Homework 3

CS 205: Discrete Structures I
Fall 2019

Due: At the beginning of the lecture on Monday, Dec 2nd 2019

Total points: 100

Name:

NetID:

Section No.:

INSTRUCTIONS:

1. Print all the pages in this document and make sure you write the solutions in the space provided below each problem. This is very important!
2. Make sure you write your name, NetID, and Section No. in the space provided above.
3. After you are done writing the solutions, staple the sheets in the correct order and bring them to class on the day of the submission (See above). No late submissions barring exceptional circumstances!
4. As mentioned in the class, you may discuss with others but my suggestion would be that you try the problems on your own first. Even if you do end up discussing, make sure you understand the solution and write it in your own words. If we suspect that you have copied verbatim, you may be called to explain the solution.

Problem 1. [10 + 10 = 20 pts]
Prove that for any two sets X and Y in some universe U,

$$X = Y \text{ if and only if } \bar{X} = \bar{Y}.$$

Use the previous statement to prove that if A and B are subsets of a universe U then

$$\bar{A} \cap \bar{B} = \emptyset \text{ if and only if } A \cup B = U.$$

Problem 2. [4 parts × 10 pts = 40 pts]
Answer the following questions showing all the steps/work involved (in other words, if you only write the answer without showing any work at all, you are at the risk of getting a zero):

1. What is pow(pow(pow(∅))), where pow(S) denotes the power set of S?

2. If $A = \{n \in \mathbb{N} |\ 1 \leq n^2 < 90\}$ and $B = \{n^2 |\ n \in \mathbb{N} \wedge (1 \leq n \leq 3)\}$ then is it true that $A - B = A \oplus B$?

3. Describe the following set (by listing all its elements within { and }):

$$\{(a, b) \in \mathbb{Z} \times \mathbb{N} |\ a^2 + b^2 < 5\}.$$

4. For all $n \in \mathbb{N}, n \geq 1$, let $[n]$ denote the set $\{1, 2, \ldots, n\}$. What is the image of the function $f : [20] \to [20]$ that is defined as follows?

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ \frac{x+1}{2} & \text{otherwise} \end{cases}$$

More space for Problem 2:

Problem 3. [20 pts]
Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ that is surjective but not injective. You must explain why your example is surjective and why it is not injective.
Hint: To show that a function $f : \mathbb{N} \to \mathbb{N}$ is surjective, you need to show that for all $y \in \mathbb{N}$ there is some $x \in \mathbb{N}$ such that $f(x) = y$.
To show that a function is *not* injective, simply show that there are two points $x_1 \neq x_2$ in the domain such that $f(x_1) = f(x_2)$.

Problem 4. [20 pts]
Let $P(x)$ be the predicate *"x is divisible by* 4*"* and $Q(x)$ be the predicate *"x is divisible by* 11*"*. Let U, the universe, be the set $\{n \in \mathbb{N} |\ 1 \leq n \leq 1000\}$. What is the cardinality of the following set?

$$\{x \in U |\ P(x) \vee Q(x)\}.$$

Hint: Observe that

$$\{x \in U |\ P(x) \vee Q(x)\} = \{x \in U |\ P(x)\} \cup \{x \in U |\ Q(x)\}.$$

More space for Problem 4: