Quiz I (CS 205 - Fall 2019)

Name:

NetID:

Section No.:

For each of the following problems, use the space provided below the problem statement to write down your answer. Write clearly and concisely. There are 4 problems in total.

1. (**20 pts**) Consider an argument whose premise is $\exists x\ (P(x) \rightarrow Q(x))$ and conclusion is $(\forall x\ \neg Q(x)) \rightarrow \neg\,(\forall x\ P(x))$. Prove that this is a valid argument using rules of inference. Show all the steps and mention the rule you use in each step.
   (Hint: What can you do when the conclusion of an argument is of the form $p \rightarrow q$?)

2. (**5 + 5 = 10 pts**) Write the following propositions in the *"If... then ..."* form:

   (a) *"A sufficient condition for $\mathcal{NP}$ to be contained in $\mathcal{BPP}$ is that SAT can be solved in randomized polynomial time".*

(b) *"A necessary condition for Graph isomorphism to be $\mathcal{NP}$-complete is that the polynomial hierarchy collapses".*

3. (**10 pts**) Suppose the domain of discourse is the set of integers and let $P(x)$ be the predicate *"x is a perfect square"*. Express $P(x)$ as a predicate formula using logical connectives, quantifiers, and other mathematical symbols (if needed).
   (Recall: x is a perfect square if x can be written as $x = n^2$ for some integer n.)

4. (**10 pts**) Is the proposition $p_1 \wedge p_2 \wedge \ldots \wedge p_{1001} \wedge (p_1 \oplus p_2 \oplus \ldots \oplus p_{1001})$ a contradiction? Provide a short explanation for your answer.
   (Recall: A contradiction is a proposition that is always false, regardless of what truth values are assigned to the variables.)