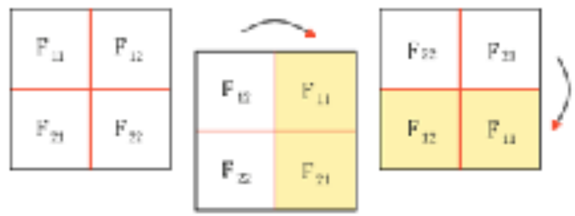
$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

Differentiating with respect to X_{11}, X_{12}, X_{21} *and* X_{22}

$$\frac{\partial O_{11}}{\partial X_{11}} = F_{11} \quad \frac{\partial O_{11}}{\partial X_{12}} = F_{12} \quad \frac{\partial O_{11}}{\partial X_{21}} = F_{21} \quad \frac{\partial O_{11}}{\partial X_{22}} = F_{22}$$

Similarly, we can find local gradients for O_{12}, O_{21} *and* O_{22}

$$\frac{\partial L}{\partial X_{11}} = \frac{\partial L}{\partial O_{11}} * F_{11}$$
$$\frac{\partial L}{\partial X_{12}} = \frac{\partial L}{\partial O_{11}} * F_{12} + \frac{\partial L}{\partial O_{12}} * F_{11}$$
$$\frac{\partial L}{\partial X_{13}} = \frac{\partial L}{\partial O_{12}} * F_{12}$$
$$\frac{\partial L}{\partial X_{21}} = \frac{\partial L}{\partial O_{11}} * F_{21} + \frac{\partial L}{\partial O_{21}} * F_{11}$$
$$\frac{\partial L}{\partial X_{22}} = \frac{\partial L}{\partial O_{11}} * F_{22} + \frac{\partial L}{\partial O_{12}} * F_{21} + \frac{\partial L}{\partial O_{21}} * F_{12} + \frac{\partial L}{\partial O_{22}} * F_{11}$$
$$\frac{\partial L}{\partial X_{23}} = \frac{\partial L}{\partial O_{12}} * F_{22} + \frac{\partial L}{\partial O_{22}} * F_{12}$$
$$\frac{\partial L}{\partial X_{31}} = \frac{\partial L}{\partial O_{21}} * F_{21}$$
$$\frac{\partial L}{\partial X_{32}} = \frac{\partial L}{\partial O_{21}} * F_{22} + \frac{\partial L}{\partial O_{22}} * F_{21}$$
$$\frac{\partial L}{\partial X_{33}} = \frac{\partial L}{\partial O_{22}} * F_{22}$$



$$\begin{bmatrix} F_{22} & F_{21} \\ F_{12} & F_{11} \end{bmatrix}$$

0	0	0	0
0	$\frac{\partial L}{\partial O_{11}}$	$\frac{\partial L}{\partial O_{12}}$	0
0	$\frac{\partial L}{\partial O_{21}}$	$\frac{\partial L}{\partial O_{22}}$	0
0	0	0	0

$$\begin{bmatrix} \frac{\partial L}{\partial X_{11}} & \frac{\partial L}{\partial X_{12}} & \frac{\partial L}{\partial X_{13}} \\ \frac{\partial L}{\partial X_{21}} & \frac{\partial L}{\partial X_{22}} & \frac{\partial L}{\partial X_{23}} \\ \frac{\partial L}{\partial X_{31}} & \frac{\partial L}{\partial X_{32}} & \frac{\partial L}{\partial X_{33}} \end{bmatrix} = \text{Full Convolution}\left(\begin{bmatrix} F_{22} & F_{21} \\ F_{12} & F_{11} \end{bmatrix}, \begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix} \right)$$

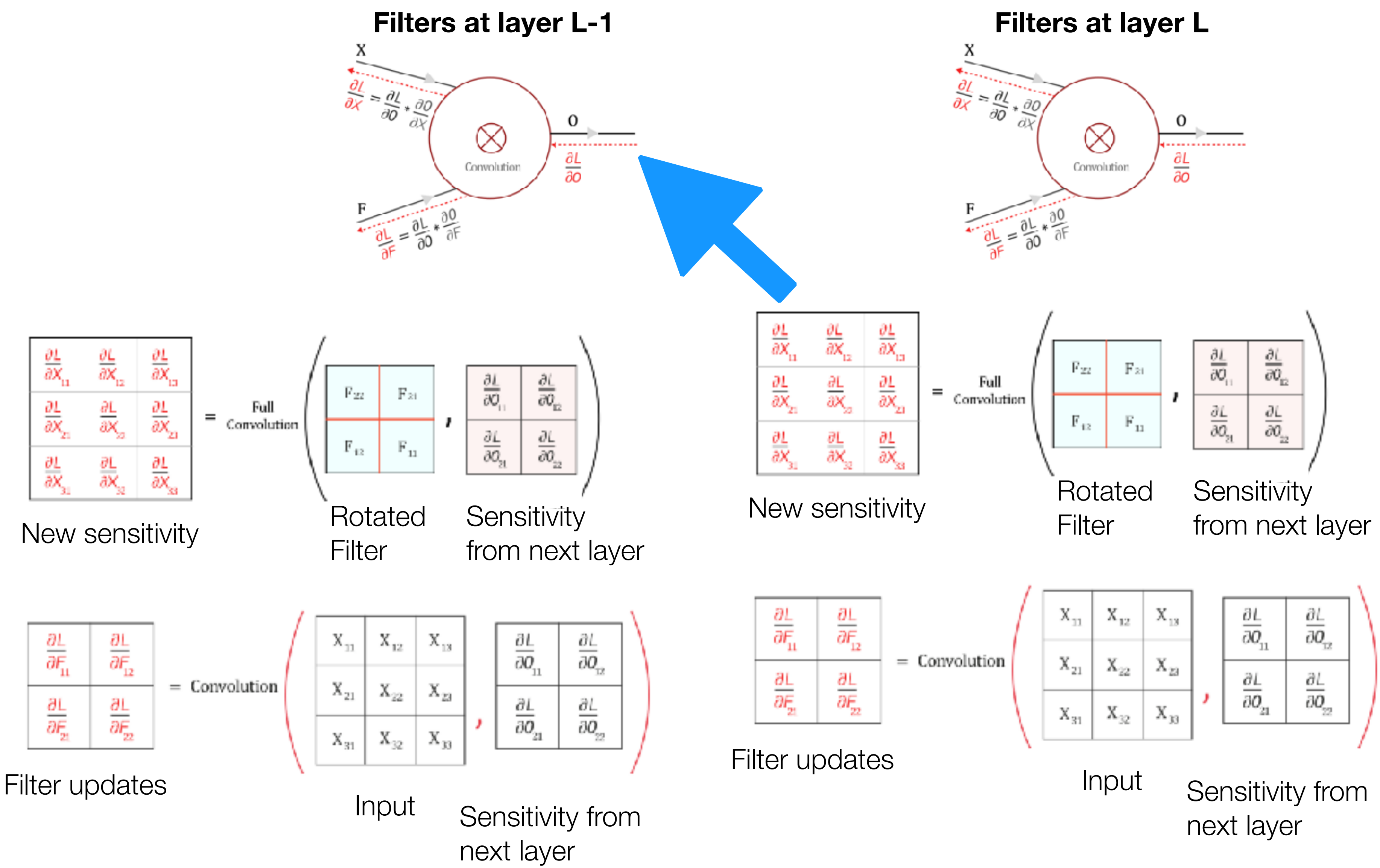
New sensitivity

Rotated Filter

Sensitivity from next layer (zero padded)

https://medium.com/@pavisj/convolutions-and-backpropagations-46026a8f5d2c

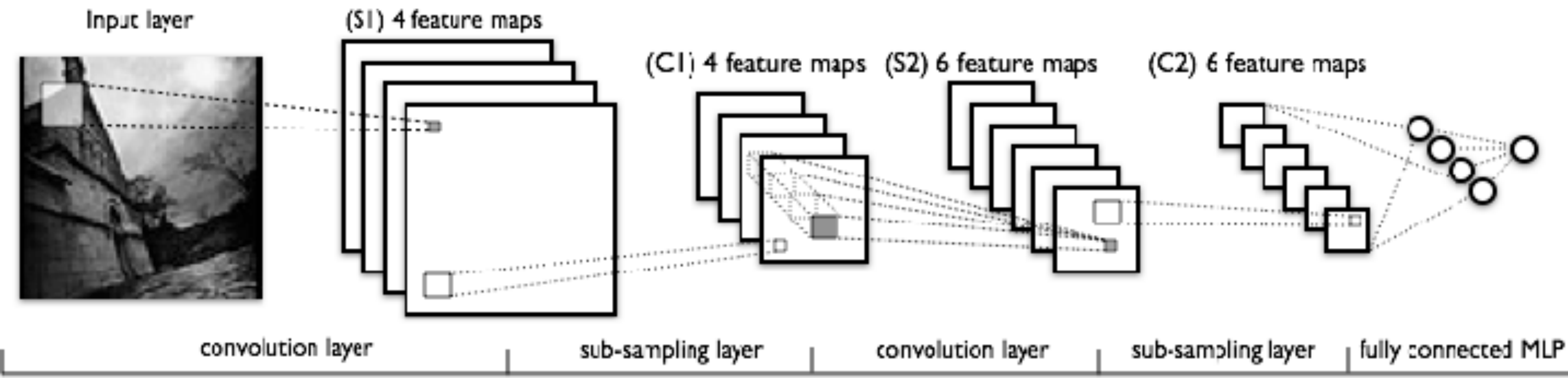
Summary

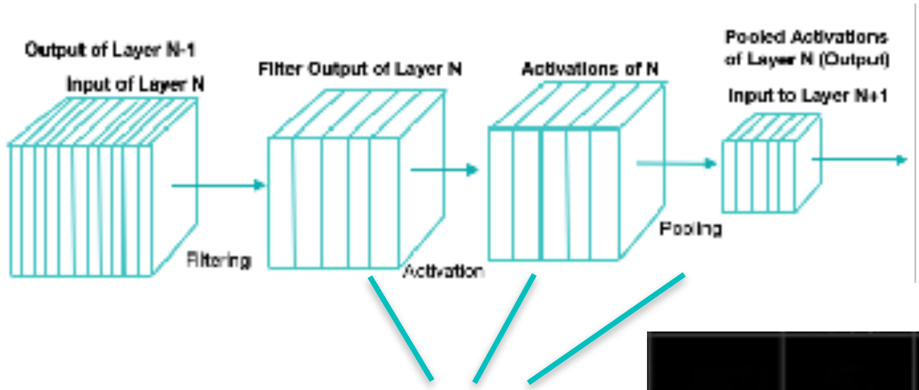

Filters at layer L-1
Filters at layer L
X
O
F
Convolution
Full Convolution
Rotated Filter
Sensitivity from next layer
New sensitivity
Filter updates
Input
Sensitivity from next layer

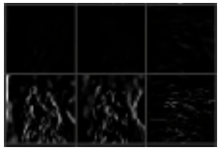
CNN Gradient

- Takeaways:
  - Derivative of a convolutional layer is calculated through two additional convolutions
    - One for filter updates
    - One for calculating a new sensitivity
  - We need to run convolution fast in order to speed up both:
    - feedforward operations (inference and training)
    - back propagation (training)
- Another great resource:
  - https://becominghuman.ai/back-propagation-in-convolutional-neural-networks-intuition-and-code-714ef1c38199

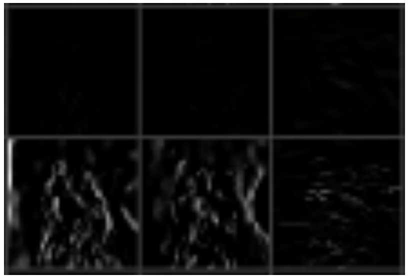
Some Example CNN Architectures







Visualize
each channel



CNN: Visuals

Deep Visualization Toolbox

yosinski.com/deepvis

#deepvis


Jason Yosinski


Jeff Clune


Anh Nguyen


Thomas Fuchs


Hod Lipson

Cornell University

University of Wyoming

Jet Propulsion Laboratory
California Institute of Technology

https://github.com/yosinski/deep-visualization-toolbox

Demo

Convolutional Neural Networks
in TensorFlow
with Keras

11. Convolutional Neural Networks.ipynb

Next Lecture

- More CNN architectures and CNN history