	Nomenclature	$\mathbf{P}_{i o j}^{max}$	3-phase maximum active powers flow of line
Acronyms	Alternative discretization with all of an Idalian	-max	segment $l_{i \to j}$.
ADMM	Alternating direction method of multipliers.	\mathbf{P}_{swing}^{max}	3-phase maximum active powers flow from the
DN DSM	Distribution network.	$\mathbf{P}^{\phi,t,pre}$	transmission grid to DN.
	Demand-side management.	$\mathbf{P}_{j,g}^{arphi,arphi,arphi}$	3-phase predicted output values of the g-th
EV	Electric vehicle.	$\mathbf{p}^{\phi,t,actual}$	renewable energy generator at node j .
ESS	Energy storage system.	$\mathbf{P}_{j,g}^{ au, au, au}$	3-phase actual output values of the g -th renew-
FCM	Fuzzy C means.	D.F.	able energy generator at node j .
GA	Genetic algorithm.	PF_c	Power factor of customer c .
GBD	Generalized Bendeers decomposition.	$\mathbf{Q}_{i\rightarrow j}^{max}$	3-phase maximum reactive powers flow of line
HEMS	Home energy management system.	o mar	segment $l_{i \to j}$.
HVAC	Heat, ventilation and air conditioning.	\mathbf{Q}_{swing}^{max}	3-phase maximum reactive powers flow from
IoT	Internet of things.	d t	the transmission grid to DN.
KKT	Karush-Kuhn-Tucker optimality conditions.	$\mathbf{q}_{j,v}^{\phi,t} \ p_{c}^{PV,t}$	Output of SVG at node j ,
LAN	Local area network.	$p_{c}^{PV,t}$	PV output power at time slot t .
LMP	Locational marginal price.	$p_c^{fix,t}$	Fixed load power of customer c at time slot t .
MILP	Mixed-integer linear programming.	$p^{ESS,max}$	Maximum output power of ESS.
PV	Photovoltaic.	$p^{ESS,min}$	Minimum output power of ESS.
SOC	State of charge.	$p^{EV,max,ch}$	Maximum charing power of EV.
SVG	Static var generator.	$p^{EV,max,d}$	Maximum discharing power of EV.
WT	Wind turbine.	$p^{EV,min,ch}$	Minimum charing power of EV.
		$p^{EV,min,d}$	Minimum discharing power of EV.
Index		$p^{HVAC,max}$	Maximum power of HVAC.
c	Index of customer.	$p^{HVAC,min}$	Minimum power of HVAC.
g	Index of generator.	$p_{j,g,s}^{\phi,t}$	Output of renewable energy in scenario s .
i, j	Index of node/bus.	$SOC^{ESS,m}$	Maximum SOC of ESS.
k	Index of cluster center.	$SOC^{ESS,m}$	in Minimum SOC of ESS.
s	Index of scenario.	$SOC^{ESS,ir}$	¹ Initial SOC of ESS.
t	Index of time.		coect Customer expected SOC of EV.
v	Index of SVG.	$SOC^{EV,ma}$	Maximum SOC of EV.
ϕ	Index of phase.	$SOC^{EV,mi}$	
Τ		$tag^{\phi,t}$	
Sets		tag	Tap position of the interconnecting transformer at time slot t .
\mathcal{C}	Set of customers.	t^{back}	
\mathcal{G}_j	Set of generators at bus j .	$t \\ t^{depart}$	EV's arrival time.
\mathcal{L}	Set of line segments.	U	EV's departure time.
$\widetilde{\mathcal{N}}$	Set of nodes.	$T_c^{set,t} \ T^{out,t}$	Set temperature at time slot t .
\mathcal{N}_j	Set of all buses located strictly downstream of	$T_c^{out,v}$ T^{max}	Outdoor temperature at time slot t .
\mathcal{I} $\mathcal{I}_{\mathcal{J}}$	bus j .	T^{min}	Maximum temperature.
\mathcal{T}	Set of time.		Minimum temperature.
	Set of time. Set of SVG at bus j .	$egin{array}{c} \mathbf{v}_{swing}^{\phi,t} \ \mathbf{v}_{i}^{max} \end{array}$	3-phase voltage at the swing node.
$\mathcal{V}_j \ \mathcal{S}$	Set of seenarios.	\mathbf{v}_i^{max}	3-phase maximum voltage at node <i>i</i> .
		\mathbf{v}_i^{min}	3-phase minimum voltage at node i .
$\mathcal{S}_K \ \mathcal{T}^{ex}$	Set of reduced scenarios.	α	Temperature sensitivity of customer.
,	Set of time slots when the node power exceeds	β	Temperature-to-heat flux conversion coeffi-
	the limit.		cient.
D 4		ΔT_c^t	Difference between outdoor and indoor tem-
Parameters			perature at time slot t .
$d_{s,k}$	Euclidean distance between scenario s and	γ	conversion coefficient between HVAC power
	center k .		and released heat flux.
e FFSS mar	EV's energy consumption (kWh per km).	$\eta^{ESS}_{}$	ESS discharging efficiency.
$E^{ESS,max}$	ESS's battery capacity.	$\eta^{EV} \ heta^{EV}$	EV discharging efficiency.
$E_{\phi}^{EV,max}$	EV's battery capacity.		EV's battery degradation cost.
\mathbf{h}_{j}^{ϕ}	3-phase households at node j .	$ heta^{ESS}$	ESS's battery degradation cost.
k	heat flux-to-temperature conversion coefficient.		
LMP^t	LMP at time slot t .	Variables	
$m_{\overline{z}}$	EV's daily driving distance.	$H^{ex,t}$	Heat flux exchange with outdoor at time slot t .
$\mathbf{M}_{i o j}^P$	3-phase resistance of line segment $l_{i \to j}$.	$H_c^{HVAC,t}$	Heat flux of HVAC at time slot t .
$egin{aligned} \mathbf{M}_{i o j}^P \ \mathbf{M}_{i o j}^Q \end{aligned}$	3-phase reactance of line segment $l_{i \to j}$.	C	
- 'J	-		

o_k	The k -th cluster center.
$p_c^{ESS,t}$	ESS discharging power at time t .
$p_c^{EV,t,ch}$	EV charging power at time t .
$p_c^{EV,t,d}$	EV discharging power at time t .
nHVAC,t	HVAC power at time t .
$n^{\phi,t}$	Power of g -th generator at node j .
$\mathbf{p}_{j.max}^{ ho, j, g}$	3-phase maximum power of node j at time t .
\mathbf{p}_{j} $\mathbf{p}_{\phi,t}$	
$\mathbf{p}_{j,c}^{\phi,t}$	3-phase active power of customer c at node j
$\mathbf{p}^{\phi,t}$	at time slot t .
$\mathbf{P}_{i o j}^{\phi,t}$	3-phase active powers flow of line segment
$\mathbf{p}^{\phi,t}$	$l_{i o j}.$
$\mathbf{P}_{j\rightarrow k}^{\phi,t}$	3-phase active powers flow of line segment
III	$l_{j o k}$.
$egin{aligned} \mathbb{P}_k \ \mathbf{Q}_{i ightarrow j}^{\phi, t} \end{aligned}$	probability of occurrence of k -th cluster center.
$\mathbf{Q}_{i\to j}^{r,r}$	3-phase reactive powers flow of line segment
$\mathbf{a}\phi \cdot t$	$l_{i o j}.$
$\mathbf{Q}_{j\to k}^{\phi,t}$	3-phase reactive powers flow of line segment
ϕt	$l_{j o k}$.
$\mathbf{q}_{j,c}^{\phi,t}$	3-phase reactive power of customer c at node
Frin t	j at time slot t .
$T_c^{in,t}$	Indoor temperature of customer c at time slot
	t.
$u_{s,k}$	Membership degree between scenario s and the
$\phi.t$	k-th cluster center.
$\mathbf{v}_{j_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{1}}}}}}}}}$	3-phase voltage at node j at time slot t .
$rac{\mathbf{v}_{j}^{\phi,t}}{\mathbf{\delta}_{c}^{p,t}}$	Dual variable of j -th node's slack problem.
cn t	
$\boldsymbol{\delta}_{c}^{\scriptscriptstyle P}$	Complementary slackness variable of maxi-
	mum HVAC power constraint.
	mum HVAC power constraint.
$oldsymbol{\delta}_{c}^{T,t}$	mum HVAC power constraint. Complementary slackness variable of maxi-
$\overline{\boldsymbol{\delta}_{c}^{T,t}}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint.
	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of mini-
$oldsymbol{\delta_c^{T,t}} \ oldsymbol{\delta_c^{p,t}}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint.
$\overline{\boldsymbol{\delta}_{c}^{T,t}}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum HVAC power constraint.
$egin{aligned} \overline{oldsymbol{\delta}_{c}^{T,t}} \ & \underline{oldsymbol{\delta}_{c}^{p,t}} \ & \underline{oldsymbol{\delta}_{c}^{T,t}} \end{aligned}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum temperature power constraint.
$oldsymbol{\delta_c^{T,t}} \ oldsymbol{\delta_c^{p,t}}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum temperature power constraint. Lagrange multipliers corresponding to the up-
$egin{aligned} \overline{oldsymbol{\delta}_{c}^{T,t}} \ & \underline{oldsymbol{\delta}_{c}^{p,t}} \ & \underline{oldsymbol{\delta}_{c}^{T,t}} \ & \overline{oldsymbol{\lambda}_{c}^{p,t}} \end{aligned}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum temperature power constraint. Lagrange multipliers corresponding to the upper bound constraints on HVAC power.
$egin{aligned} \overline{oldsymbol{\delta}_{c}^{T,t}} \ & \underline{oldsymbol{\delta}_{c}^{p,t}} \ & \underline{oldsymbol{\delta}_{c}^{T,t}} \end{aligned}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum temperature power constraint. Lagrange multipliers corresponding to the upper bound constraints on HVAC power. Lagrange multipliers corresponding to the upper bound constraints on the up
$egin{aligned} \overline{oldsymbol{\delta}_{c}^{T,t}} \ & \underline{oldsymbol{\delta}_{c}^{p,t}} \ & \underline{oldsymbol{\delta}_{c}^{T,t}} \ & \overline{oldsymbol{\lambda}_{c}^{p,t}} \ & \overline{oldsymbol{\lambda}_{c}^{T,t}} \end{aligned}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum temperature power constraint. Lagrange multipliers corresponding to the upper bound constraints on HVAC power. Lagrange multipliers corresponding to the upper bound constraints on indoor temperature.
$egin{aligned} \overline{oldsymbol{\delta}_{c}^{T,t}} \ & \underline{oldsymbol{\delta}_{c}^{p,t}} \ & \underline{oldsymbol{\delta}_{c}^{T,t}} \ & \overline{oldsymbol{\lambda}_{c}^{p,t}} \end{aligned}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum temperature power constraint. Lagrange multipliers corresponding to the upper bound constraints on HVAC power. Lagrange multipliers corresponding to the upper bound constraints on indoor temperature. Lagrange multipliers corresponding to the
$egin{aligned} \overline{oldsymbol{\delta}_{c}^{T,t}} \ & \underline{oldsymbol{\delta}_{c}^{p,t}} \ & \underline{oldsymbol{\delta}_{c}^{T,t}} \ & \overline{oldsymbol{\lambda}_{c}^{p,t}} \ & \overline{oldsymbol{\lambda}_{c}^{T,t}} \end{aligned}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum temperature power constraint. Lagrange multipliers corresponding to the upper bound constraints on HVAC power. Lagrange multipliers corresponding to the upper bound constraints on indoor temperature. Lagrange multipliers corresponding to the lower bound constraints on HVAC power tem-
$egin{aligned} \overline{oldsymbol{\delta}_{c}^{T,t}} \ \overline{oldsymbol{\delta}_{c}^{T,t}} \ \overline{oldsymbol{\lambda}_{c}^{T,t}} \ \overline{oldsymbol{\lambda}_{c}^{T,t}} \ \overline{oldsymbol{\lambda}_{c}^{T,t}} \ \overline{oldsymbol{\lambda}_{c}^{T,t}} \end{aligned}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum temperature power constraint. Lagrange multipliers corresponding to the upper bound constraints on HVAC power. Lagrange multipliers corresponding to the upper bound constraints on indoor temperature. Lagrange multipliers corresponding to the lower bound constraints on HVAC power temperature.
$egin{aligned} \overline{oldsymbol{\delta}_{c}^{T,t}} \ & \underline{oldsymbol{\delta}_{c}^{p,t}} \ & \underline{oldsymbol{\delta}_{c}^{T,t}} \ & \overline{oldsymbol{\lambda}_{c}^{T,t}} \ & \overline{oldsymbol{\lambda}_{c}^{T,t}} \ & \underline{oldsymbol{\lambda}_{c}^{T,t}} \ & \underline{oldsymbol{\lambda}_{c}^{T,t}} \end{aligned}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum temperature power constraint. Lagrange multipliers corresponding to the upper bound constraints on HVAC power. Lagrange multipliers corresponding to the upper bound constraints on indoor temperature. Lagrange multipliers corresponding to the lower bound constraints on HVAC power temperature. Lagrange multipliers corresponding to the
$egin{aligned} \overline{oldsymbol{\delta}_{c}^{T,t}} \ & \underline{oldsymbol{\delta}_{c}^{p,t}} \ & \underline{oldsymbol{\delta}_{c}^{T,t}} \ & \overline{oldsymbol{\lambda}_{c}^{T,t}} \ & \overline{oldsymbol{\lambda}_{c}^{T,t}} \ & \underline{oldsymbol{\lambda}_{c}^{T,t}} \ & \underline{oldsymbol{\lambda}_{c}^{T,t}} \end{aligned}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum temperature power constraint. Lagrange multipliers corresponding to the upper bound constraints on HVAC power. Lagrange multipliers corresponding to the upper bound constraints on indoor temperature. Lagrange multipliers corresponding to the lower bound constraints on HVAC power temperature. Lagrange multipliers corresponding to the lower bound constraints on indoor temperature.
$egin{aligned} \overline{oldsymbol{\delta}_{c}^{T,t}} \ \overline{oldsymbol{\delta}_{c}^{T,t}} \ \overline{oldsymbol{\lambda}_{c}^{T,t}} \ \overline{oldsymbol{\lambda}_{c}^{T,t}} \ \overline{oldsymbol{\lambda}_{c}^{T,t}} \ \overline{oldsymbol{\lambda}_{c}^{T,t}} \end{aligned}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum temperature power constraint. Lagrange multipliers corresponding to the upper bound constraints on HVAC power. Lagrange multipliers corresponding to the upper bound constraints on indoor temperature. Lagrange multipliers corresponding to the lower bound constraints on HVAC power temperature. Lagrange multipliers corresponding to the lower bound constraints on indoor temperature. Variable of slack problem of node <i>j</i> at time
$egin{aligned} \overline{oldsymbol{\delta}_{c}^{T,t}} \ & \underline{oldsymbol{\delta}_{c}^{p,t}} \ & \underline{oldsymbol{\delta}_{c}^{T,t}} \ & \overline{oldsymbol{\lambda}_{c}^{T,t}} \ & \overline{oldsymbol{\lambda}_{c}^{T,t}} \ & \underline{oldsymbol{\lambda}_{c}^{T,t}} \ & \underline{oldsymbol{\lambda}_{c}^{T,t}} \end{aligned}$	mum HVAC power constraint. Complementary slackness variable of maximum temperature power constraint. Complementary slackness variable of minimum HVAC power constraint. Complementary slackness variable of minimum temperature power constraint. Lagrange multipliers corresponding to the upper bound constraints on HVAC power. Lagrange multipliers corresponding to the upper bound constraints on indoor temperature. Lagrange multipliers corresponding to the lower bound constraints on HVAC power temperature. Lagrange multipliers corresponding to the lower bound constraints on indoor temperature.

problem of node j.

charging power of ESS.

charging power of EV.

ing power of EV.

Optimal value of the slack problem's dual

Simplex multiplier corresponding to the dis-

Simplex multiplier corresponding to the charg-

Simplex multiplier corresponding to the dis-

Simplex multiplier corresponding to the power

problem of customer c at node j.

 $\Pi_{i,c}^*$

 $oldsymbol{\pi}_{j,c}^{ESS,\phi,t}$

 $\pmb{\pi}_{j,c}^{EV,\phi,t,ch}$

of HVAC. $\mu^{t,ch}$ Indicator of EV's charging status at time slot $\mu^{t,d}$ Indicator of EV's discharging status at time slot t. **Function** F The Lagrangian function of the slave problem in the two-layer Stackelberg game of HEMS. \mathscr{L} The objective function part of the Lagrangian function for the slave problem in the two-layer Stackelberg game of HEMS. The constraints part of the Lagrangian function for the slave problem in the two-layer Stackelberg game of HEMS.

APPENDIX A. TWO-LAYER STACKELBERG GAME OF HEMS-EQUIVALENT KKT OPTIMALITY CONDITIONS

The objective function of FCM.

As mentioned in Section VIII, to facilitate the derivation of the KKT optimality conditions, we reformulate the lower-level problem in matrix form, where the objective function can be rewritten as:

$$\begin{split} \min \mathscr{F}(\mathbf{p}_{c}^{HVAC,t}) &= \sum_{t \in \mathcal{T}} \left[\alpha_{c}^{t} \right]_{1 \times 24} \cdot \\ \left[\left\| \left[T_{c}^{in,t} \right]_{1 \times 24}^{T} - \left[T_{c}^{set,t} \right]_{1 \times 24}^{T} \right\|^{2} \right] \end{split} \tag{A1}$$

where $\left[\alpha_c^t\right]$, $\left[T_c^{in,t}\right]$, and $\left[T_c^{set,t}\right]$ are 1×24 vectors, representing the customer's temperature sensitivity, indoor temperature, and expected temperature settings, respectively, over a 24-hour period.

The Lagrangian function of the slave problem ${\mathscr L}$ can be expressed as:

$$\begin{split} & \mathscr{L}(\mathbf{p}_{c}^{HVAC,t}, \underline{\boldsymbol{\lambda}_{c}^{p,t}}, \overline{\boldsymbol{\lambda}_{c}^{p,t}}, \underline{\boldsymbol{\lambda}_{c}^{T,t}}, \overline{\boldsymbol{\lambda}_{c}^{T,t}}) = \mathscr{F}(\mathbf{p}_{c}^{HVAC,t}) \\ & + \left[\underline{\boldsymbol{\lambda}_{c}^{p,t}} \cdot \left(\left[p^{HVAC,min} \right]_{1\times24}^{T} - \left[p^{HVAC,t}_{c} \right]_{1\times24}^{T} \right) \right] \\ & + \left[\overline{\boldsymbol{\lambda}_{c}^{p,t}} \cdot \left(\left[p^{HVAC,t}_{c} \right]_{1\times24}^{T} - \left[p^{HVAC,max} \right]_{1\times24}^{T} \right) \right] \\ & + \left[\underline{\boldsymbol{\lambda}_{c}^{T,t}} \cdot \left(\left[T^{min} \right]_{1\times24}^{T} - \left[T^{in,t}_{c} \right]_{1\times24}^{T} \right) \right] \\ & + \left[\overline{\boldsymbol{\lambda}_{c}^{T,t}} \cdot \left(\left[T^{in,t}_{c} \right]_{1\times24}^{T} - \left[T^{max} \right]_{1\times24}^{T} \right) \right] \end{split} \tag{A2}$$

where $\underline{\lambda}_{c}^{p,t}$, $\overline{\lambda}_{c}^{p,t}$, $\underline{\lambda}_{c}^{T,t}$, and $\overline{\lambda}_{c}^{T,t}$ are 1×24 vectors, denoting the Lagrange multipliers for each of the 24 time slots.

According to the KKT optimality conditions, the necessary conditions for obtaining the optimal solution are:

$$\frac{\partial \mathcal{L}}{\partial (\mathbf{p}_{c}^{HVAC,t})} = \frac{\partial \mathcal{F}}{\partial (\mathbf{p}_{c}^{HVAC,t})} + \frac{\partial \mathcal{G}}{\partial (\mathbf{p}_{c}^{HVAC,t})} = 0 \quad (A3)$$

where \mathscr{G} represents the inequality constraints of the slave problem, which are constraints(58), (59).

For the convenience of the following derivation, we will express $\frac{\partial \mathscr{F}}{\partial (\mathbf{p}_c^{HVAC,t})}$ and $\frac{\partial \mathscr{G}}{\partial (\mathbf{p}_c^{HVAC,t})}$ in the form of 1×24 matrices as shown below.

$$\frac{\partial \mathcal{F}}{\partial (\mathbf{p}_{c}^{HVAC,t})} = \sum_{t \in \mathcal{T}} \left[\frac{\partial \mathcal{F}}{\partial (\mathbf{p}_{c}^{HVAC,t})} \right]_{1 \times 24} \tag{A4}$$

$$\frac{\partial \mathcal{G}}{\partial (\mathbf{p}_c^{HVAC,t})} = \sum_{t \in \mathcal{T}} \left[\frac{\partial \mathcal{G}}{\partial (\mathbf{p}_c^{HVAC,t})} \right]_{1 \times 24} \tag{A5}$$

The expression for $\left[\frac{\partial \mathscr{F}}{\partial (\mathbf{p}_c^{HVAC,t})}\right]$ is shown below:

$$\begin{bmatrix}
\frac{\partial \mathscr{F}}{\partial (\mathbf{p}_{c}^{HVAC,t})}
\end{bmatrix} = 2 \left[\alpha_{c}^{t}\right]_{1\times24} \odot \left[\left\|\left[T_{c}^{in,t}\right]_{1\times24} - \left[T_{c}^{set,t}\right]_{1\times24}\right\|\right]$$

$$\odot \left[\frac{\partial T_{c}^{in,t}}{\partial (p_{c}^{HVAC,t})}\right]_{1\times24} \tag{A6}$$

To facilitate subsequent derivations, we express the partial derivative operations in the following matrix form:

$$\frac{\partial}{\partial(\mathbf{p}_{c}^{HVAC,t})} = \left[\frac{\partial}{\partial(p_{c}^{HVAC,1})} \cdots \frac{\partial}{\partial(p_{c}^{HVAC,t})} \cdots \frac{\partial}{\partial(p_{c}^{HVAC,24})}\right]_{1 \times 24} \tag{A7}$$

The vector $\mathbf{T}_{j,c}^{in,t}$ can be expanded as:

$$\mathbf{T}_{c}^{in,t} = \begin{bmatrix} T_{c}^{in,1} + k[\beta(T_{c}^{out,1} - T_{c}^{in,1}) - \gamma p_{c}^{HVAC,1}] & \dots \\ T_{c}^{in,t} + k[\beta(T_{c}^{out,t} - T_{c}^{in,t}) - \gamma p_{c}^{HVAC,t}] & \dots \\ T_{c}^{in,24} + k[\beta(T_{c}^{out,24} - T_{c}^{in,24}) - \gamma p_{c}^{HVAC,24}] \end{bmatrix}_{24 \times 1}^{T}$$
(A8)

Due to the temporal coupling of heat transfer effects, the indoor temperature at a given time slot is related to the HVAC power consumption of all previous time slots, which can be expressed as a functional relationship.

$$T_c^{in,t} = f(p_c^{HVAC,1}, p_c^{HVAC,2}, ..., p_c^{HVAC,t-1})$$
 (A9)

Then, the partial derivative of the indoor temperature $\mathbf{T}_c^{in,t}$ with respect to the decision variable $\mathbf{p}_c^{HVAC,t}$ can be obtained for each time slot as:

$$\mathbf{T}_{c}^{in,t} \otimes \frac{\partial}{\partial (\mathbf{p}_{j,c}^{HVAC,t})} = -\begin{bmatrix} \gamma & \cdots & (1-k\beta)\gamma \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma \end{bmatrix}_{24 \times 24}^{T}$$
(A10)

$$\mathcal{CO} = \sum_{t \in \mathcal{T}} \mathbf{T}_{c}^{in,t} \otimes \frac{\partial}{\partial (\mathbf{p}_{j,c}^{HVAC,t})} = -$$

$$\begin{bmatrix} \sum_{t \in \mathcal{T}} (\gamma & \cdots & (1 - k\beta)\gamma) \\ \sum_{t \in \mathcal{T}} (\vdots & \ddots & \vdots) \\ \sum_{t \in \mathcal{T}} (0 & \cdots & \gamma) \end{bmatrix}^{T}$$

$$= -\begin{bmatrix} \sum_{t=1} (\gamma + (24 - 1)(1 - k\beta)\gamma) \\ \cdots \\ \sum_{t \in \mathcal{T}} (\gamma + (24 - t)(1 - k\beta)\gamma) \\ \cdots \end{bmatrix}^{T}$$

$$= \begin{bmatrix} \sum_{t=1} (\gamma + (24 - t)(1 - k\beta)\gamma) \\ \cdots \\ \cdots \end{bmatrix}^{T}$$

$$= \begin{bmatrix} \frac{\partial T_{c}^{in,t}}{\partial (p_{c}^{HVAC,t})} \end{bmatrix}_{1 \times 24}$$

where the matrix in A10 has diagonal elements of γ , lower triangular elements of 0, and upper triangular elements of $(1 - k\beta)\gamma$. The coefficient of the partial derivative of indoor temperature with respect to HVAC power at each time slot t is the sum of the elements in the t-th row of the matrix, denoted as \mathcal{CO} .

Similarly, taking the partial derivative of the inequality constraints with respect to the decision variables yields the mathematical expression of $\left[\frac{\partial \mathscr{G}}{\partial (\mathbf{p}_{c}^{HVAC,t)}}\right]$:

$$\begin{bmatrix} \frac{\partial \mathcal{G}}{\partial (\mathbf{p}_{c}^{HVAC,t})} \end{bmatrix} = -\begin{bmatrix} \frac{\lambda_{c}^{p,1}}{\cdots} \\ \frac{\lambda_{c}^{p,24}}{\cdots} \end{bmatrix}_{24\times1}^{T} + \begin{bmatrix} \overline{\lambda_{c}^{p,1}} \\ \cdots \\ \overline{\lambda_{c}^{p,24}} \end{bmatrix}_{24\times1}^{T} \\
-\mathcal{CO} \odot \begin{bmatrix} \frac{\lambda_{c}^{T,1}}{\cdots} \\ \frac{\lambda_{c}^{T,24}}{\cdots} \end{bmatrix}_{24\times1}^{T} + \mathcal{CO} \odot \begin{bmatrix} \overline{\lambda_{c}^{T,1}} \\ \cdots \\ \overline{\lambda_{c}^{T,24}} \end{bmatrix}_{24\times1}^{T}$$
(A12)

Substituting $\left[\frac{\partial T^{in,t}}{\partial (p^{HV}_c AC,t)}\right]$ and $\left[\frac{\partial \mathscr{G}}{\partial (\mathbf{p}^{HV}_c AC,t)}\right]$ into the Lagrangian equation yields the mathematical expression for the KKT optimality conditions. By incorporating these into the master problem, the two-layer Stackelberg game can be transformed into a single-level optimization problem.

 $\begin{array}{c} \text{Appendix B. Parameter Settings in Numerical} \\ \text{Tests} \end{array}$

A. HVAC Parameter Settings

TABLE B1 HVAC PARAMETER SETTINGS

Parameter	Value	Parameter	Value
$k({}^{\circ}C \cdot m^2/kW)$	1	$p^{HVAC,max}(kW)$	2
$\beta(kW/^{\circ}C\cdot m^2)$	1.2	$T^{min}(^{\circ}C)$	22
$\gamma(1/m^2)$	6	$T^{max}(\circ C)$	26
$p^{HVAC,min}(kW)$	0.5	, ,	height

As shown in table B1, where k is the heat flux-to-temperature conversion coefficient for changes in heat flux inside the house, with units of m, set to 1 in this paper. β is the temperature-to-heat flux conversion coefficient for the temperature difference between indoors and outdoors, which is related to the size and material of the house, also with units of m, set to 1.2 here. γ is the conversion coefficient between HVAC power and released heat flux, with units of m, the numerical values is 6. The power range for the HVAC system is between 0.5kW and 2kW, while the temperature range is between $22^{\circ}C$ and $26^{\circ}C$.

B. EV Parameter Settings

Table B2 lists the parameters related to the EV. The power range for both charging and discharging of EV are set between 0 and 6 kW, with efficiency of 90%. The customer's expected EV SOC value is 0.95, EV's battery capacity $E^{EV,max}$ is 60 kWh, with an energy consumption e of 0.25 kWh per km. The battery degradation cost θ^{EV} is 0.01 USD per kW.

C. ESS Parameter Settings

The parameters related to the ESS are listed in table B3. The discharge power range is 0-5 kW, with an efficiency of 90% and a battery capacity $E^{ESS,max}$ of 30 kWh. The battery degradation cost of ESS θ^{ESS} is 0.01 USD per kW.

TABLE B2 EV PARAMETER SETTINGS

Parameter	Value	Parameter	Value
$p^{EV,min,ch}(kW)$	0	$SCO^{EV,min}$	0.0
$p^{EV,max,ch}(kW)$	6	$SCO^{EV,expect}$	0.95
$p^{EV,min,c}(kW)$	0	$E^{EV,max}(kWh)$	60
$p^{EV,max,c}(kW)$	6	e(kWh/km)	0.25
$\eta^{EV}(\%)$	90%	$\theta^{\dot{E}V}(\$/kW^2)$	0.01
$SOC^{EV,max}$	0.95		

TABLE B3 ESS PARAMETER SETTINGS

Parameter	Value	Parameter	Value
$p^{ESS,min}(kW)$	0	$\theta^{ESS}(\$/kW^2)$	0.01
$p^{ESS,max}(kW)$	5	$SOC^{\grave{E}SS,min}$	0.0
$\eta^{ESS}(\mathring{\%})$		$SOC^{ESS,max}$	1.0
$E^{ESS,max}(kWh)$	30		

D. SADN Parameter Settings

TABLE B4 SADN PARAMETER SETTINGS

Parameter	Value	Parameter	Value
v_i^{min}	0.95	P_{swing}^{min}	1.5
v_i^{max}	1.05	Q_{swing}^{max}	0.75
$P_{i \to j}^{max,3\phi}$	1.5	$q_{83}^{\phi,t}$	0.2
$Q_{i \to j}^{max,3\phi}$	0.75	$q_{88}^{a,t}$	0.05
$P_i^{max,\phi}$	0.75	$q_{90}^{b,t}$	0.05
$O^{max,\phi}$	0.375	$q_{92}^{c,t}$	0.05
$tag^{\phi,t}$	4		

Table B4 contains parameters of SADN, all in per-unit values. The base value for voltage is 480 V, and the base value for power is 1 MVA. To ensure the reliability of electricity supply, the node voltage fluctuation must not exceed 5%. Therefore, parameters v_i^{min} and v_i^{max} are set to 0.95 and 1.05, respectively. In the SADN, there are both 3-phase and singlephase transmission lines. The active power limit $P_{i o j}^{max,3\phi}$ for the 3-phase transmission lines is 1.5, and the reactive power limit $Q_{i \to j}^{max,3\phi}$ is 0.75. For the single-phase transmission lines, the active power limit $P_{i \to j}^{max,\phi}$ is 0.75, and the reactive power limit $Q_{i o j}^{max,\phi}$ is 0.375. The transformer's active power limit per phase P_{swinq}^{min} is 1.5, and the reactive power limit Q_{swinq}^{max} is 0.75. The transformer tags are set to 4 for all 3 phases, the voltage value on the low-voltage side is 1.025 according to (11). A 3-phases SVG is aggregated at node 83, capable of supplying 0.2 reactive power per phase. At nodes 88, 90, and 92, the single-phase SVGs can supply 0.05 reactive power to phases a, b, and c, respectively.

E. Node 76 A-Phase Customers Parameter Settings

Table B5 lists the parameters for the six households of node 76 A-phase, as generated in Section ??. These parameters include the EV return time t^{back} , EV departure time t^{depart} , daily driving distance m, indoor temperature setting T^{set} , and sensitivity to temperature α . Due to the varying electricity

usage behavior of the customers on the previous day, the remaining energy in the ESS also differs.

TABLE B5
PARAMETER SETTINGS OF HOUSEHOLD AT NODE 76

	t^{back}	t^{depart}	T^{set}	α	m	$SCO^{EV,t^{back}}$	$SOC^{ESS,in}$
H1	6 p.m.	9 a.m.	$24^{\circ}C$	0.94	111.75 km	0.484	0.500
H2	7 p.m.	10 a.m.	$24.5^{\circ}C$	1.08	19.91 km	0.867	0.250
H3	5 p.m.	10 a.m.	$24.7^{\circ}C$	1.01	126.61 km	0.423	0.333
H4	5 p.m.	9 a.m.	$24.4^{\circ}C$	1.05	61.52 km	0.694	0.417
H5	5 p.m.	9 a.m.	$24.2^{\circ}C$	0.98	35.99 km	0.801	0.200
					168.06 km	0.250	0.583

F. Households Location in SADN

The aggregation of households at each node in SADN is shown in table B6, where phase A aggregates 71 households, phase B aggregates 49 households, and phase C aggregates 60 households.

TABLE B6 HOUSEHOLDS IN SADN

Phase A		Phase B		Phase C		
Node	Number of	Node	Number of	Node	Number of	
name	households	name	households	name	households	
1	2	2	1	4	2	
7	1	12	1	5	1	
9	2	22	2	6	2 2	
11	2	35	1	34	2	
10	1	38	1	16	2	
19	2	39	1	17	1	
20	2	43	2	24	2	
28	2	47	2	31	1	
33	2	48	4	30	2	
29	2	49	3	32	1	
35	1	56	1	41	1	
37	2	58	1	47	2	
42	1	59	1	48	4	
45	1	64	4	49	2	
47	2	65	2	50	2 2	
46	1	76		62	2	
48	3	77	5 2	65	2 3	
49	2	86	1	66	4	
51	1	80	2	73		
52	2	87	2 2	76	2 5	
53	2	90	2	74	2	
55	1	95	1	75	2 2	
60	1	96	1	84	1	
63	2	99	2	83	1	
65	2	106	2	85	2	
68	1	107	2	92	2	
69	2			100	2	
70	1			102	1	
71	2			103	2	
76	6			104	2	
79	2					
82	2					
88						
94	2 2					
98	2					
109	2					
111	1					
112	1					
113	2					
114	1					
114	1					