Midterm exam

CS 205: Discrete Structures I
Fall 2019

Total points: 100 (regular credit) + 40 (extra credit)

Duration: 1 hour

Name:

Section No.:

NetID:

INSTRUCTIONS:

1. There are 7 problems in all, 5 for regular credit and 2 for extra credit. You have 1 hour to attempt these problems. The points for each problem are specified along with the problem statement. To get full points for a problem, you must give details for all the steps involved in solving the problem AND arrive at the correct answer. Giving partial details or arriving at the wrong answer will result in a partial score.

2. Make sure you write your solutions ONLY in the space provided below each problem. There is plenty of space for each problem. You can use the back of the sheets for scratchwork.

3. You may refer to physical copies of any books or lecture notes you want to during the exam. However, the use of any electronic devices will lead to cancellation of your exam and a zero score, with the possibility of the authorities getting involved.

4. Make sure you write your name, NetID, and section number in the space provided above.

5. If we catch you cheating, or later suspect that your answers were copied from someone else, you will be given a zero on the exam, and might even be reported to the authorities!

Regular credit: 100 pts, 7 problems

Problem 1. [20 pts]
Show that the following compound proposition is a tautology.

$$((r \to (q \to p)) \land (q \land r)) \to p.$$

If you choose to use truth tables be sure to include columns for all intermediate propositions that occur in the expression (i.e., columns for $(q \land r)$, $(q \to p)$, $r \to (q \to p)$, etc.)

Problem 2. [$5 \times 4 = 20$ pts]
State which of the following statements are true and which of them are false. Give very short explanations for your answers.

1. If 2.5 is irrational then 72 is an odd number.

2. The sum of any three even numbers must be even.

3. $p \wedge (\neg p) \equiv \mathbf{F}$ only if $2 \times 2 = 5$.

4. $(p_1 \vee p_2) \rightarrow (p_1 \wedge p_2)$ is a tautology.

5. Assuming that the domain is the set of all integers, the expression $\forall x\ \exists y\ (x + y = 5)$ is true.

More space for Problem 2:

Problem 3. [20 pts]
Consider an argument with premises

- $(p \to q) \wedge (r \vee s)$
- $\neg(q \vee s)$

and conclusion $r \wedge \neg p$. Show that it is a valid argument. Give all steps and mention the rules of inference being used.

Problem 4. [20 pts]
Prove that for all integers $n \geq 0$ $n^3 - n$ is divisible by 6. You must give a formal proof with all steps. Feel free to use the following facts if you need to:

$$\forall n \geq 0,\ n(n+1) = n^2 + n \textit{ is even.}$$

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b).$$

Problem 5. [20 pts]
Prove that if α is irrational then $\frac{1+\alpha}{1-\alpha}$ must also be irrational.

Extra credit: 40 pts, 2 problems

Problem 8. [20 pts]
Assuming that both $\sqrt{2}$ and $\sqrt{3}$ are irrational, prove that both $\sqrt{2}+\sqrt{3}$ and $\sqrt{2}-\sqrt{3}$ must also be irrational.

Problem 9. [20 pts]
Use strong induction to prove that every positive integer $n \geq 1$ can be written as

$$n = 2^k \ell,$$

where $k \geq 0$ is some integer and ℓ is an odd integer.