Midterm exam

CS 205: Discrete Structures I
Fall 2019

(Sample)

Total points: 100 + 30 (extra credit)

Duration: 1 hour

Name:

Section No.:

NetID:

INSTRUCTIONS:

1. There are 9 problems in all, 7 for regular credit and 2 for extra credit. You have 1 hour to attempt these problems. The points for each problem are specified along with the problem statement. To get full points for a problem, you must give details for all the steps involved in solving the problem AND arrive at the correct answer. Giving partial details or arriving at the wrong answer will result in a partial score.

2. Make sure you write your solutions ONLY in the space provided below each problem. There is plenty of space for each problem. You can use the back of the sheets for scratchwork.

3. You may refer to physical copies of any books or lecture notes you want to during the exam. However, the use of any electronic devices will lead to cancellation of your exam and a zero score, with the possibility of the authorities getting involved.

4. Make sure you write your name, NetID, and section number in the space provided above.

5. If we catch you cheating, or later suspect that your answers were copied from someone else, you will be given a zero on the exam, and might even be reported to the authorities!

Regular credit: 100 pts, 7 problems

Problem 1. [10 + 3 = 13 pts]
Write the truth table for the compound proposition:

$$((\neg r) \land (p \to q) \land (q \to r)) \to (\neg p).$$

What do you call this kind of proposition?

Problem 2. [$5 \times 3 = 15$ pts]
State which of the following statements are True and which of them are False. Give very short explanations for your answers.

1. If $2 + 2 = 5$ then $1 < 2$.
2. If $1 < 2$ then $2 + 2 = 5$.
3. The product of two rational numbers is always rational.
4. $(p_1 \oplus p_2) \rightarrow (p_1 \vee p_2)$ is a tautology.
5. Assuming that the domain is natural numbers, $\forall x \; \exists y \; (x = y^2)$.

Problem 3. [15 pts]
Consider an argument with premises

- $(p \vee q) \rightarrow r$
- $(\neg p) \rightarrow (\neg a)$
- $b \vee a$
- $\neg b$

and conclusion r. Show that it is a valid argument. Give all steps and mention the rules of inference being used.

Problem 4. [15 pts]
Prove that, for all integers $n \geq 0$, $n(n+1)(n+2)$ is divisible by 3. You must give a formal proof with all steps.

Problem 5. [10 pts]
Prove that if x^{2019} is odd then x must be odd.

Problem 6. [15 pts]
Consider an argument with premises:

- $((\exists x\ P(x)) \wedge (\exists x\ Q(x))) \rightarrow p$
- $P(c)$ for some c
- $\neg p$

and conclusion

$$\forall x\ (\neg Q(x)).$$

Show that this argument is valid and give a proof thereof showing all the steps and the rules of inference used in each step.

Problem 7. [8 + 9 pts]

In this problem you will prove that there are no positive integers $x, y, z \geq 1$ that satisfy the equation $x^3 + y^3 + z^3 = 28$.

1. (**8 pts**) First prove that if $x^3 + y^3 + z^3 = 28$ then $(x \geq 3) \vee (y \geq 3) \vee (z \geq 3)$.
2. (**9 pts**) Now using the statement proved in the previous part and the fact that $x, y, z \geq 1$ conclude that there are no integers $x, y, z \geq 1$ that satisfy the given equation.

Extra credit: 30 pts, 2 problems

Problem 8. [15 pts]
Consider a rectangular board of size 60 inches $\times$ 10 inches. Suppose that 745 darts are thrown at the board and all of them end up landing on the board. Prove that there must be two darts that end up landing within 1.5 inches of each other. You may assume that $\sqrt{2} = 1.414$.

Problem 9. [15 pts]
Prove that for every pair of rational numbers a and b such that $0 < a < b < 1$ there is an irrational number c such that $a < c < b$.