Theoretical Neuroscience I

Exercise 7: Conditional probability and signal-detection theory

Due: 18 December 2019

You will characterize the noisy responses of a sensory neuron to stimuli of two types and of different strengths. You will compute the conditional probability of responses given a stimulus type, determine how informative response are about stimulus type, and compute the conditional likelihood of stimulus types given a response.

The exercise covers material presented in Lectures 12 to 14.

Sensory stimuli and responses

Generate a sequence of stimuli of different strength and type. Randomly assign to each stimulus one of two types, A or B. Additionally, assign to all stimulus a fixed strength, s. Specifically, create an input matrix with 2 rows (for the two types) and n columns (for a sequence of length n):

$$S_{in} = \begin{pmatrix} 0 & s & s & 0 & \dots & s & 0 \\ s & 0 & 0 & s & \dots & 0 & s \end{pmatrix}$$

The non-zero stimuli in the top row are of type A and the non-zero stimuli in the bottom row of type B.

You are provided with a Matlab-function **SensResp** which produces a variable response x to stimulus sequences in this format. Matlab-function **Example** illustrates how to use this function.

When supplied with above input S_{in}, **SensResp** produces a vector of responses

$$R_{out} = \begin{pmatrix} r_1 & r_2 & r_3 & r_4 & \dots & r_{n-1} & r_n \end{pmatrix}$$

of size $[1, n]$

Assignment I: conditional response probability

Stimulate repeatedly (at least 500 times) with stimuli A and B of strength $s = 2$. Use Matlab-function **hist** to obtain a histogram of the fractional probability of different

size responses. Plot the conditional response probability $P(x|A)$ for stimulus A and, separately, the conditional response probability $P(x|B)$!

Repeat for stimuli of strength $s = 7$ and strength $s = 20$!

Discuss your results! How well do the responses distinguish between the two stimuli for different values of s?

Assignment II: ROC analysis

Quantify the informativeness of the neuron's responses about stimulus type for $s = 20$! Specifically, perform an ROC analysis by placing a 'decision criterion' at different response levels, covering the entire range from the smallest to the largest responses. For each 'criterion levels', compute the fraction of responses above that level in $P(x|A)$ and, separately, in $P(x|B)$. Plot the 'fraction above' in the A-distribution against the 'fraction above' in the B-distribution, for all 'criterion levels'. Compute the area under the resulting curve, as a fraction of the entire plot area! This is the reliability (fraction correct) with which stimulus type (A or B) can be inferred from the neurone's response at $s = 20$.

Assignment III: Stimulus likelihood

In the previous assignment, you obtained the conditional probabilities $P(x|A)$ and $P(x|B)$ for stimulus strength $s = 20$. Building on these results, obtain the marginal probability $P(x)$ assuming $P(A) = P(B) = 0.5$, as follows

$$P(x) = \frac{1}{2}\,P(x|A) + \frac{1}{2}\,P(x|B)$$

Finally, apply Bayes' rule to obtain the conditional likelihoods $L(A|x)$ and $L(B|x)$, as follows

$$L(A|x) = \frac{P(x|A) \cdot P(A)}{P(x)}, \qquad L(B|x) = \frac{P(x|B) \cdot P(B)}{P(x)}$$

Plot $L(A|x)$ and $L(B|x)$ as a function of observed response x in the same plot! Discuss your results!