

## Quiz I [Version 2] (CS 205 - Fall 2019) [Solutions]

Name:

NetID:

Section No.:

For each of the following problems, use the space provided below the problem statement to write down your answer. Write clearly and concisely. There are 4 problems in total.

1. (10 pts) Does  $p_1 \wedge p_2 \wedge \dots \wedge p_{100} \wedge (p_1 \oplus p_2 \oplus \dots \oplus p_{100})$  have a satisfying assignment? If yes, provide any satisfying assignment and explain in a concise manner why it is a satisfying assignment, and if no, provide a short explanation for why there is no satisfying assignment. (Recall: A satisfying assignment is an assignment of truth values to the variables that makes the proposition true.)

Answer:

If any of the variables is False, then the whole proposition becomes False because of the conjunction. Thus, the only possibility for a satisfying assignment is when all the variables are True. In this case, the truth value of the proposition is determined by the truth value of the expression

$$(p_1 \oplus p_2 \oplus \dots \oplus p_{100})$$

in the case when all variables are True. Since this is an XOR of an even number of True values, we know (from the HW) that its truth value will be False, and thus the Truth value of the entire proposition will be False.

Thus the proposition has no satisfying assignment.

2. (10 pts) Suppose the domain of discourse is the set of real numbers and let  $R(x)$  be the predicate  $x$  is a rational number. Express  $R(x)$  as a predicate formula using logical connectives, quantifiers, and other mathematical symbols (if needed). (Recall:  $x$  is a rational number if there are integers  $p, q$  such that  $q \neq 0$  and  $x = \frac{p}{q}$ .)

Answer:

$$R(x) = \exists p \in \mathbb{Z} \exists q \in \mathbb{Z} \left( (q \neq 0) \wedge (x = \frac{p}{q}) \right)$$

OR

Let  $I(x) =$  “ $x$  is an integer” then we can write  $R(x)$  as

$$R(x) = \exists p \exists q \left( I(p) \wedge I(q) \wedge (q \neq 0) \wedge (x = \frac{p}{q}) \right)$$

3. (5 + 5 = 10 pts) Write the following propositions in the “If...then...” form:

- (a) “A necessary condition for  $\mathcal{NP}$  to be contained in  $\mathcal{BPP}$  is that the polynomial hierarchy collapses”.

Answer: “If  $\mathcal{NP}$  is contained in  $\mathcal{BPP}$  then the polynomial hierarchy collapses”.

(b) “Graph isomorphism is  $\mathcal{NP}$ -complete only if the polynomial hierarchy collapses”.

Answer: “If Graph isomorphism is  $\mathcal{NP}$ -complete then the polynomial hierarchy collapses”.

4. (20 pts) Consider an argument whose premise is  $\forall x (Q(x) \rightarrow P(x))$  and conclusion is  $((\exists x Q(x)) \rightarrow (\exists x Q(x) \wedge P(x)))$ . Prove that this is a valid argument using rules of inference. Show all the steps and mention the rule you use in each step.  
(Hint: What can you do when the conclusion of an argument is of the form  $p \rightarrow q$ ?)

Proving that the given argument is valid is equivalent to proving that the argument

$$\begin{array}{l} \forall x (Q(x) \rightarrow P(x)) \\ \exists x Q(x) \\ \text{---} \text{---} \text{---} \\ \therefore \exists x (Q(x) \wedge P(x)) \end{array}$$

So we will focus on proving that the latter argument is valid.

- (a)  $\forall x (Q(x) \rightarrow P(x))$  (Premise)
- (b)  $\exists x Q(x)$  (Premise)
- (c)  $Q(c)$  for some  $c$  (Existential Instantiation on (b))
- (d)  $Q(c) \rightarrow P(c)$  for the same  $c$  as in (c) (Universal Instantiation on (a))
- (e)  $P(c)$  (Modus Ponens on (c) and (d))
- (f)  $Q(c) \wedge P(c)$  (Conjunction of (c) and (e)).
- (g)  $\exists x (Q(x) \wedge P(x))$  (Existential Instantiation on (f); can do this since the statement (f) is true for some  $c$  in the domain)

Since (g) is the conclusion, we have proved that the argument is valid.