Quiz I (CS 205, Fall 2019)

Name:

NetID:

Section No.:

For each of the following problems, use the space provided below the problem statement to write down your answer. Write clearly and concisely. There are 4 problems in total.

1. (**10 pts**) Does the proposition $p_1 \wedge p_2 \wedge \ldots \wedge p_{100} \wedge (p_1 \oplus p_2 \oplus \ldots \oplus p_{100})$ have a satisfying assignment? If yes, provide any satisfying assignment and explain in a concise manner why it is a satisfying assignment, and if no, provide a short explanation for why there is no satisfying assignment.
   (Recall: A satisfying assignment is an assignment of truth values to the variables that makes the proposition true)

2. (**10 pts**) Suppose the domain of discourse is the set of real numbers and let $R(x)$ be the predicate *"x is a rational number"*. Express $R(x)$ as a predicate formula using logical connectives, quantifiers, and other mathematical symbols (if needed).
   (Recall: x is a rational number if there are integers p, q such that $q \neq 0$ and $x = \frac{p}{q}$.)

3. (**5 + 5 = 10 pts**) Write the following propositions in the *"If... then ..."* form:

   (a) *"A necessary condition for $\mathcal{NP}$ to be contained in $\mathcal{BPP}$ is that the polynomial hierarchy collapses".*

   (b) *"Graph isomorphism is $\mathcal{NP}$-complete only if the polynomial hierarchy collapses".*

4. (**20 pts**) Consider an argument whose premise is $\forall x \ (Q(x) \rightarrow P(x))$ and conclusion is $(\exists x \ Q(x)) \rightarrow (\exists x \ (P(x) \wedge Q(x))$. Prove that this is a valid argument using rules of inference. Show all the steps and mention the rule you use in each step.
   (Hint: What can you do when the conclusion of an argument is of the form $p \rightarrow q$?)