

Quiz III (CS 205 - Fall 2019) (Solutions)

Name:

NetID:

Section No.:

For each of the following problems, use the space provided below the problem statement to write down your answer. Write clearly and concisely. There are 3 problems in total.

1. (10 pts) Consider the following statement

$$\text{If } Y \subseteq X \text{ then } \bar{X} \subseteq \bar{Y}.$$

Below is an incomplete proof of the statement. Complete the proof.

Proof: Assume that $Y \subseteq X$. Recall that in order to show that $\bar{X} \subseteq \bar{Y}$ we need to prove that

$$a \in \bar{X} \rightarrow a \in \bar{Y}.$$

Let a be an arbitrary element of the universe such that $a \in \bar{X}$. This is equivalent to the statement

$$a \notin X.$$

(Now use the fact that $a \notin X$ and $Y \subseteq X$ to show that $a \in \bar{Y}$. This would complete the proof)

Since $Y \subseteq X$ and $a \notin X$, a cannot be contained in Y and so $a \notin Y$. This is equivalent to

$$a \in \bar{Y}.$$

This completes the proof

2. (10 + 10 = 20 pts) For each of the following statements, state whether you think the statement is True or False and provide an explanation for your answer.

- (a) Let $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4, 5\}$. Then it is possible to define a function $f : A \rightarrow B$ such that f is bijective.

Solution: True. Consider the function $f : A \rightarrow B$ such that

$$f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4, f(e) = 5.$$

Clearly f is a bijection.

- (b) Let $U = \{1, 2, 3, 4, 5\}$ then there is a set $S \subseteq U$ such that $\emptyset \notin \text{pow}(S)$.

Solution: False. Let $S \subseteq U$ be an arbitrary subset of U . Since $\emptyset \subseteq S$, it must be the case that

$$\emptyset \in \text{pow}(S),$$

and so the statement is False.

3. (20 pts) There are 150 students in CS 205: 120 know Python, 40 know Rust, and 30 know both Python and Rust. How many student are there in CS 205 that know either Python or Rust?

Solution: Let A be the set of students who know Python, B be the set of students who know Rust. Then $A \cap B$ is the set of students who know both Python and Rust, and $A \cup B$ is the set of students who know either Python or Rust. Using the given data, we can conclude

$$|A \cup B| = |A| + |B| - |A \cap B| = 120 + 40 - 30 = 130,$$

and so the number of students who know either Python or Rust is 130.