Theoretical Neuroscience I

Exercise 8: Maximum likelihood inference

Due: Wednesday, 8 January 2020

You will observe the noisy responses of four sensory neurons and infer the most likely stimulus to have elicited the responses in question. To this end, you are also provided with 'prior knowledge' as to how each of the four neurons is likely to respond to different stimuli.

All possible stimulus values are assumed to occur equally often, so that *maximum likelihood* inference is sufficient.

The exercise covers material presented in Lectures 14 to 15.

Sensory inputs

As sensory inputs, you will use sequences of n values s_i, formatted as an input vector

$$S_{in} = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & \ldots & s_{n-1} & s_n \end{pmatrix}$$

of size $[1, n]$. Possible values s_i are *integers* in the range $5 \leq s_i \leq 20$;

Sensory responses

You can obtain the noisy sensory responses x_1, x_2, x_3, x_4 to such inputs produced by *four neurons* $N1, N2, N3, N4$ from Matlab-function **SensResp4**. Matlab-function **Example4** illustrates the use of this function. Specifically, when provided with a sensory input of size $[1, n]$, **SensResp4** produces a matrix of responses

$$R_{out} = \begin{pmatrix} x_{1,1} & x_{2,1} & x_{3,1} & x_{4,1} & \ldots & x_{n-1,1} & x_{n,1} \\ x_{1,2} & x_{2,2} & x_{3,2} & x_{4,2} & \ldots & x_{n-1,2} & x_{n,2} \\ x_{1,3} & x_{2,3} & x_{3,3} & x_{4,3} & \ldots & x_{n-1,3} & x_{n,3} \\ x_{1,4} & x_{2,4} & x_{3,4} & x_{4,4} & \ldots & x_{n-1,4} & x_{n,4} \end{pmatrix}$$

of size $[4, n]$. Individual response vectors $r_i = \begin{pmatrix} x_{i,1} \\ x_{i,2} \\ x_{i,3} \\ x_{i,4} \end{pmatrix}$ are of size $[4, 1]$ and represent the four different responses to s_i.

Assignment I: Joint probability and conditional likelihood

You are provided with ‘prior knowledge’ as to how neurone are likely to respond to different simuli in terms of the function RespPDF(). Given a row vector of possible (integer) stimulus values *sin* and a row vector of possible (integer) response values *xout*, this function provides a matrix of the joint probabilities $P_k(x, s)$, or PN_XandS, of observing any particular combination of stimulus and response, as well as vector of the expected response values $\langle x_k(s) \rangle$, or *EXN_S*, separately for each of the four neurons. The following code illustrates the use of this function:

$$\begin{array}{lc}
>> & sin = 5 : 20; \\
>> & xout = 0 : 30; \\
>> & [PN_XandS, EXN_S] = \mathrm{RespPDF}(sin, xout); \\
>> & [Nneuron, Nxout, Nsin] = \mathrm{size}(PN_XandS); \\
>> & [Nneuron, Nsin] = \mathrm{size}(EXN_S); \\
>> & \mathrm{ShowRespPDF}(sin, xout, PN_XandS, EXN_S);
\end{array}$$

From this joint probability, compute the marginal probability of responses for each of the four neurons:

$$P_n(x) = \sum_s P_n(x, s) \qquad\qquad \sum_x P_n(x) = 1$$

Compute further the conditional likelihood of stimuli s, given particular responses x_1, x_2, x_3, and x_4 of the four neurons

$$P_1(s|x_1), \qquad P_2(s|x_2), \qquad P_3(s|x_3), \qquad P_4(s|x_4)$$

by way of normalising to unity, separately for each value of x_i:

$$\sum_s P_n(s|x_i) = 1 \qquad\qquad \forall x_i$$

Visualize the conditional likelihood with **ShowRespPDF** and identify the *preferred stimulus* of each neuron!

Assignment II: Stimulus likelihood

Assuming that you have observed a response $r_A = \begin{pmatrix} 11 \\ 8 \\ 3 \\ 1 \end{pmatrix}$, compute the *joint* conditional log-likelihood of different stimuli having caused this response:

$$\log P(s|r_A) = \log P_1(s|x_1 = 11) + \log P_2(s|x_2 = 8) + \log P_3(s|x_3 = 3) + \log P_4(s|x_4 = 1)$$

Plot this log-likelihood as a function of s!

Repeat for an observed response $r_B = \begin{pmatrix} 1 \\ 4 \\ 10 \\ 4 \end{pmatrix}$, and $r_C = \begin{pmatrix} 2 \\ 0 \\ 5 \\ 8 \end{pmatrix}$!

Assignment III: Stimulus decoding

For each possible stimulus s, with $5 \leq s \leq 20$, obtain from **SensResp4** 100 population responses $r_k = \begin{pmatrix} x_{k,1} \\ x_{k,2} \\ x_{k,3} \\ x_{k,4} \end{pmatrix}$.

From every r_k, compute the *joint* conditional log-likelihood $\log P(s|r_k)$ and the associated ML estimate s_{ML} (*i.e.*, the value of s for which $\log P(s|r_k)$ is maximal).

Averaging over the 100 repetitions, compute the average $\langle s_{ML} \rangle$ and standard deviation $\sqrt{\langle s_{ML}^2 \rangle - \langle s_{ML} \rangle^2}$

Plot average and standard deviation as a function of the true stimulus s! Are some stimuli encoded better or worse than others?

List of Matlab functions

Example4.m
SetParams.m
SensResp4.m
RespPDF.m
ShowRespPDF.m