Theoretical Neuroscience I

Exercise 2: Single-compartment model

Due: Wednesday, 6 November 2019

How does an input current to a cell change its membrane potential?

1 State variables, initial condition, and parameters

In a single-compartment model (SCM), a cell is assumed to be a single, membrane-enclosed volume. The important parameters of the cell membrane are resistance r_m, capacitance c_m, and time constant τ_m:

$$r_m = 0.9\, M\Omega mm^2, \qquad c_m = 12\, nF/mm^2, \qquad \tau_m = r_m\, c_m = 10.8\, ms$$

The *state* of a cell at time t is determined by the membrane potential $V_m(t)$ and the input current $i_e(t)$. The initial membrane potential V_0 is also important. We assume that

$$V_0 = V_m(t = 0) = 0\, mV$$

Several dependent state variables are also of interest, include the membrane current $i_r(t)$, the capacitor current $i_c(t)$, and the equilibrium potential $V_\infty(t)$.

2 Dynamic equation

The membrane potential changes over time according to the dynamic equation

$$\frac{dV_m(t)}{dt} = \frac{1}{\tau_m}\left[r_m \cdot i_e(t) - V_m(t)\right]$$

Take a close look to understand what this means! When the $i_e(t)$ is zero, a positive $V_m(t)$ decreases over time, whereas a negative $V_m(t)$ increases over time (‘exponential relaxation’). Additionally, a positive $i_e(t)$ increases, and a negative $i_e(t)$ decreases, the membrane potential.

3 Numerical integration

To solve this equation numerically, we use either the **Euler discretization** of the dynamic equation

$$\frac{V_m(t+\Delta t) - V_m(t)}{\Delta t} = \frac{1}{\tau_m}\left[r_m \cdot i_e(t) - V_m(t)\right]$$

or

$$V_m(t+\Delta t) = V_m(t)\left(1 - \frac{\Delta t}{\tau_m}\right) + r_m \cdot i_e(t)\,\frac{\Delta t}{\tau_m}$$

or the following, slightly more exact discritization:

$$V_m(t+\Delta t) = V_m(t)\,\exp\left(-\frac{\Delta t}{\tau_m}\right) + r_m \cdot i_e(t)\left[1 - \exp\left(-\frac{\Delta t}{\tau_m}\right)\right]$$

In other words, knowing i_e and V_m at time t, we can compute V_m at time $t + \Delta t$. This assumes that i_e remains constant over the short time interval Δt.

This *iterative rule* allows us to start from the initial condition $V_m(0) = V_0$ and to compute $V_m(\Delta t)$, $V_m(2\Delta t)$, ..., one after the other.

4 Assignments

You are provided with three Matlab-files, **Ie_example.m**, **V_example.m** and **ExerSCM.m**. Starting from these, please carry out the following steps:

1. Define parameters r_m, c_m, τ_m, and V_0.
2. Define an appropriate time vector, ranging from $0\,ms$ to $T = 250\,ms$ in steps of $\Delta t = 0.05\,ms$.
3. Define a constant input current $i_e = 25\,nA/mm^2$ for all t.
4. Plot the input current against time (first axis).
5. Iteratively compute the membrane potential V_m. Starting with $0\,mV$, use a for-loop to iterate through all time steps.
6. Plot the membrane voltage against time (second axis). You may use subplot() to combine several plots in one figure.

You have now completed the most difficult part of this exercise. As your script is now complete, you can investigate the effects of alternative input currents:

1. Simulate the membrane potential for a step current of low frequency:

$$i_e(t) = i0 \cdot \operatorname{sign}\left[\sin(2\pi f t)\right], \qquad f = 10\,Hz, \qquad \operatorname{sign}(x) = \begin{cases} 1, & \text{if} x \geq 0 \\ -1, & \text{if} x < 0 \end{cases}$$

2. Simulate the membrane potential for a sinusoidal current of low frequency:

$$i_e(t) = i0 \cdot \sin(2\pi f t), \quad f = 10\,Hz$$

3. Simulate the membrane potential for a sinusoidal current of medium frequency:

$$i_e(t) = i0 \cdot \sin(2\pi f t), \quad f = 100\,Hz$$

4. Finally, simulate the membrane potential for high-frequency random input current (use the corresponding two lines of Matlab code from Ie_example.m)!

5 Optional assignments

1. Calculate also capacitor current $i_c(t)$, membrane current $i_r(t)$, and equilibrium potential $V_\infty(t)$.

$$i_c(t) = c_m \frac{\Delta V_m(t)}{\Delta t}$$
$$i_r(t) = \frac{V_m(t)}{r_m}$$
$$V_\infty(t) = r_m\, i_e(t)$$

2. Plot the currents $i_r(t)$ and $i_c(t)$ in the first axis (subplot(), hold on, legend()).

3. Plot the potential $V_\infty(t)$ in the second axis (subplot(), hold on, legend()).