CS7IS2: Artificial Intelligence

Lecture 2: Informed search

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Uninformed search: summary

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

Informed search

- Estimate “desirability” of each node in the frontier and expand the most desirable unexpanded node
- Heuristic search – estimate of the path cost from each node
  - n(n) – heuristic function
  - Vs uninformed search = blind/exhaustive exploration of the full search space
- Same as with uninformed searches – informed search algorithms only differ by what node do they choose to expand next

Best-first search

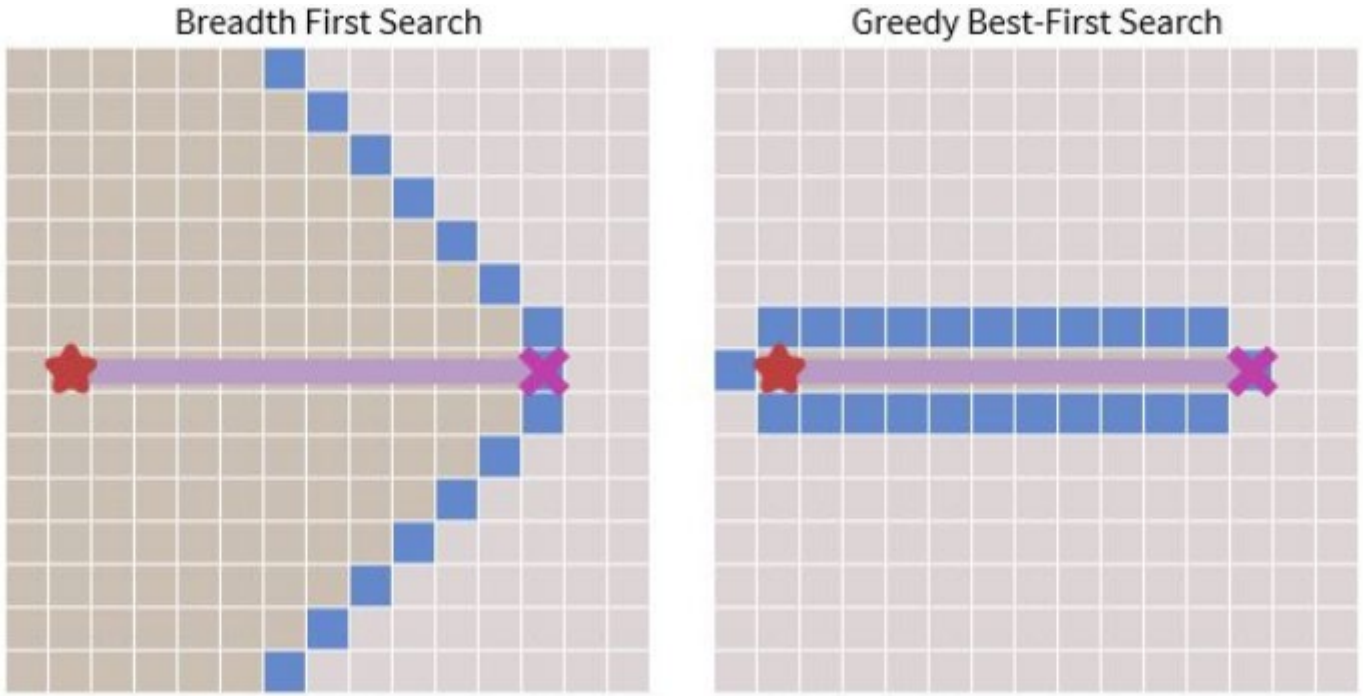
- Node selected for expansion based on a function f(n)
- f(n) is a cost estimate, and the nodes with the lowest f(n) are expanded first
- Choice of f(n) = search strategy
- Special-cases: greedy search and A* search

Greedy best-first search (GBFS)

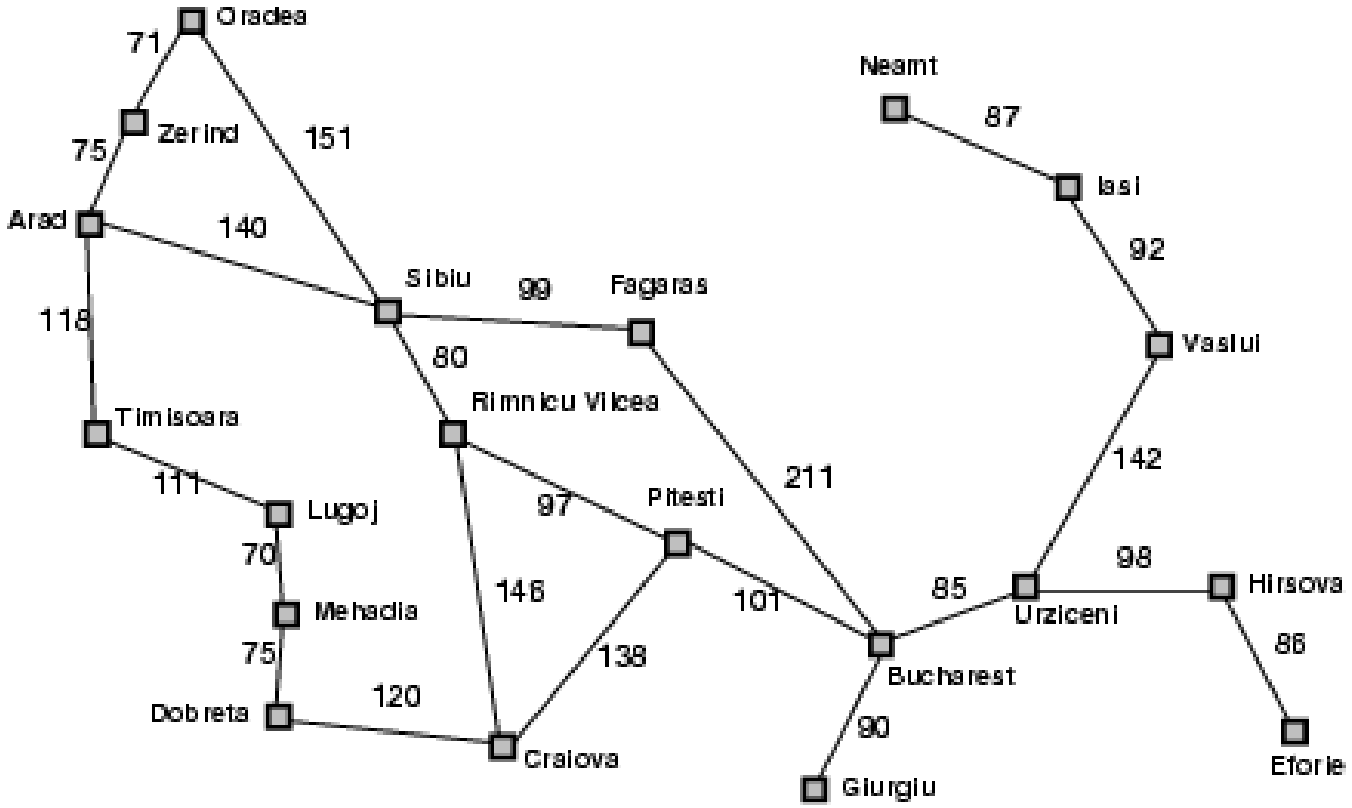
- Expand the node **that appears to be closest** to the goal, under assumption that it is the closest to the goal
- f(n) = h (n)

Greedy best-first search

› Each step moving closer to the target, e.g.,



Greedy best-first search – Romania example



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
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Neamt	234
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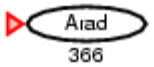
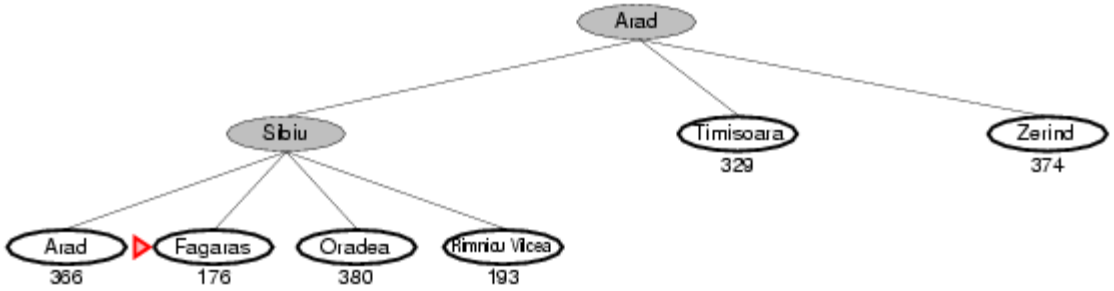
Urziceni	80
Vaslui	199
Zerind	374

Greedy best-first search example

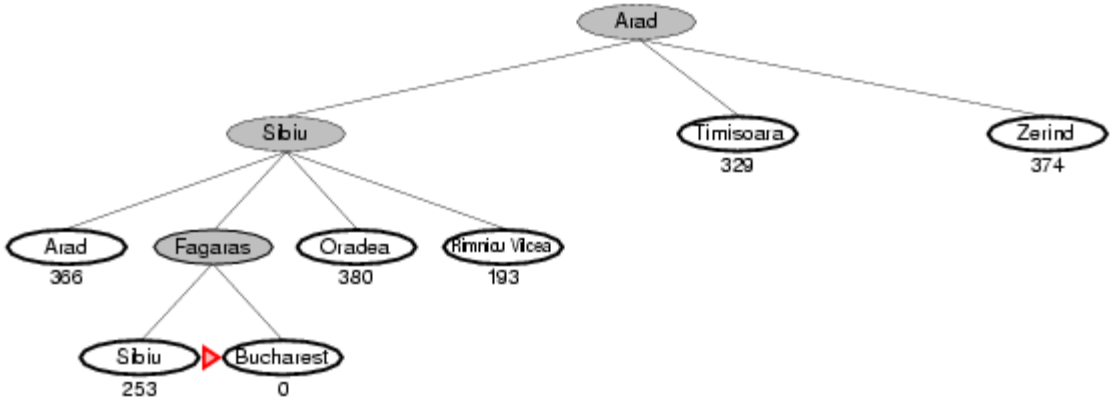


Greedy best-first search example

Greedy best-first search example



Greedy best-first search example



Properties of greedy best-first search

Complete?

– No, can get stuck in loops, e.g., Iasi -> Neamt -> Iasi -> Neamt ->

Complete in finite space with repeated-state checking

Time?

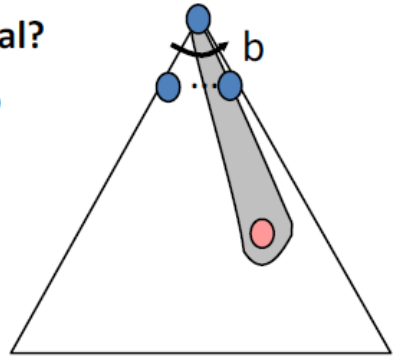
– $O(b^m)$, but a good heuristic can give dramatic improvement

Space?

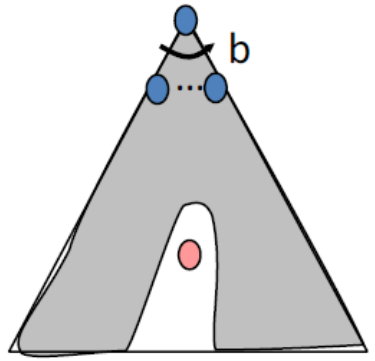
– $O(b^m)$, keeps all nodes in memory

Optimal?

– No



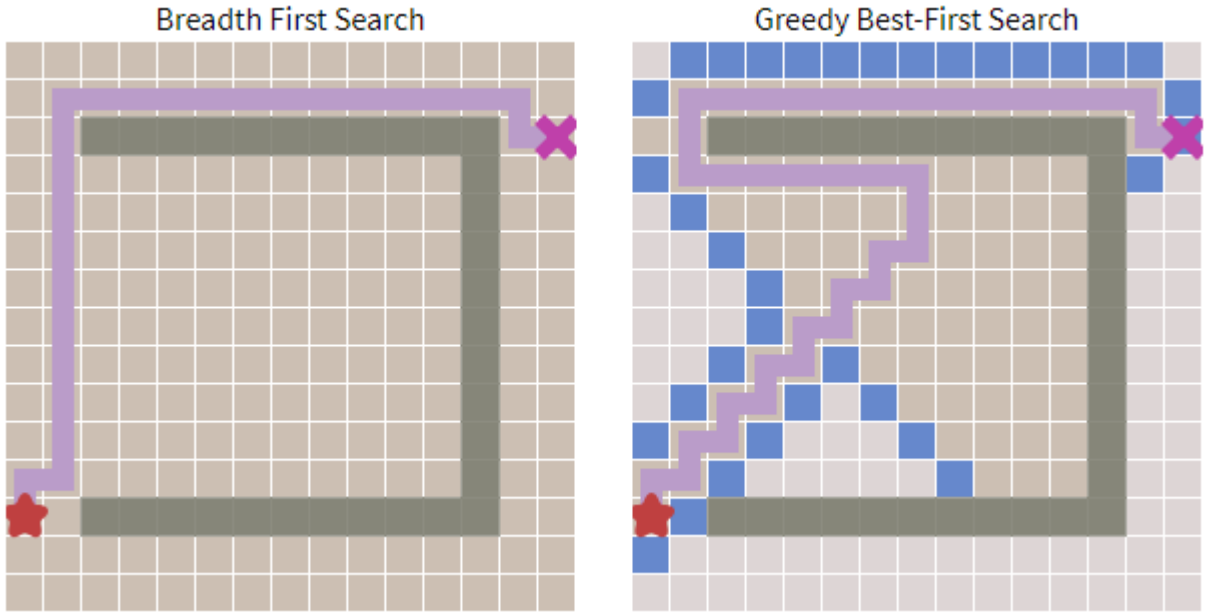
Common case: takes you straight to the (wrong) goal



Worst case: badly-guided DFS

Optimality

- Demo of BFS vs Greedy best-first search
- https://cs.stanford.edu/people/abisee/tutorial/greedy.html



Uniform-cost search (UCS): revise

Expand least-cost unexpanded node

- g(n) cost function for a given path to the node n

Implementation:

- Fringe = priority queue ordered by path cost, lowest first

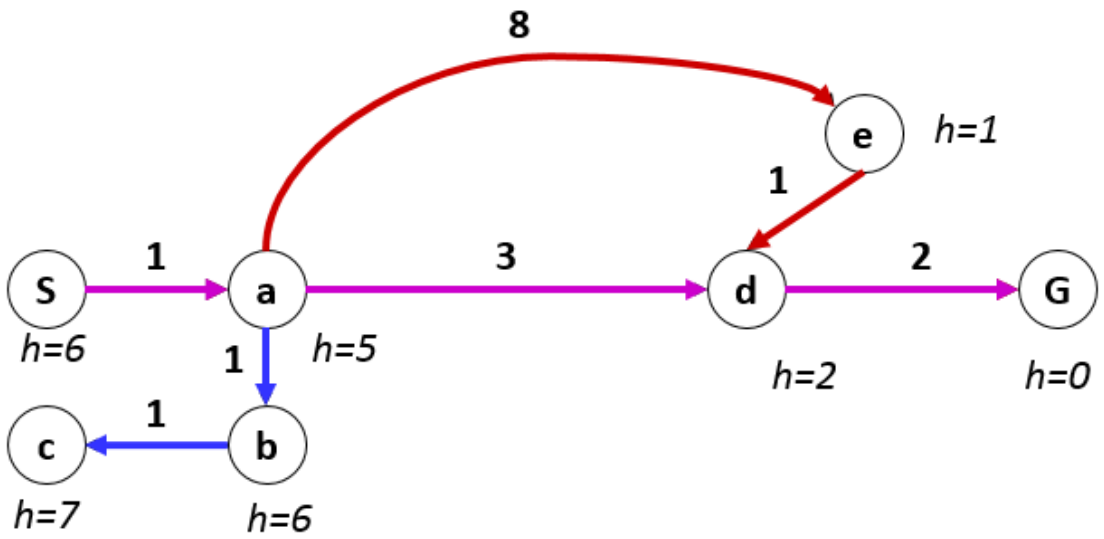
Goal test is applied to a node when it is selected for expansion

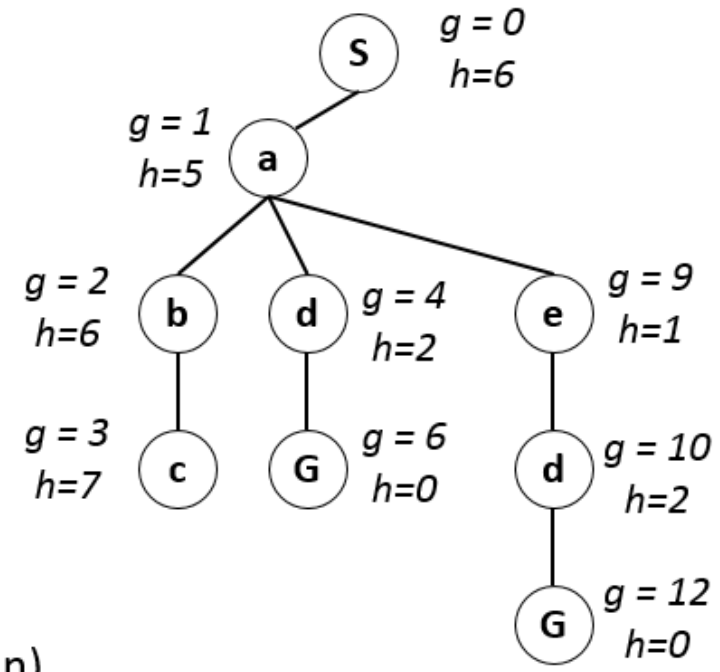
A test is added in case a better path is found to a node currently on the frontier

Equivalent to breadth-first if step costs all equal

A* - Combine UCS with GBFS

- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)





- A* Search orders by the sum: f(n) = g(n) + h(n)

Example: Teg Grenager

A*

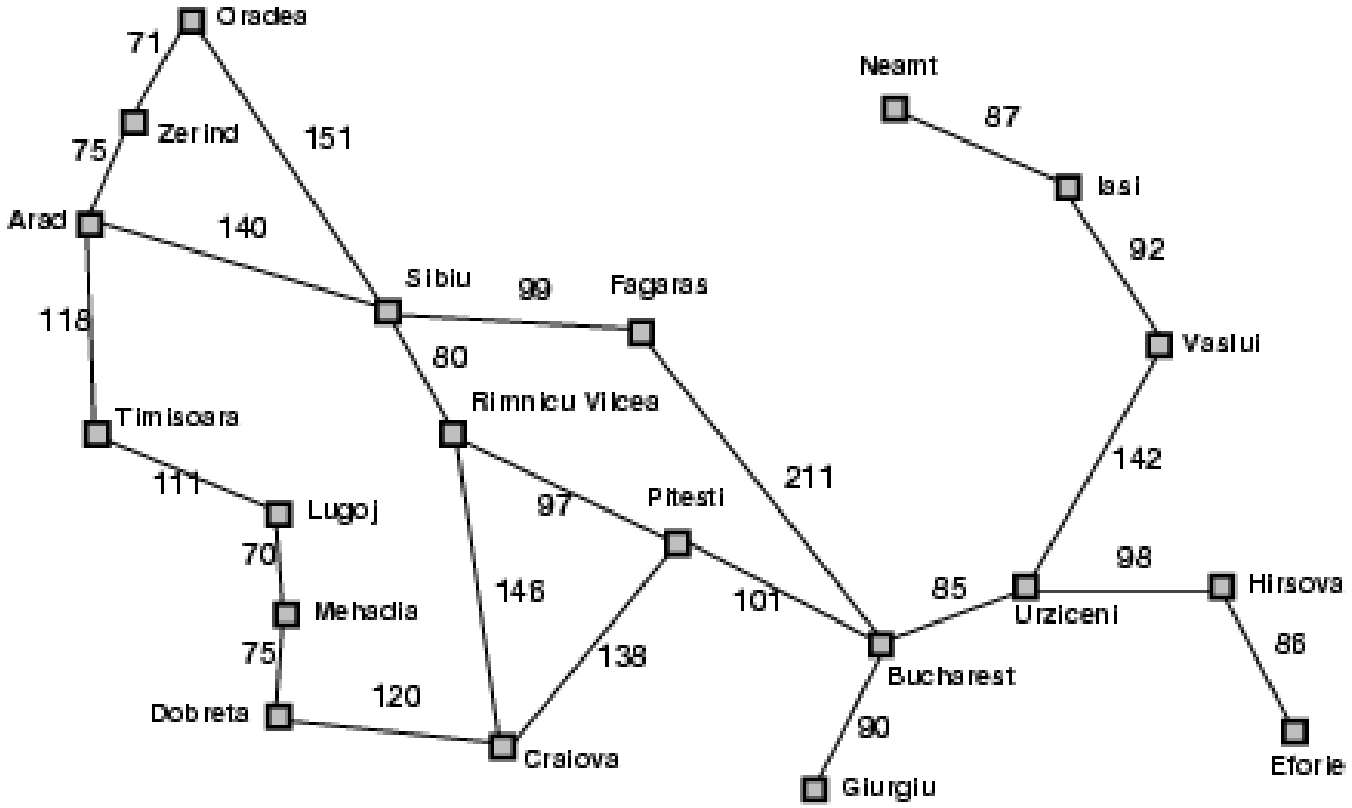
f(n) = Current path cost + estimated remaining path cost

- actual cost of path so far + estimated cost to goal
  - f(n) = g(n) + h(n)

- This helps avoid local maxima traps
- May be slower than Greedy Best-First
  - But guarantees shortest path

- Main idea: avoid paths that have already been expensive

A* – Romania example



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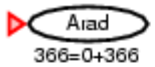
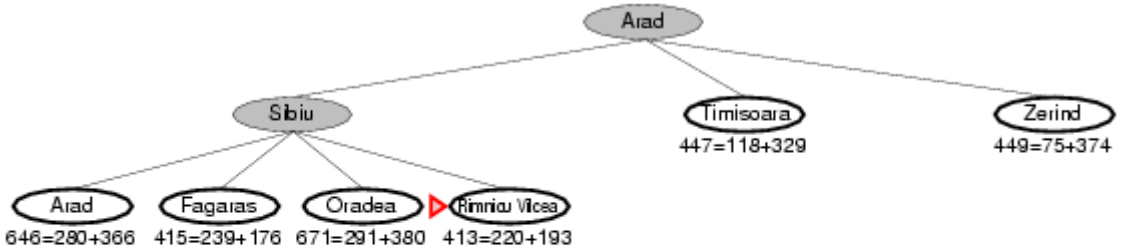
A* search example

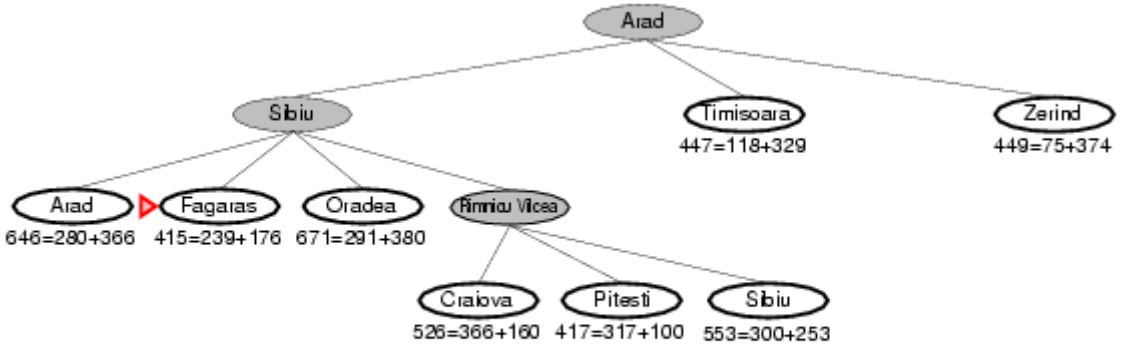


A* search example

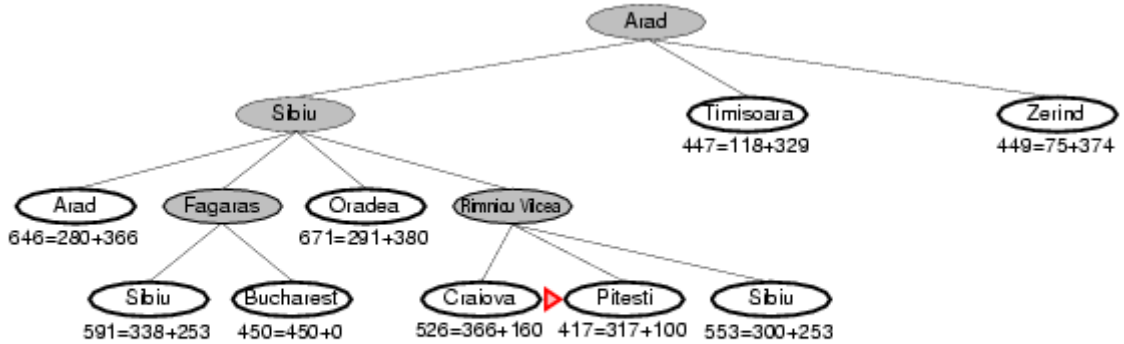
A* search example



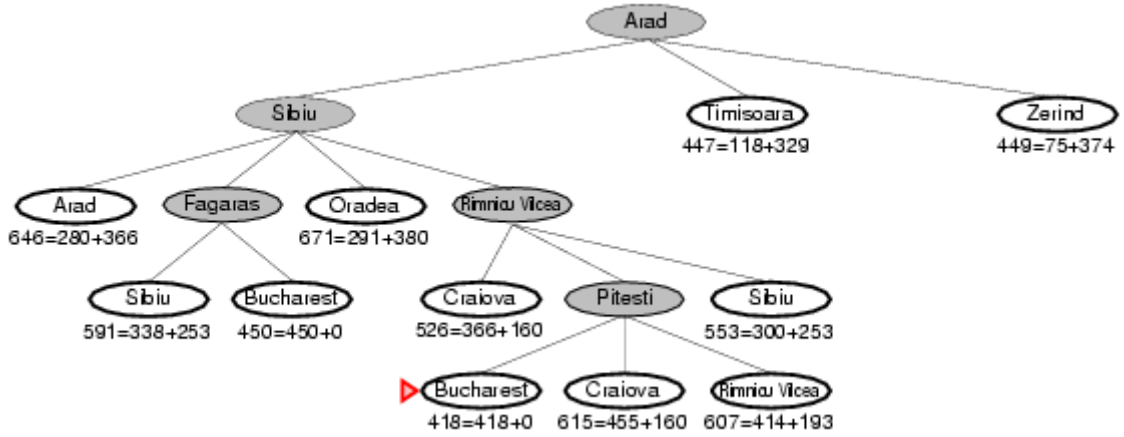
A* search example



A* search example

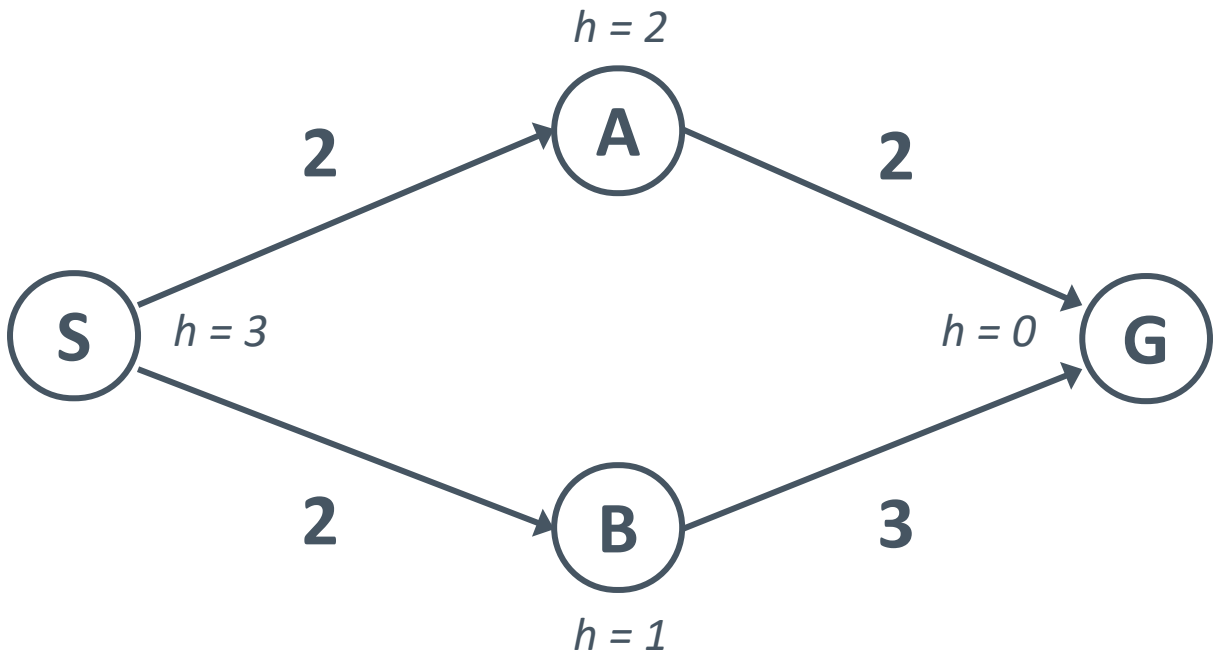


A* search example



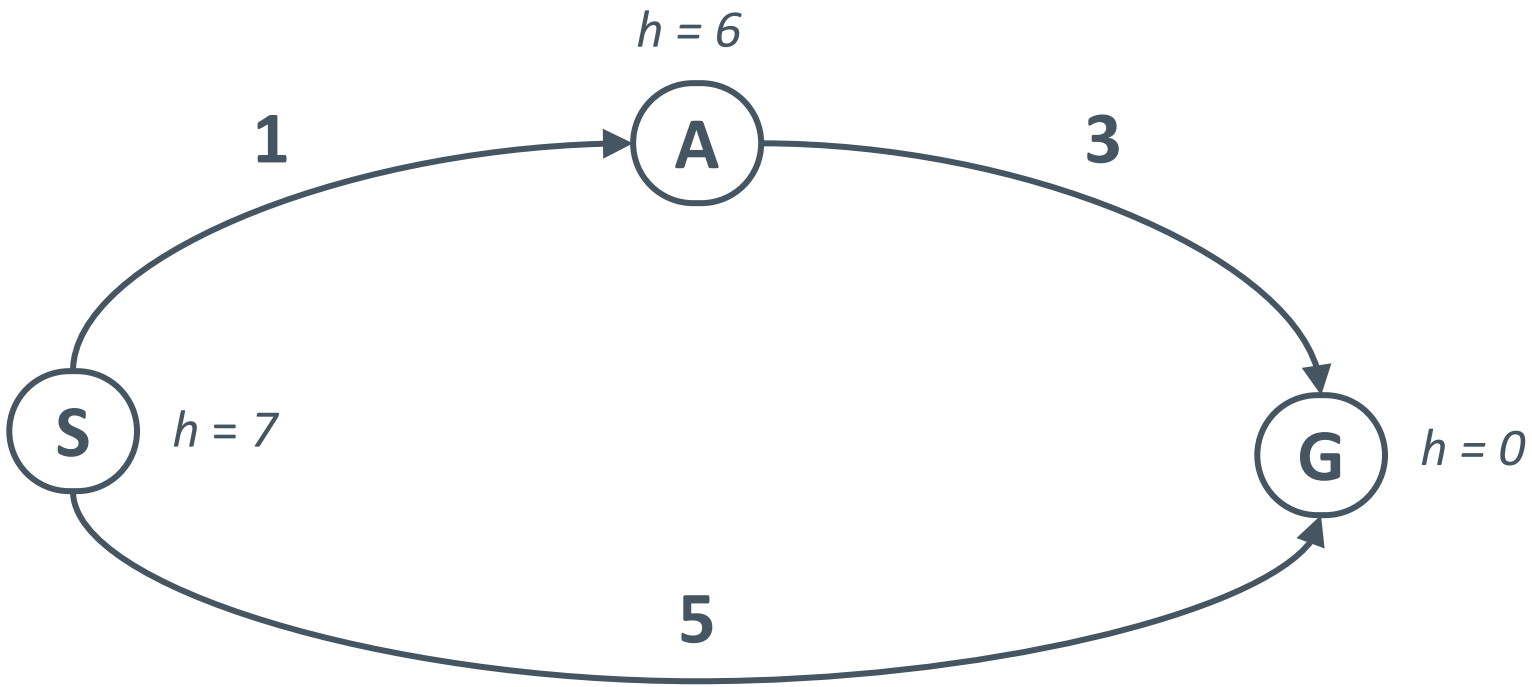
When should A* terminate?

› Should we stop when we enqueue a goal?



› No: only stop when we dequeue a goal

Is A* Optimal?



- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs in order for A* to be optimal

Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node n,

  $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.

- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic

- Example: *Romania example straight line distance* - never overestimates the actual road distance

- Theorem: If $h(n)$ is admissible, A* using `TREE-SEARCH` is optimal

Admissible heuristics

- Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe

Properties of A*

Complete?

- Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?

- Exponential in [relative error in h length of soln.]

Space?

- Keeps all nodes in memory

Optimal?

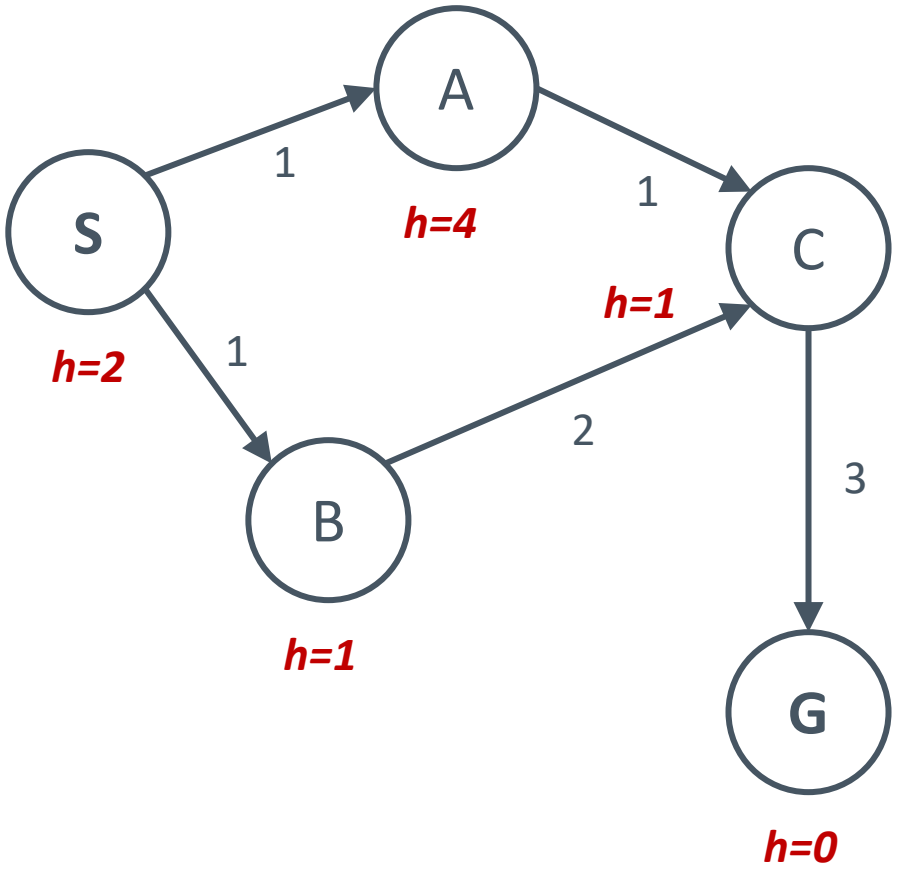
- Yes, cannot expand f_{i+1} until f_i is finished

A* applications

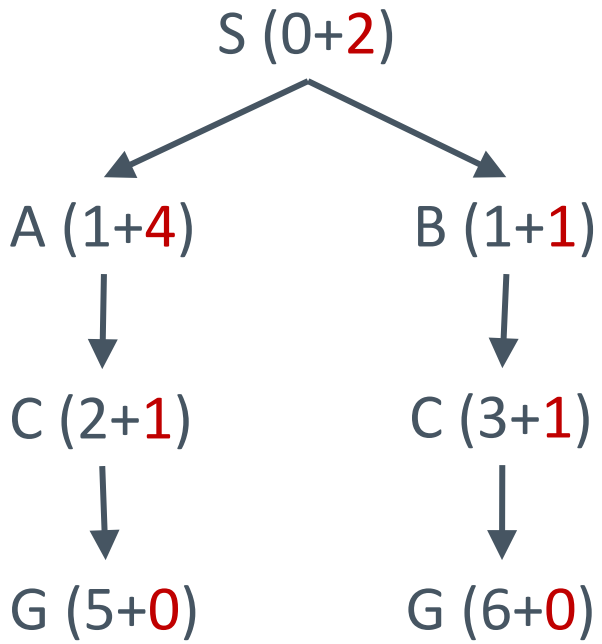
- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition

A* Graph Search Gone Wrong?

State space graph

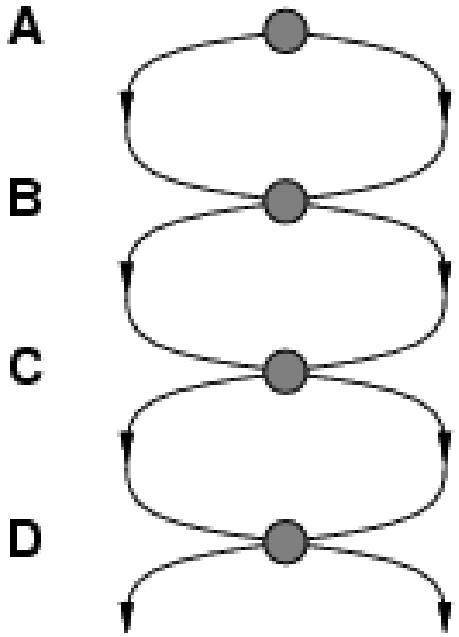


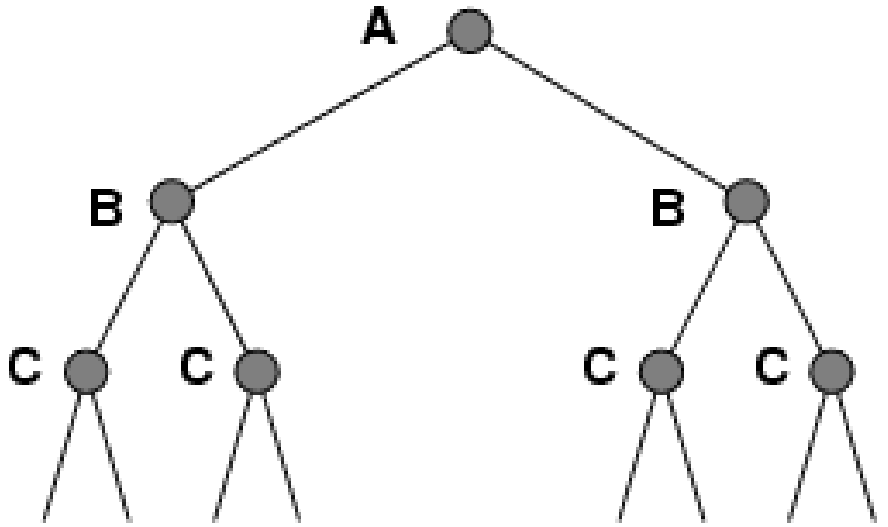
Search tree



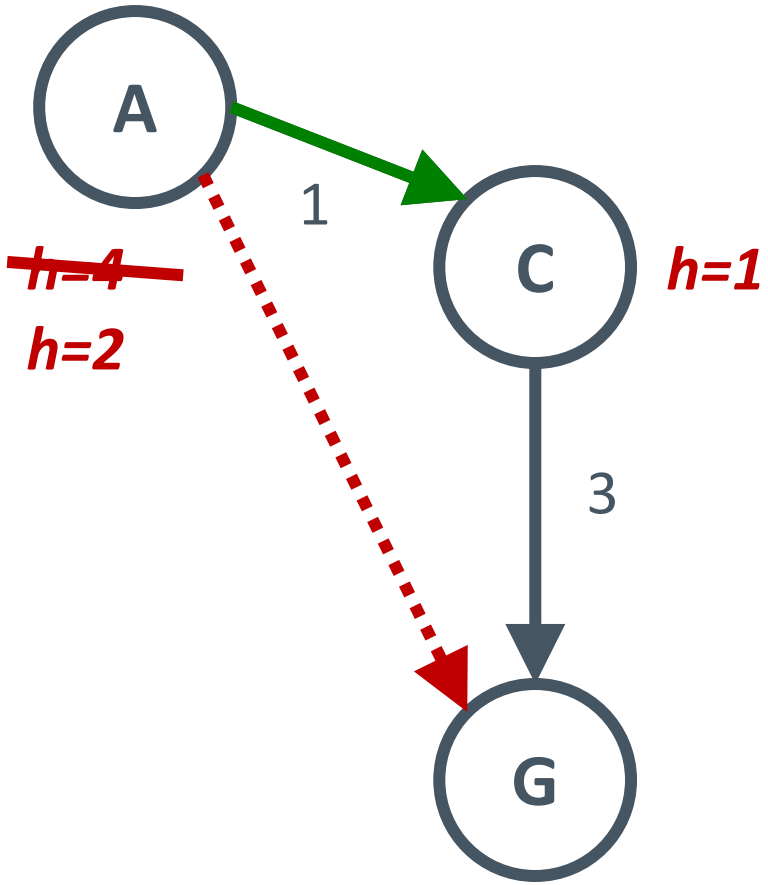
Repeated states

› Reminder: graph search keeps track of closed/repeated states





Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G
  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)

- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)
  - A* graph search is optimal

Optimality

- Tree search:
  - A* is optimal if heuristic is admissible
  - UCS is a special case (h = 0)

- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)

- Consistency implies admissibility

- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems

Generating heuristics?

- Effect of heuristic accuracy on performance
- How to generate admissible heuristics?
  - From relaxed problems
  - From sub-problems
  - From experience

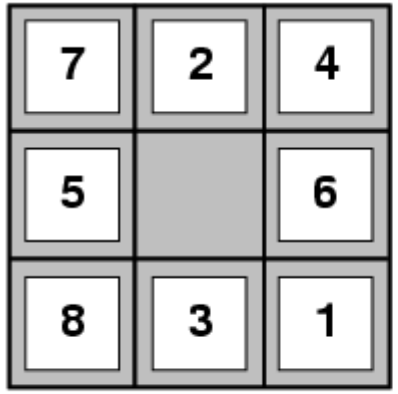
Heuristic accuracy vs performance

- Effective branching factor b* - if a total number of nodes generated by A* is N, and the solution depth is d, then b* is the branching factor that a uniform tree of depth d would have to have in order to contain N+1 nodes
- N+1 = 1 + b* + (b*)^2 + … + (b*)^d
- Well-designed heuristic would have b* closer to 1 -> allows large problems to be solved at a reasonable computational cost
- Evaluate usefulness of a heuristic experimentally on a small set of problems

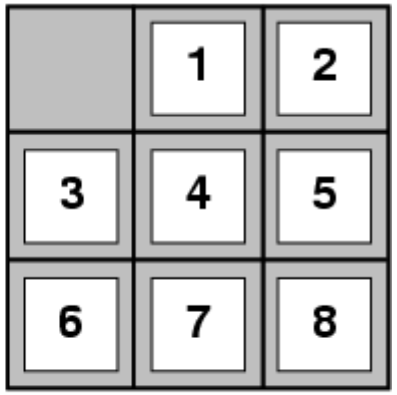
Heuristic dominance

› Ideas for heuristics?

How to tell the difference between two states, in terms of which is closer to the goal?



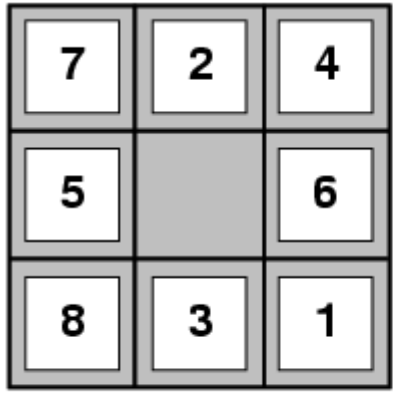
Start State



Goal State

Heuristic dominance

› How to compare 2 heuristics?



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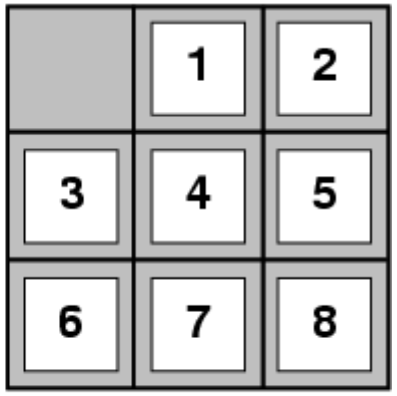


Goal State

- E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles

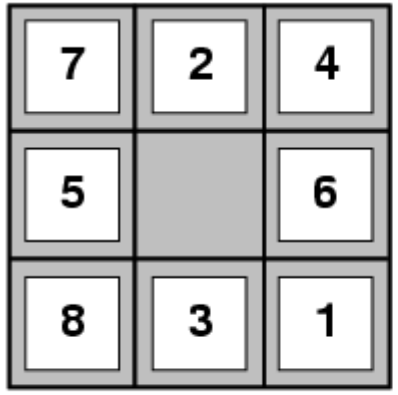
$h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)

Takes into account every "wrong" tile, and how "wrong" it is.
Higher "wrong" score = worse state to reach goal

Heuristic dominance

› How to compare 2 heuristics?



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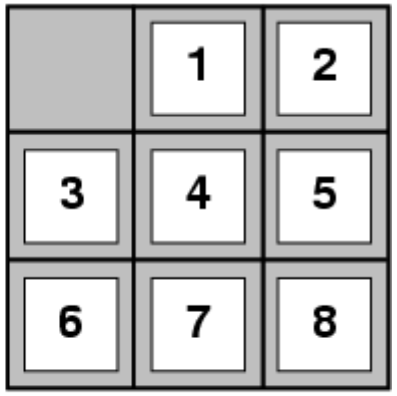


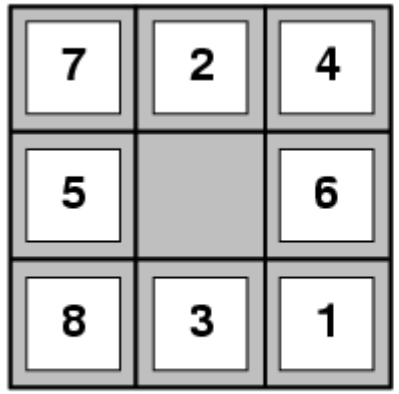
Goal State

Calculate h1 and h2 given the start state in the picture?

- h1 (S) = 8
- h2 (S) = 3+1+2+2+2+3+3+2 = 18

Heuristic dominance

- Which heuristic to choose?



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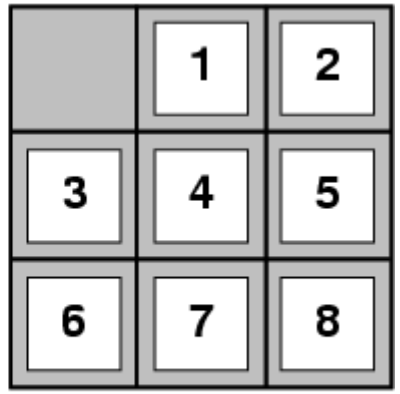
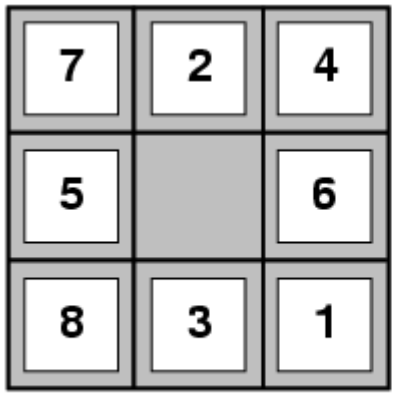


Goal State

	Average nodes expanded when the optimal path has...		
	...4 steps	...8 steps	...12 steps
TILES	13	39	227
MANHATTAN	12	25	73

Heuristic dominance



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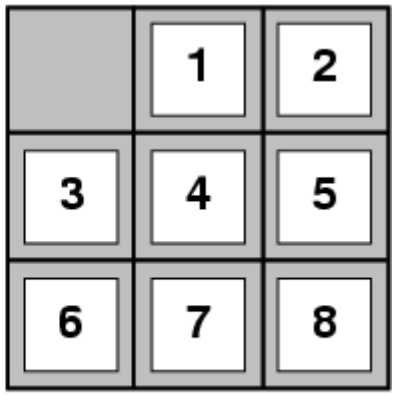

Goal State

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Heuristic dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible) then h_2 dominates h_1
- h_2 is better for search

Generating heuristics from relaxed problems

- Admissible heuristics can be derived from the exact solution cost of a **relaxed version** of the problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then h1 (n) gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then h2 (n) gives the shortest solution
- **Key point**: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Generating heuristic

- From sub-problems:
  - Eg 8-puzzle problem
  - Subproblem: getting tiles 1-4 into correct position (instead of all 8)
  - The cost of optimal solution is clearly a lower bound on the cost of the complete problem
- Learning from experience:
  - Solving a lot of 8-puzzle problems – given experience of a number of states and costs to the path, predict the cost from remaining states

Summary of search strategies

Strategy	What to select from frontier	complete/ halts?	space
DFS	First node added	No	Linear
BFS	Last node added	Yes	Exponential
Best first	Globally minimal h(p)	No	Exponential
Lowest Cost first	Minimal cost (p)	Yes	Exponential
A*	Minimal cost (p) +h(p)	Yes	Exponential

Heuristics

› Ideas for a heuristic to find a solution in:
  – a maze
  – Connect 4 game
  – Chess

Manhattan distance to maze exit ignoring walls => not perfect but better than nothing.