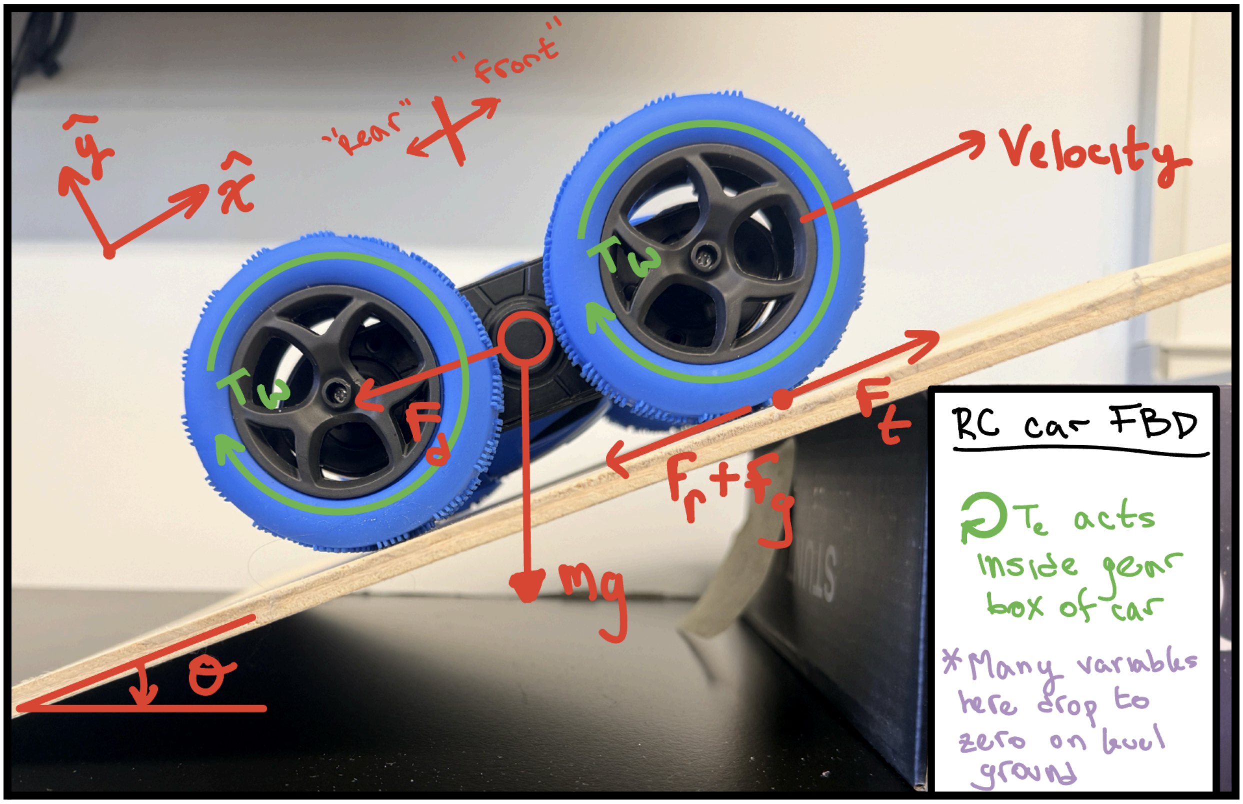
ODE's and Transfer Functions for an RC Car System:



*The components and variables pictured here are referenced throughout the ODE, TF, and State Space solving.

RC Car ODEs:

Solving the Longitudinal Dynamics ODEs:

Force balance:

$$m\frac{dv}{dt} = F_t - F_d \text{ where } F_d = F_{rolling} + F_{drag} + F_{grade/gravity\ component}$$

**For a small RC car on flat ground F_d is effectively zero because of little to no drag. As for F_{drag} and $F_{grade/gravity\ component}$, they are already zero when the car is on level ground.

Conditions:

- If the wheel roll without slip that gives: $\omega_{wheels} = \frac{v}{r}$
- Sum of torques (Newton's Law of Rotation):
    - $J\dot{\omega} = T_w - rF_d$
    - Where:
        - T_w: Torque delivered to the wheel from the internal circuitry
        - rF_d: Resisting Torque from ground at wheel base
        - $J\dot{\omega}$: Simply Inertia Times angular acceleration (ie. General Torque equation).
- More details on conditions were outlined in http://doi.org/10.56958/jesi.2018.3.3.251 (they were not outlined more explicitly in this project s.t. focusing on the system mechanics was of greater focus)

After solving for $\frac{dv}{dt}$ and taking into account necessary assumptions for an RC car, the force balance solves to:

$$(J_w + mr^2 + i_g^{\,2} J_e)\,\dot{v} = r i_g T_e + r^2 F_d$$

Where:

- v = vehicle speed
- T_e = engine (motor) torque
- r = wheel radius
- i_g = gear ratio
- J_w = wheel inertia
- J_e = engine inertia

But when drag is zero and surface is flat:

$$(J_w + mr^2 + i_g^{\,2} J_e)\,\dot{v} = r i_g T_e$$

If we express the engine motor's torque, $T_e = K_t i_g$, in terms of wheel torque we get $T_w = i_g T_e$.

Then, dividing both sides by r^2 sets the right side of the equation equal to F_t which is exactly equal to $\frac{iT_e}{r}$. Now if we define the entire left side as M_{eq} we are left with a form of Newton's second law:

$$M_{eq}\dot{v} = F_t$$

Where:

- $M_{eq} = \frac{(J_w + mr^2 + i^2 J_e)}{r^2}$
- $F_t = \frac{i_g T_e}{r}$

Solving the Motor and Internal Circuitry ODEs:

Similarly to class, the internal rotating DC motors can be modeled as:

- Electrical:
  - $L\frac{di}{dt} + Ri = V - K\omega$
- Torque Production:
  - $T_e = K_e i$
  - i=current
- Mechanical Rotation:
  - $I\dot{\omega} + b\omega = T_e - T_d$ where $\omega = \frac{v}{r}$ (no slip)

Where:

- V= voltage input
- T_d= disturbance torque from environment
- i = current
- ω = rotor speed
- b = damping
- T_e = applied torque of the system
- I = rotor inertia

This means if:

$L\frac{di}{dt} = V - Ri - K_e\frac{v}{r}$ ← where ω $= \frac{v}{r}$

Then we know $F_t = \frac{i_g K_t}{r} i$

Combining both ODEs Yields:

$$\dot{v} = \frac{1}{M_{eq}}\left(\frac{i_g K_t}{r}\right)$$

State Spaces:

Taking the ODEs into account and picking the states, that leaves:

$$x_1 = i\ (motor\ current)$$
$$x_2 = v\ (RC\ car\ speed)$$
$$u = V\ (applied\ motor\ voltage)$$

First take $L\frac{di}{dt} = V - Ri - K_e\frac{v}{r}$ and isolate $\frac{di}{dt}$ to get:

$$\dot{x}_1 = \frac{u}{L} - \frac{R}{L}x_1 - \frac{K_e}{Lr}x_2$$

For the vehicle ODE, simply take $\dot{v} = \frac{1}{M_{eq}}\left(\frac{i_g K_t}{r}\right)$ and account for $\dot{v}$:

$$\dot{x}_2 = \frac{1}{M_{eq}}\left(\frac{i_g K_t}{r}\right)$$

In state space model form:

$$\dot{x} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_e}{Lr} \\ \frac{i_g K_t}{M_{eq} r} & 0 \end{bmatrix} x + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

Transfer functions:

Below is the laplace work to develop the transfer functions which combine the electrical and motor ODEs. The red assumption at the bottom was made from: http://doi.org/10.56958/jesi.2018.3.3.251

Laplace + TFs:

Electrical ODE:

- $L\frac{di}{dt} + Ri = \overset{u \text{ value}}{V} - k_e\frac{v}{R}$

$i \nearrow$

- $LsI(s) = u(s) - \frac{K_e}{R}v(s) - RI(s)$

Motor Dynamics ODE

- $\dot{v} = \frac{1}{M_{eq}}\left(\frac{i_g K_t}{r}\right)i$
- $M_{eq}sV(s) = \underbrace{\frac{i_g k_t}{r}}_{\text{"}z\text{"}} I(s)$

$$\Rightarrow I(s) = \frac{M_{eq}sV(s)}{z}$$

if $G(s) = \frac{V(s)}{u(s)}$,

$$LsI(s) = u(s) - \frac{K_e}{R}v(s) - RI(s)$$

$$LsI(s) + RI(s) = u(s) - \frac{K_e}{R}v(s)$$

$$I(s)(Ls+R) = \text{" \quad "}$$

$$\frac{M_{eq}sV(s)}{z}(Ls+R) = u(s) - \frac{K_e}{R}v(s)$$

$$v(s)\left[\frac{M_{eq}s}{z}(Ls+R) + \frac{K_e}{R}\right] = u(s)$$

$$\boxed{G(s) = \frac{V(s)}{U(s)} = \frac{1}{\frac{M_{eq}s}{z}(Ls+R) + \frac{K_e}{R}} \quad \text{*where } z = \frac{i_g k_t}{r}}$$

*from what I can tell from our sources:

$$\boxed{G(s) \approx \frac{1}{\tau s+1}}$$

from approximation