Theoretical Neuroscience I

Exercise 3:
Leaky Integrate & Fire Neuron

Due: Wednesday, 13 November, 2019

The leaky integrate & fire model (LIF model) of a neuron adds two new elements to the single-compartment model (SCM) of a cell: a **reversal potential** and a **spiking threshold**.

1 State variables and parameters

A neuron is modeled as a membrane-enclosed volume, with membrane resistance r_m, capacitance c_m, and time-constant τ_m:

$$r_m = 1.5\, M\Omega mm^2, \qquad c_m = 20\, nF/mm^2, \qquad \tau_m = r_m\, c_m = 30\, ms$$

Its state at time t is given by membrane potential $V_m(t)$ and input current $i_e(t)$.

Additional model parameters are the reversal potential E_L, reset potential V_{reset}, and spiking threshold V_{thresh}:

$$E_L = V_{reset} = -65\, mV, \qquad V_{thresh} = -50\, mV$$

The initial condition is

$$V_0 = V_m(t = 0) = -65\, mV$$

2 Dynamical equation

The membrane potential changes over time according to the dynamic equation

$$\frac{dV_m(t)}{dt} = \frac{1}{\tau_m} \left[r_m \cdot i_e(t) + E_L - V_m(t) \right]$$

provided the membrane potential remains below threshold, $V_m(t) < V_{thresh}$.

When $V_m(t) \geq V_{thresh}$, $V_m(t)$ is instantaneously ‘reset’ to V_{reset} whenever.

Note that this mechanism requires that the threshold is higher than the reset potential:

$$V_{th} > E_L \geq V_{reset}$$

3 Numerical integration

We do **not** use the Euler discretization, but solve the dynamical equation analytically for small time intervals Δt, assuming that $i_e(t)$ remains constant over this time.

$$V_m(t + \Delta t) = V_m(t)\, \exp\left(-\frac{\Delta t}{\tau_m}\right) + [i_e(t) \cdot r_m + E_L]\left[1 - \exp\left(-\frac{\Delta t}{\tau_m}\right)\right]$$

This rule holds as long as $V_m(t + \Delta t)$ remains below V_{thresh}. Whenever it reaches or exceeds this value we reset according to

$$V_m(t + \Delta t) = V_{reset}$$

4 Assignments

You are provided with one Matlab file, **if_example.m**. Starting from this, please carry out the following steps:

1. Define parameters.
2. Define an appropriate time vector, ranging from $0\,ms$ to $T = 500\,ms$ in steps of $\Delta t = 0.1\,ms$.
3. Define a constant input current $i_0 = 12\,nA/mm^2$ for all t.
4. Plot the input current against time (first axis).
5. Iteratively compute the membrane potential V_m. Don't forget the spiking mechanism!
6. Plot the membrane voltage against time (second axis). You may use subplot() to combine several plots in one figure.
7. Add code to store the times of any spikes (when $V(t + \Delta t)$ was reset) in a vector t_{spike}.
8. Compute yet another vector with the inter-spike-intervals t_{isi} (use tisi = diff(tspike)).

You can now investigate the effects of other input currents.

1. Simulate the membrane potential for a sinusoidal current of low frequency:

$$i_e(t) = i_0 \cdot \sin(2\pi f\, t), \quad f = 4\,Hz$$

2. Simulate the membrane potential for a sinusoidal current of medium frequency:

$$i_e(t) = i_0 \cdot \sin(2\pi f\, t), \quad f = 20\,Hz$$

3. Simulate the membrane potential for a ramping current:

$$i_e(t) = i_0 \cdot t/150\,ms$$

5 Optional assignment

Analytically solve the dynamical equation over a time interval Δt, to derive the numerical iteration rule provided in Section 3.