Analysis of Quarantine Implementation in an Epidemic

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Abstract

Quarantining is a popular intervention method when trying to control the spread of a disease. The focus of the model from the paper, *Renaissance model of an epidemic with quarantine*, by Dobay, Gall, Rankin, and Bagheri is the role that the carrying capacity of a quarantine facility has on the effectiveness of controlling a syphilis outbreak in Zurich during the 16th century. The writers of the original paper explored this by supplementing a modified SIR model with various carrying capacities and explored the resulting outcomes. The modified SIR model adds additional susceptible and infected classes to represent those patients in the quarantine facility.

While the carrying capacity of quarantine facilities is an interesting and important aspect to explore when analyzing how to effectively contain a disease, it is also important to see what would result from the quarantine facility being open for different durations of time or rather at what point in the outbreak would be the most effective to open the facility. It is not feasible to have a quarantine facility open indefinitely due to costs and lack of use after it is no longer needed. Therefore, we intend to expand upon the aforementioned paper by exploring what point during a given disease outbreak might be the most effective time to have a quarantine facility.

Introduction

The case of a syphilis epidemic in Zurich that occurred in the 16th century provides a good basis for analyzing the effectiveness of a quarantine facility. The officials controlled the outbreak by closing the city to travel and advising anyone who was symptomatic to avoid public spaces. Eventually the city opened a quarantine facility called the Blatternhaus (the name came from the German name for syphilis at the time). Individuals who were showing any variation of symptoms of syphilis or who tested positive were brought to this facility. The results of this specific outbreak and the paper allow us to conclude that a quarantine facility is an effective measure in reducing the severity of an epidemic. However the quarantine facility's level of effectiveness depends on many factors, such as carrying capacity, detection rate, force of infection, and duration of the quarantine. Without the ability to properly identify who is truly infected, the effectiveness of our quarantine is greatly diminished. Specifically with syphilis, there is a latent stage where those who have previously had symptoms of syphilis will no longer exhibit those symptoms but will still be highly contagious. The latent stage can complicate the quarantine process in this case because it is possible for a patient to be released from quarantine because they no longer have symptoms and then spread the disease in the community. Similarly, it is possible someone has symptoms of syphilis but is not actually infected, so they are susceptible in quarantine which can diminish the quarantine's purpose. An effective detection rate is crucial to accurately track the true stage of the outbreak so that it is possible to better determine when to activate quarantine procedures so that the quarantine process is the most successful, and have an accurate sense of who actually needs to be in the quarantine facility to determine when it is safe to release quarantined patients back into the community.

Methods

One of the simplest methods for modeling an epidemic is a SIR model. We modify the basic SIR model so that we have two additional classes to represent the individuals in quarantine: one class for susceptibles quarantined as a result of a false positive and one for quarantined infected individuals. We assume that individuals go into quarantine at a rate proportional to the amount of space in the quarantine facility. We represent this space by $k_{cd}=C-S_B-I_B$ where C represents the carrying capacity, S_B gives us the number of quarantined susceptibles and the number of infected in quarantine is I_B.

These additional classes give us a modified SIR model of:

$dS/dt = -k_1*S*I - k_{cd}*k_3*S + k_4*S_B$

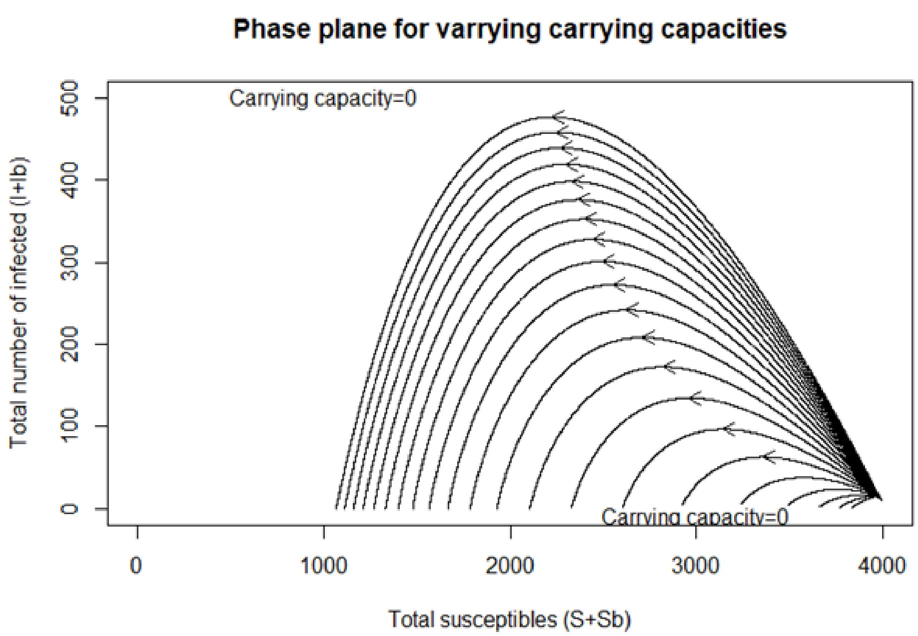
$dS_B/dt = -k_5*S_B*I_B + k_{cd}*k_3*S - k_4*S_B$

$dI/dt = k_1*S*I - k_2*I - k_{cd}*k_6*I + k_7*I_B$

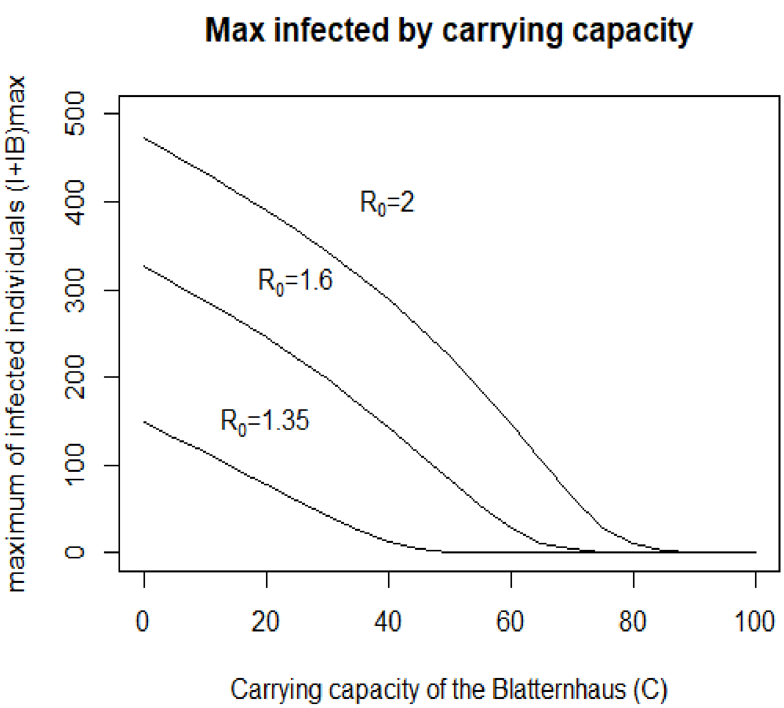
$dI_B/dt = k_5*S_B*I_B + k_{cd}*k_6*I - k_7*I_B - k_8*I_B$

$dR/dt = k_2*I + k_8*I_B$

The additional parameters k1, k2, k3, k4, k5, k6, k7, and k8 represent different rates: k1 is the per capita force of infection, k2 is the recovery rate, k3 is the rate of quarantine of falsely detected individuals, k4 represents removal from quarantine before full recovery, k5 stands for the rate at which susceptibles in quarantine become infected while in Blatternhaus, k6 is the rate of quarantine of true detections, k7 represents the exclusion rate of individuals from quarantine, and k8 represents the recovery/removal rate after quarantine. To analyze the effects that altering the carrying capacity will have on a population we look back to the original paper and case study. We start with a population size of 4000 and 1 infected individual. Using the following parameters: k1=5*10^-6, k2=1/90, k3=0, k4=0,k5=0, k6=1*10^-5, k7=1*10^-5, and k8=1/70 we can begin to numerically solve our model. These parameters and *R* allow us to find solutions for times 0 to 2500 by intervals of 0.1. As a result of evaluating the effects of different carrying capacities we can recreate a phase plane of these capacities (Figure 1). This figure features a phase line for varying carrying capacities in the range of 0 to 100 by intervals of 5. These phase lines

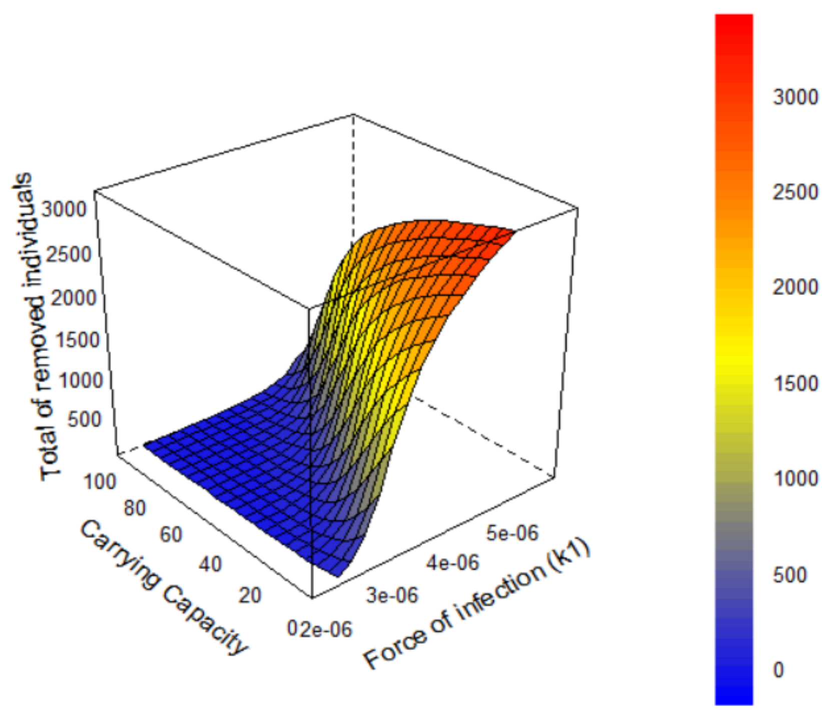


outline a clear pattern. When we increase our quarantine facility's carrying capacity we lower the peak total number of infected in the epidemic.

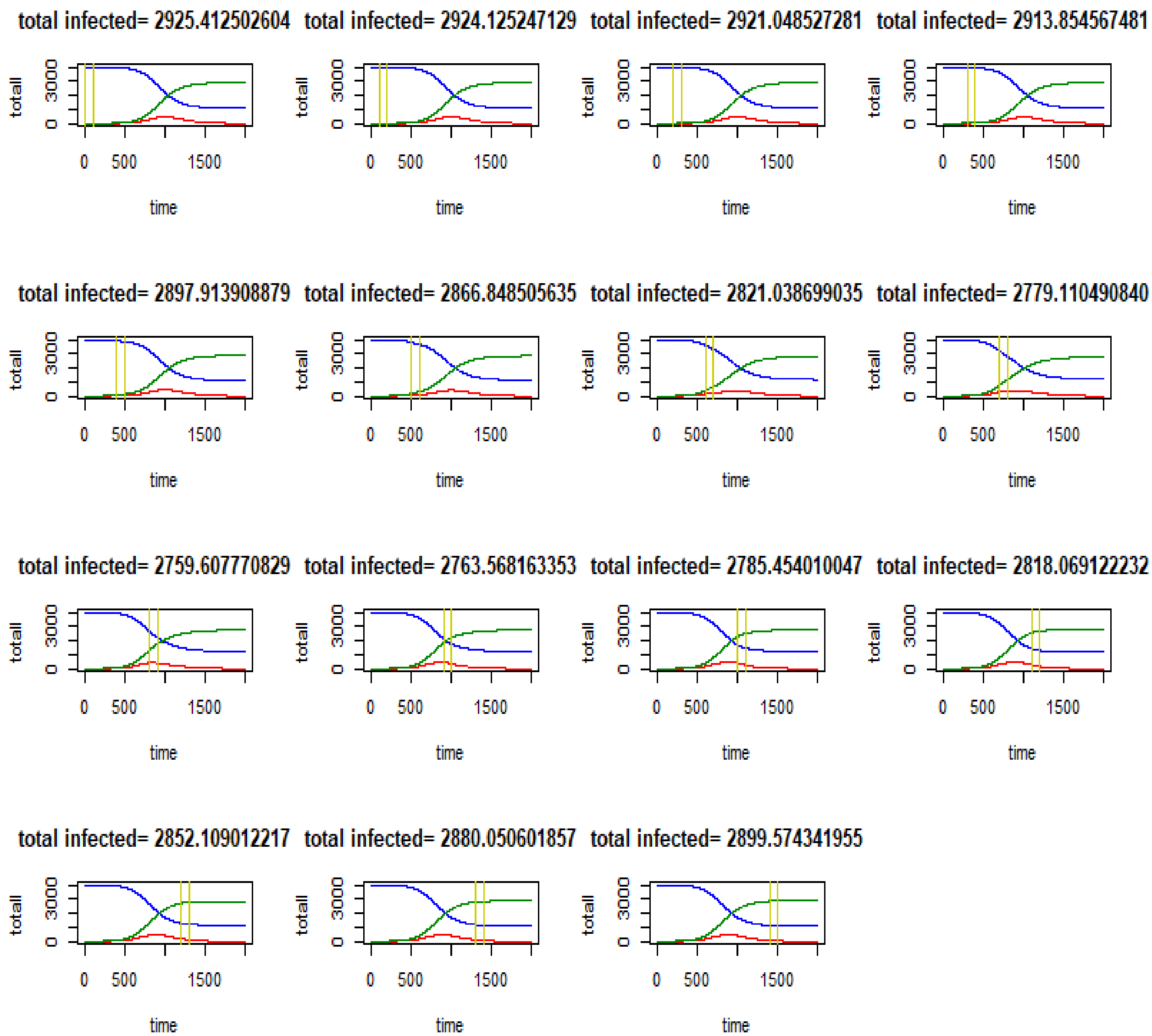


Next, we recreated a figure containing three different lines for successively smaller R_0s. R_0 represents the basic reproduction rate of the disease. In this model we define this rate as, $R_0=k_1*S_0/(k_2+k_6*C)$ where S_0 defines the entire population of susceptibles. The following figure is made by taking three values for R_0, 1.35, 1.6, and 2, and using them to solve for subsequent values of the recovery rate, k_2. From there we ran our simulation in *R* with our R_0 and k_2 values to find the maximum number of infected individuals at different quarantine carrying capacities.

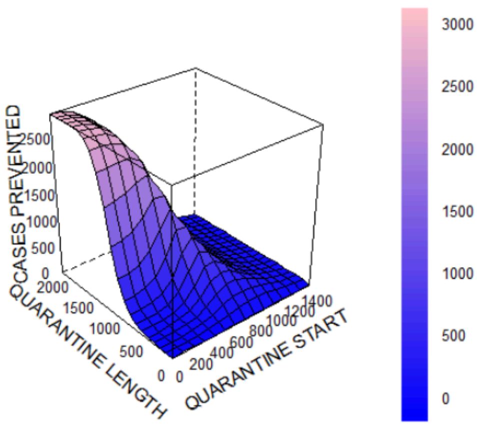
To further look into the effects of carrying capacity we recreated the graph from the paper in which they plotted the number of removed individuals over various carrying capacities and forces of infection. This graph gives us insight into which areas a change of the force of infection will create the most drastic impact on the total number of cases in the population.

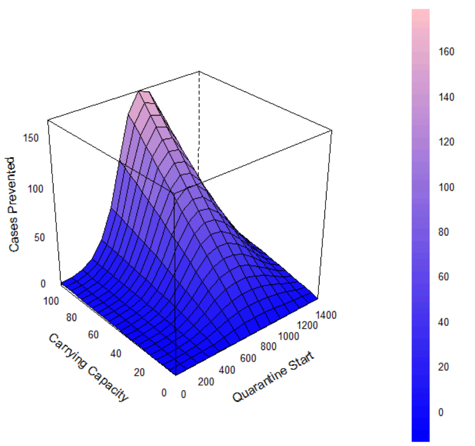


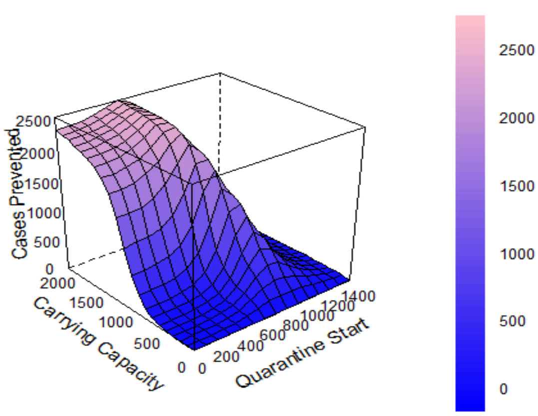
To expand the original model we wanted to see when the implementation of a quarantine facility is most effective. To do this, we wrote a program in *R* that would produce a graph of each of the 15 potential periods over which we could implement the quarantine. The quarantines take place over 100 units of time, during which we shift from a basic SIR model to the quarantine model.The quarantine uses a carrying capacity of 100 people for our population of 4000. To evaluate the effectiveness of a given quarantine during a certain timespan we can calculate the total number of cases that occur in each of those particular scenarios. This total number of cases was found by looking at the number of removed individuals at the end of each iteration.



This analysis also led to the question of how the length of the quarantine period would affect our total number of cases. To answer this we ran our analyses across quarantine lengths 0 to 2000 by intervals of 100 and quarantine starting times 0 to 1400 by intervals of 100 (Figure 5). Rather than looking at the total number of cases we decided for our analysis it would be better to look at the number of prevented cases.







We found the number of cases prevented by looking at a base case where there was no quarantine. From there we then subtracted the total cases in our other scenarios from that base total. We can see that for this carrying capacity the quarantine length does have an effect on cases prevented. We can see that the time to start a quarantine with a small quarantine length is roughly at the peak of the epidemic (around 881 in this model.) However, we see that when you have a longer quarantine length you are eventually able to start your quarantine sooner, this becomes the case very near to where the quarantine is able to entirely prevent the disease.

We also analyzed if changing the quarantine carrying capacity would have any effect on the time at which to implement the quarantine. We looked at this effect similarly to how we looked at the effect of quarantine length in Figure 5. From Figure 6, we can see that for the carrying capacities discussed we do not see any significant change on the most effective quarantine start time. We can however see that if we increase the carrying capacity greatly we can begin to observe an interesting interaction. We can see that from Figure 7, once the carrying capacity is beyond 1000, the most effective time for a quarantine begins to shift closer to the beginning of the outbreak.

Results

The results from our analyses show that the carrying capacity of the quarantine facility has a large impact on the total number of cases of an outbreak. From our phase plane we can see as we increase the carrying capacity of the quarantine facility we see increasing returns in the effectiveness of our quarantine until we hit an upper bound. These results are shown again in Figure 2 where we see an increasingly negative slope that eventually levels off as the carrying capacity increases even further. These two observations suggest an upper limit on the effectiveness of increasing the carrying capacity. At a certain level, with the latent stage effect and other considerations, there would be too many susceptibles exposed within quarantine for the facility to be effective. After further studying the example of 16th century Zurich we can conclude that a carrying capacity of around 90 could entirely prevent an outbreak.

In reference to the most effective time to implement the quarantine, it is evident that if the quarantine is applied too early in the outbreak, there is very little impact on the overall total number of individuals infected in this model. This is the case because we can not eliminate the spread of the infection early on, but this may not hold in a real-life situation. We can also see that if we implement the quarantine too late, the bulk of the outbreak has already run its course and therefore quarantine ends up being relatively useless. While we still observe a decrease in overall cases when quarantine goes into effect late, it is negligible compared to quarantining at the other times we analyzed. Looking at the most effective time to establish a quarantine it appears to be just before the peak of the epidemic. Establishing your quarantine at the peak of the epidemic in these cases prevents the most number of cases by causing the epidemic to crash at a greatly accelerated rate. This is further supported even when looking at different quarantine lengths, we can see that for smaller quarantine lengths the ideal time to establish your quarantine does not change. We do however see that if your quarantine length is long enough to sufficiently prevent the disease then it is obviously worth starting a quarantine immediately.

In regards to carrying capacity's effect on prevention of new cases, we see similar results on quarantine length. If you have a quarantine facility that has a smaller carrying capacity and can only be run for a short period of time, then the ideal time for this facility to be open is at the peak of the epidemic. However if we assume that the quarantine is done by containing individuals at home rather than at a quarantine facility (larger capacity case), we can begin to see a case for beginning quarantines earlier. If the quarantine does not have the physical limitations of a quarantine facility but can remain as effective, we can see that once you reach a threshold (roughly ¼ of the population in this case) the time at which you can prevent the most cases begins to shift to earlier quarantine starting times. This result further emphasizes the importance of carrying capacity of a quarantine that the paper previously discussed. Being able to reduce the total cases and doing it at a quicker pace is an exciting outcome.

Discussion

The results of our analyses are roughly in line with what common sense dictates. If during an outbreak, you have a method for effectively removing infected individuals from your population during the course of their infection, they present a lower risk of spreading the infection to susceptible individuals and thus the overall impact of the disease will diminish. One particularly interesting result from our analysis is that there are diminishing returns on the carrying capacity of a quarantine facility. This observation makes practical sense because if you already have enough space to bring all of the infected people into the quarantine facility, then, from the perspective of our model, having extra space available in the facility would not bring any further benefit. There may be factors beyond the ones we have considered in the model such as the reduced possibility that susceptibles in quarantine would become infected if there is more space available in the facility. Another factor that could make the quarantine more effective but that was not accounted for in the model is having a more full proof method of detection in order to ensure that those who are actually infected are the ones that are quarantined.

The implications of the most “effective” time to introduce quarantine are interesting but with these analyses, we start to lose some of the bigger picture. The direct interpretation of our results is that we should delay quarantine until the infection starts to ramp up in intensity (likely meaning a larger force of infection) however, this interpretation entirely disregards the benefits a population could observe from beginning a quarantine earlier than that. In the case of an earlier quarantine,we could observe a shift in the curve or rather a smaller number of total cases, which would still be effective in its own right but doesn’t necessarily fit with the model we created. The implementation of early quarantine could result in a greater preparedness for an epidemic. Shifting the curve allows us more time to prepare hospitals or possibly produce other preventative measures such as a vaccine. Regardless, it seems that the most effective way to control an outbreak is to be prepared ahead of time and have a course of action ready so that it can be dealt with at the right time to reduce the spread.

Another limitation of the paper’s model is that they do not account for the possibility of recovered individuals re-entering the susceptible class. As this is not addressed in our paper, our thought is that this step was removed because the proportion of people who became infected and saw full recovery was probably very small. In the 16th century, testing was not accurate and the medical community did not achieve a meaningful breakthrough in syphilis treatment until about 1910. So, most of these infected individuals likely entered the tertiary stage and died or carried the disease with them until death. However if we were to further analyze the model and make it more similar to modern day, we would want to consider adding that aspect into our model to account for that. Further, the paper does not account for the incubation period of the disease and how this plays into quarantine length. Syphilis has a 21 day incubation period on average with a maximum period of 90 days. Since there is a large delay between exposure/contraction and expression of symptoms, it would be interesting to see how that truly impacts the efficiency of quarantine as infected people can be in the population without knowing they are infected. All of these limitations can most likely be a case of limited medical capabilities during the 16th century so if we were studying a more modern case, those would be important aspects to account for in the model and the conclusion of the efficacy of quarantine.

An interesting way to expand our analysis would be to see the effect of introducing a wide array of other preventative measures. With COVID-19, we see that there has been a process in which each method of prevention was introduced but it could be insightful to create a full-fledged outbreak response timeline for our model. Along with quarantine, there are many other methods of outbreak control such as social distancing, protective facial coverings, etc. One of the most well-known and effective ways to intervene in an outbreak is to create a vaccine which builds immunity in the population. We think it would be interesting, if we had more time to expand our research, to see the interaction between quarantine, vaccination, and other prevention strategies and analyze how each step would influence the efficacy of the other. Intuitively, any prevention method would have some positive impact on reducing the size of the outbreak, but adding any of them into our model could provide insight into what is the best order to implement preventative measures in this case.

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