Quiz IV (CS 205 - Fall 2019)

Name:

NetID:

Section No.:

For each of the following problems, use the space provided below the problem statement to write down your answer. Write clearly and concisely. There are 3 problems in total.

1. (**10 pts**) Let $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$, i.e. the set of positive integers, and let $\mathbb{Q}^+$ be the set of all positive rational numbers (0 is not included). Show that there is a surjective function $f : \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Q}^+$. You must prove that the function you state as an example is surjective. Is the function you provided as an example also injective? Why or why not?

2. (**10 + 10 = 20 pts**) For each of the following statements, state whether you think the statement is True or False and provide an explanation for your answer.

   (a) Let A, B, C be finite sets such that there is an injective function $f : A \to B$ and a surjective function $g : C \to B$. Then $|A| \leq |B| \leq |C|$.

   (b) Let $E = \{0, 2, 4, \ldots\}$, i.e. the set of *all* even natural numbers, and $O = \{1, 3, 5, \ldots\}$, i.e. the set of *all* odd natural numbers. Then $|E| \neq |O|$, i.e. there is no bijection between the two sets.

3. (**20 pts**) Consider the infinite sequence given by the following recurrence:

$$a_0 = 0$$

$$a_n = a_{n-1} + 2n - 1 \text{ for } n \geq 1.$$

Compute the first few terms of the sequence using the recurrence. Observe a pattern in the values and try to guess a formula for a_n (the formula should be purely in terms of n). Use induction to prove that the formula you guessed is correct.