Appendix A. Analysis of the Optimality of the Personalized-Preserving GBD

Consider the following optimization problem:

$$\min \sum_{i}^{n} f_i(x_i) \tag{A1}$$

$$\text{s.t. : } g_j(x_i) \leq 0 \quad j = 1, 2, ..., J \tag{A2}$$

$$h_k(x_i) = 0 \quad k = 1, 2, ..., K \tag{A3}$$

$$v_l(x) \leq 0 \quad l = 1, 2, ..., L \tag{A4}$$

where (A1) is the objective function, which consists of n independent sub objective functions $f_1(x_1)$ to $f_n(x_n)$. Constraints (A2) and (A3) represent J inequality constraints and K equality constraints corresponding to each x_i, respectively. However, constraint (A4) involves all the decision variables x_1 to x_n, denoted as x. When directly solving the above optimization problem, as the number of sub-problems increases, the computational complexity and solving time will also increase.

To achieve decentralized optimization while avoiding sharing the specific values of x_i, we use the proposed privacy-preserving GBD to handle the optimization problem by reformulating it into an upper problem and a slack problem.

Upper Problem:

$$\min \sum_{i}^{n} f_i(x_i) \tag{A5}$$

$$\text{s.t. : } g_j(x_i) \leq 0 \quad m = 1, 2, ..., J \tag{A6}$$

$$h_k(x_i) = 0 \quad n = 1, 2, ..., K \tag{A7}$$

$$\text{Benders feasibility cuts} \tag{A8}$$

Slack Problem

$$\min \Pi = \sum_{1}^{L} \epsilon_l \tag{A9}$$

$$\text{s.t. : } v_l(x) - \epsilon_l \leq 0 \quad l = 1, 2, ..., L \tag{A10}$$

$$0 \leq \epsilon_i \tag{A11}$$

where ϵ_l is the slack variable of the l-th constraint, the objective of the slack problem is to minimize the values of the slack variables. When the objective function value of slack problem is zero, it indicates that the constraints are not violated, and the corresponding solution x^* at this point is the optimal solution to the original problem.

Assume $\hat{x}$ is the initial solution of the upper-level problem, let Π^* be the optimal value of the slack problem's objective function. While $\Pi^* > 0$, it means that some constraints has been violated. Then We introduce the following Benders feasibility cut to guide x so that Π^* becomes zero.

$$\Pi^* + \Delta\Pi \leq 0 \tag{A12}$$

By performing a first-order Taylor expansion of Π at $\hat{x}$, the relationship between $\hat{x}$ and $\Delta\Pi$ is obtained as follows:

$$\Delta\Pi = \sum_{i}^{n} \frac{\partial \Pi}{\partial x_i}(x_i - \hat{x_i}) \tag{A13}$$

where $\frac{\partial \Pi}{\partial x_i}$ is called the simplex multiplier of x_i. The complete form of the Benders feasibility cut is:

$$\Pi^* + \sum_{i}^{n} \frac{\partial \Pi}{\partial x_i}(x_i - \hat{x_i}) \leq 0 \tag{A14}$$

By incorporating (A14) as a constraint in the upper problem and solving for $\hat{x}$ again, if Π^* is greater than 0, we continue to generate a Benders feasibility cut and add it to the upper problem as a constraint, then solve for $\hat{x}$ again. When the value of Π^* is 0, it indicates that constraints (A2), (A3), and (A4) are all satisfied, the Benders feasibility cuts are equivalent to constraint (A4), making the upper problem equivalent to the original problem., and the corresponding $\hat{x}$ is the optimal solution x^*.

Appendix B. Calculation of the Simplex Multipliers

The specific calculation process for simplex multipliers $\pi_{j,c}^{HVAC,\phi,t}$, $\pi_{j,c}^{EV,\phi,t,ch}$, $\pi_{j,c}^{EV,\phi,t,d}$ and $\pi_{j,c}^{ESS,\phi,t}$ is as follow:

$$\begin{aligned}\boldsymbol{\pi}_{j,c}^{HVAC,\phi,t} &= \frac{\partial \Pi_j}{\partial p_{j,c}^{HVAC,\phi,t}} = \frac{\partial(\sum_{t\in\mathcal{T}} y_j^{\phi,t}(p_{j,c}^{\phi,t} - p_j^{\phi,t,max}))}{\partial p_{j,c}^{HVAC,\phi,t}} \\ &= \frac{\partial(y_j^{\phi,t} p_{j,c}^{\phi,t})}{\partial p_{j,c}^{HVAC,\phi,t}} = y_j^{\phi,t} \frac{\partial p_{j,c}^{\phi,t}}{\partial p_{j,c}^{HVAC,\phi,t}} = y_j^{\phi,t}\end{aligned} \tag{B1}$$

$$\begin{aligned}\boldsymbol{\pi}_{j,c}^{EV,\phi,t,ch} &= \frac{\partial \Pi_j}{\partial p_{j,c}^{EV,\phi,t}} = \frac{\partial(\sum_{t\in\mathcal{T}} y_j^{\phi,t}(p_{j,c}^{\phi,t} - p_j^{\phi,t,max}))}{\partial p_{j,c}^{EV,\phi,t,ch}} \\ &= \frac{\partial(y_j^{\phi,t} p_{j,c}^{\phi,t})}{\partial p_{j,c}^{EV,\phi,t,ch}} = y_j^{\phi,t} \frac{\partial p_{j,c}^{\phi,t}}{\partial p_{j,c}^{EV,\phi,t,ch}} = y_j^{\phi,t}\end{aligned} \tag{B2}$$

$$\begin{aligned}\boldsymbol{\pi}_{j,c}^{EV,\phi,t,d} &= \frac{\partial \Pi_j}{\partial p_{j,c}^{EV,\phi,t,d}} = \frac{\partial(\sum_{t\in\mathcal{T}} y_j^{\phi,t}(p_{j,c}^{\phi,t} - p_j^{\phi,t,max}))}{\partial p_{j,c}^{EV,\phi,t,d}} \\ &= \frac{\partial(y_j^{\phi,t} p_{j,c}^{\phi,t})}{\partial p_{j,c}^{EV,\phi,t,d}} = y_j^{\phi,t} \frac{\partial p_{j,c}^{\phi,t}}{\partial p_{j,c}^{EV,\phi,t,d}} = -y_j^{\phi,t}\eta^{EV}\end{aligned} \tag{B3}$$

$$\begin{aligned}\boldsymbol{\pi}_{j,c}^{ESS,\phi,t} &= \frac{\partial \Pi_j}{\partial p_{j,c}^{ESS,\phi,t}} = \frac{\partial(\sum_{t\in\mathcal{T}} y_j^{\phi,t}(p_{j,c}^{\phi,t} - p_j^{\phi,t,max}))}{\partial p_{j,c}^{ESS,\phi,t}} \\ &= \frac{\partial(y_j^{\phi,t} p_{j,c}^{\phi,t})}{\partial p_{j,c}^{ESS,\phi,t}} = y_j^{\phi,t} \frac{\partial p_{j,c}^{\phi,t}}{\partial p_{j,c}^{ESS,\phi,t}} = -y_j^{\phi,t}\eta^{ESS}\end{aligned} \tag{B4}$$

Appendix C. Bi-level Stackelberg Game of HEMS-Equivalent KKT Optimality Conditions

As mentioned in Section V-B, the objective function can be rewritten as:

$$\min \mathscr{F}(p_{j,c}^{HVAC,\phi,t}) = \sum_{t\in\mathcal{T}} \alpha_{j,c}^{\phi,t} \| T_{j,c}^{in,\phi,t} - T_{j,c}^{set,\phi,t} \|^2 \tag{C1}$$

The Lagrangian function of the slave problem $\mathscr{L}$ can be expressed as:

$$\mathop{\mathscr{L}}_{t\in\mathcal{T}}(p_c^{HVAC,t},\underline{\lambda_{j,c}^{p,\phi,t}},\overline{\lambda_{j,c}^{p,\phi,t}},\underline{\lambda_{j,c}^{T,\phi,t}},\overline{\lambda_{j,c}^{T,\phi,t}})=\mathop{\mathscr{F}}_{t\in\mathcal{T}}(p_{j,c}^{HVAC,\phi,t})+$$
$$\sum_{t\in\mathcal{T}}\underline{\lambda_{j,c}^{p\phi,,t}}(p^{HVAC,min}-p_{j,c}^{HVAC,\phi,t})+$$
$$\sum_{t\in\mathcal{T}}\overline{\lambda_{j,c}^{p,\phi,t}}(p_{j,c}^{HVAC,\phi,t}-p^{HVAC,max})+$$
$$\sum_{t\in\mathcal{T}}\underline{\lambda_{j,c}^{T,\phi,t}}(T^{min}-T_{j,c}^{in,\phi,t})+\sum_{t\in\mathcal{T}}\overline{\lambda_{j,c}^{T,\phi,t}}(T_{j,c}^{in,\phi,t}-T^{max}) \tag{C2}$$

According to the KKT optimality conditions, the necessary conditions for obtaining the optimal solution are:

$$\frac{\partial\mathscr{L}}{\partial(p_{j,c}^{HVAC,\phi,t})}=\frac{\partial\mathscr{F}}{\partial(p_{j,c}^{HVAC,\phi,t})}+\frac{\partial\mathscr{G}}{\partial(p_{j,c}^{HVAC,\phi,t})}=0 \tag{C3}$$

where $\mathscr{F}$ is the objective function, $\mathscr{G}$ represents the inequality constraints of the slave problem, which are constraints (18), (19).

To facilitate subsequent derivations, we express the partial derivative operations in the following matrix form:

$$\frac{\partial}{\partial(p_{j,c}^{HVAC,\phi,t})}=\left[\frac{\partial}{\partial(p_{j,c}^{HVAC,\phi,1})}\cdots\frac{\partial}{\partial(p_{j,c}^{HVAC,\phi,t})}\cdots\frac{\partial}{\partial(p_{j,c}^{HVAC,\phi,24})}\right]_{1\times24} \tag{C4}$$

The indoor temperature $T_{j,c}^{in,t}$ at different time slot t can be expanded as:

$$T_c^{in,\phi,t}=$$
$$\begin{bmatrix} T_{j,c}^{in,\phi,1}+k[\beta(T_{j,c}^{out,\phi,1}-T_{j,c}^{in,\phi,1})-\gamma p_{j,c}^{HVAC,\phi,1}] \\ \cdots \\ T_{j,c}^{in,\phi,t}+k[\beta(T_{j,j,c}^{out,\phi,t}-T_{j,c}^{in,\phi,t})-\gamma p_{j,c}^{HVAC,\phi,t}] \\ \cdots \\ T_{j,c}^{in,\phi,24}+k[\beta(T_{j,c}^{out,\phi,24}-T_{j,c}^{in,\phi,24})-\gamma p_{j,c}^{HVAC,\phi,24}] \end{bmatrix}_{24\times1} \tag{C5}$$

Due to the temporal coupling of heat transfer effects, the indoor temperature at a given time slot is related to the HVAC power consumption of all previous time slots, which can be expressed as a functional relationship as:

$$T_{j,c}^{in,\phi,t}=f(p_{j,c}^{HVAC,\phi,1},p_{j,c}^{HVAC,\phi,2},...,p_{j,c}^{HVAC,\phi,t-1}) \tag{C6}$$

Then, the partial derivative of the indoor temperature $T_c^{in,t}$ with respect to the decision variable $p_c^{HVAC,t}$ can be obtained for each time slot as:

$$\left[\frac{\partial T_{j,c}^{in,\phi,t}}{\partial(p_{j,c}^{HVAC,\phi,t})}\right]_{24\times24}=-\begin{bmatrix}\gamma & \cdots & (1-k\beta)\gamma \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma\end{bmatrix}_{24\times24} \tag{C7}$$

where the matrix in C7 has diagonal elements of γ, lower triangular elements of 0, and upper triangular elements of $(1-k\beta)\gamma$. The coefficient of the partial derivative of indoor temperature with respect to HVAC power at each time slot t is the sum of the elements in the t-th row of the matrix.

Similarly, taking the partial derivative of the inequality constraints with respect to the decision variables yields the mathematical expression of $\frac{\partial\mathscr{G}}{\partial(p_c^{HVAC,\phi,t})}$ and $\frac{\partial\mathscr{F}}{\partial(p_c^{HVAC,\phi,t})}$:

$$\frac{\partial\mathscr{G}}{\partial(p_c^{HVAC,\phi,t})}=-\underline{\lambda_{j,c}^{p,\phi,t}}+\overline{\lambda_{j,c}^{p,\phi,t}}+$$
$$\underline{\lambda_{j,c}^{T,\phi,t}}\times\sum_t(\gamma+(24-t)(1-k\beta)\gamma)-$$
$$\overline{\lambda_{j,c}^{T,\phi,t}}\times\sum_t(\gamma+(24-t)(1-k\beta)\gamma) \tag{C8}$$

$$\frac{\partial\mathscr{F}}{\partial(p_c^{HVAC,\phi,t})}=-\sum_t(\gamma+(24-t)(1-k\beta)\gamma)\times$$
$$2\alpha_{j,c}^{\phi,t}(T_{j,c}^{in,\phi,t}-T_{j,c}^{set,\phi,t}) \tag{C9}$$

Substituting $\frac{\partial\mathscr{G}}{\partial(p_c^{HVAC,\phi,t})}$ and $\frac{\partial\mathscr{F}}{\partial(p_c^{HVAC,\phi,t})}$ into (C3) yields the mathematical expression for the KKT optimality conditions. By incorporating these into the master problem, the bi-level Stackelberg game can be transformed into a single-level optimization problem.

Appendix D. Parameter Settings in Numerical Tests

A. HVAC Parameter Settings

TABLE D1
HVAC PARAMETER SETTINGS

Parameter	Value	Parameter	Value
$k(°C\cdot m^2/kW)$	1	$p^{HVAC,max}(kW)$	2
$\beta(kW/°C\cdot m^2)$	1.2	$T^{min}(°C)$	22
$\gamma(1/m^2)$	6	$T^{max}(°C)$	26
$p^{HVAC,min}(kW)$	0.5		

As shown in table D1, where k is the heat flux-to-temperature conversion coefficient for changes in heat flux inside the house, with units of m, set to 1 in this paper. β is the temperature-to-heat flux conversion coefficient for the temperature difference between indoors and outdoors, which is related to the size and material of the house, also with units of m, set to 1.2 here. γ is the conversion coefficient between HVAC power and released heat flux, with units of m, the numerical values is 6. The power range for the HVAC system is between 0.5kW and 2kW, while the temperature range is between $22°C$ and $26°C$.

B. EV Parameter Settings

TABLE D2
EV PARAMETER SETTINGS

Parameter	Value	Parameter	Value
$p^{EV,min,ch}(kW)$	0	$SCO^{EV,min}$	0.0
$p^{EV,max,ch}(kW)$	6	$SCO^{EV,expect}$	0.95
$p^{EV,min,c}(kW)$	0	$E^{EV,max}(kWh)$	60
$p^{EV,max,c}(kW)$	6	$e(kWh/km)$	0.25
$\eta^{EV}(\%)$	90%	$\theta^{EV}(\$/kW^2)$	0.01
$SOC^{EV,max}$	0.95		

Table D2 lists the parameters related to the EV. The power range for both charging and discharging of EV are set between

0 and 6 kW, with efficiency of 90%. The customer's expected EV SOC value is 0.95, EV's battery capacity $E^{EV,max}$ is 60 kWh, with an energy consumption e of 0.25 kWh per km. The battery degradation cost θ^{EV} is 0.01 USD per kW.

C. ESS Parameter Settings

The parameters related to the ESS are listed in table D3. The discharge power range is 0-5 kW, with an efficiency of 90% and a battery capacity $E^{ESS,max}$ of 30 kWh. The battery degradation cost of ESS θ^{ESS} is 0.01 USD per kW.

TABLE D3
ESS PARAMETER SETTINGS

Parameter	Value	Parameter	Value
$p^{ESS,min}(kW)$	0	$\theta^{ESS}(\$/kW^2)$	0.01
$p^{ESS,max}(kW)$	5	$SOC^{ESS,min}$	0.0
$\eta^{ESS}(\%)$	90%	$SOC^{ESS,max}$	1.0
$E^{ESS,max}(kWh)$	30		

D. SADN Parameter Settings

TABLE D4
SADN PARAMETER SETTINGS

Parameter	Value	Parameter	Value
v_i^{min}	0.95	P_{swing}^{min}	1.5
v_i^{max}	1.05	Q_{swing}^{max}	0.75
$P_{i\to j}^{max,3\phi}$	1.5	$q_{83}^{\phi,t}$	0.2
$Q_{i\to j}^{max,3\phi}$	0.75	$q_{88}^{a,t}$	0.05
$P_{i\to j}^{max,\phi}$	0.75	$q_{90}^{b,t}$	0.05
$Q_{i\to j}^{max,\phi}$	0.375	$q_{92}^{c,t}$	0.05
$tag^{\phi,t}$	4		

Table D4 contains parameters of SADN, all in per-unit values. The base value for voltage is 480 V, and the base value for power is 1 MVA. To ensure the reliability of electricity supply, the node voltage fluctuation must not exceed 5%. Therefore, parameters v_i^{min} and v_i^{max} are set to 0.95 and 1.05, respectively. In the SADN, there are both 3-phase and single-phase transmission lines. The active power limit $P_{i\to j}^{max,3\phi}$ for the 3-phase transmission lines is 1.5, and the reactive power limit $Q_{i\to j}^{max,3\phi}$ is 0.75. For the single-phase transmission lines, the active power limit $P_{i\to j}^{max,\phi}$ is 0.75, and the reactive power limit $Q_{i\to j}^{max,\phi}$ is 0.375. The transformer's active power limit per phase P_{swing}^{min} is 1.5, and the reactive power limit Q_{swing}^{max} is 0.75. The transformer tags are set to 4 for all 3 phases, the voltage value on the low-voltage side is 1.025 according to (11). A 3-phases SVG is aggregated at node 83, capable of supplying 0.2 reactive power per phase. At nodes 88, 90, and 92, the single-phase SVGs can supply 0.05 reactive power to phases a, b, and c, respectively.

E. Node 76 A-Phase Customers Parameter Settings

Table D5 lists the parameters for the six households of node 76 A-phase, as generated in Section IV. These parameters include the EV return time t^{back}, EV departure time t^{depart}, daily driving distance d, indoor temperature setting T^{set}, and sensitivity to temperature α. Due to the varying electricity usage behavior of the customers on the previous day, the remaining energy in the ESS also differs.

TABLE D5
PARAMETER SETTINGS OF HOUSEHOLD AT NODE 76 PHASE A

	t^{back}	t^{depart}	T^{set}	α	d	$SCO^{EV,t^{back}}$	$SOC^{ESS,in}$
H1	7 p.m.	10 a.m.	24.5°C	1.08	19.91 km	0.867	0.250
H2	6 p.m.	9 a.m.	24°C	0.94	111.75 km	0.484	0.500
H3	5 p.m.	10 a.m.	24.7°C	1.01	126.61 km	0.423	0.333
H4	5 p.m.	9 a.m.	24.4°C	1.05	61.52 km	0.694	0.417
H5	5 p.m.	9 a.m.	24.2°C	0.98	35.99 km	0.801	0.200
H6	6 p.m.	10 a.m.	24.6°C	0.96	168.06 km	0.250	0.583

F. Households Location in SADN

The aggregation of households at each node in SADN is shown in table D6, where phase A aggregates 71 households, phase B aggregates 49 households, and phase C aggregates 60 households.

TABLE D6
HOUSEHOLDS IN SADN

Phase A		Phase B		Phase C	
Node name	Number of households	Node name	Number of households	Node name	Number of households
1	2	2	1	4	2
7	1	12	1	5	1
9	2	22	2	6	2
11	2	35	1	34	2
10	1	38	1	16	2
19	2	39	1	17	1
20	2	43	2	24	2
28	2	47	2	31	1
33	2	48	4	30	2
29	2	49	3	32	1
35	1	56	1	41	1
37	2	58	1	47	2
42	1	59	1	48	4
45	1	64	4	49	2
47	2	65	2	50	2
46	1	76	5	62	2
48	3	77	2	65	3
49	2	86	1	66	4
51	1	80	2	73	2
52	2	87	2	76	5
53	2	90	2	74	2
55	1	95	1	75	2
60	1	96	1	84	1
63	2	99	2	83	1
65	2	106	2	85	2
68	1	107	2	92	2
69	2			100	2
70	1			102	1
71	2			103	2
76	6			104	2
79	2				
82	2				
88	2				
94	2				
98	2				
109	2				
111	1				
112	1				
113	2				
114	1				