Quiz III (CS 205 - Fall 2019) (Solutions)

Name:

NetID:

Section No.:

For each of the following problems, use the space provided below the problem statement to write down your answer. Write clearly and concisely. There are 3 problems in total.

1. (**10 pts**) Consider the following statement

$$\textit{If } X \subseteq Y \textit{ then } \bar{Y} \subseteq \bar{X}.$$

Below is an incomplete proof of the statement. Complete the proof.
Proof: Assume that $X \subseteq Y$. Recall that in order to show that $\bar{Y} \subseteq \bar{X}$ we need to prove that

$$a \in \bar{Y} \rightarrow a \in \bar{X}.$$

Let a be an arbitrary element of the universe such that $a \in \bar{Y}$. This is equivalent to the statement

$$a \notin Y.$$

(Now use the fact that $a \notin Y$ and $X \subseteq Y$ to show that $a \in \bar{X}$. This would complete the proof)
Since $X \subseteq Y$ and $a \notin Y$, it is clear that $a \notin X$. This is equivalent to saying

$$a \in \bar{X}.$$

This finishes the proof.

2. (**10 + 10 = 20 pts**) For each of the following statements, state whether you think the statement is True or False and provide an explanation for your answer.

   (a) Let $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4\}$. Then it is possible to define a function $f : A \rightarrow B$ such that f is an injective function.
   Solution: False. If there is an injective function $f : A \rightarrow B$ then each element of A is mapped to a unique of element of B by f, i.e. $f(a), f(b), f(c), f(d), f(e)$ should all be distinct. This is impossible since $|B| = 4$.

   (b) Let $A = \text{pow}(\emptyset)$ then $\emptyset \subseteq A$ and $\emptyset \in A$.
   Solution: True. $A = \text{pow}(\emptyset) = \{\emptyset\}$ and so $\emptyset \in A$. Also since the emptyset if a subset of every set, we also have $\emptyset \subseteq A$.

3. (**20 pts**) There are 131 students in CS 205: 100 like chocolate ice cream, 50 like vanilla ice cream, and 20 like both chocolate and vanilla ice-cream. How many student are there in CS 205 that like neither chocolate ice cream nor vanilla ice cream?
Solution: Let A be the set of students who like chocolate ice cream, B be the set of students who like vanilla ice cream. We know that

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

$A \cap B$ is the set of students who like both chocolate and vanilla flavors. Using the given data, we can conclude that

$$|A \cup B| = 100 + 50 - 20 = 130.$$

$A \cup B$ is the set of all the students who like either chocolate or vanilla (or both). This means the number of who students who like neither chocolate nor vanilla is

$$131 - 130 = 1.$$