MATH 210 : Differential Equations.

A differential equation is an equation involving an unknown ~~fg~~ function $y(t)$ and its derivatives $y', y'', y''', \ldots$

The order of a differential equation is the ~~highest order der~~ order of the highest order derivative appearing in the equation.

~~Ex~~ A differential equation is linear if $y, y', y'', \ldots$ appear on their own and not inside another function.

Ex: $y' = y$ First order linear

... and we can solve it!

$y(t) = Ce^t$, any constant C

Ex: $y' = y^2$ First order nonlinear

... and we can solve it!

It is separable!

$y' = y^2 \Rightarrow \frac{1}{y^2} y' = 1 \Rightarrow \int \frac{1}{y^2} dy = \int dt$

$\Rightarrow -\frac{1}{y} = t + C \Rightarrow y = \frac{-1}{t+C}$, C constant.

Ex: $y'' + (y')^2 + ty = \cos(t)$

Second order nonlinear

Can you solve it? Impossible!
exactly

$\Rightarrow$ There are many kinds of DEs.
Most of them are impossible to solve explicitly, exactly.
... but we can always find numerical approximations of solutions.

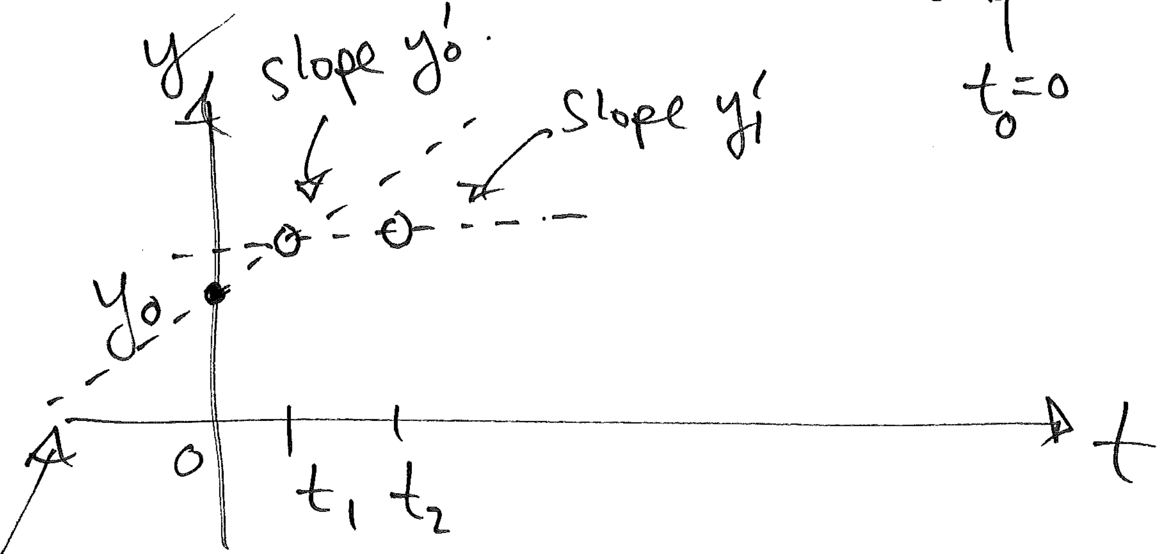
Euler's method for first order equations

Consider a first order equation

$y' = f(t, y) \leftarrow$ some function of t and y.

with initial conditions $y(0) = y_0$

$$\underset{t_0 = 0}{\uparrow}$$



Note that the equation $y' = f(t, y)$ tells us everything about the <u>slope</u> of <u>$y(t)$</u>.

$\Rightarrow$ $y'(0) = f(0, y_0) = y_0'$

~~Note that~~ Note that <u>this</u> is the tangent line to $y(t)$ at $(0, y_0)$.

$\Rightarrow$ We approximate $y(t_1)$ by the tangent line

$$y(t_1) \cong \underbrace{y(0) + t_1 y'(0, y_0)}_{y_1}$$

Do the same for $t = t_1$, $y = y_1$

$$y(t_2) \simeq \underbrace{y_1 + (t_2 - t_1)\, y'(t_1, y_1)}_{y_2}$$

<u>Euler's method</u>: $y' = f(t, y)$, $y(t_0) = y_0$

$$\Rightarrow \quad y_{n+1} = y_n + h\, f(t_n, y_n)$$

with step size h.
