MAE 3260 Final Group Work

Title: Analysis of System Dynamics for Quadcopter Toy

Quadcopter Toy: FAO Schwarz RC X-Treme Aero High Performance Vehicle

Abstract: Our team is studying the system dynamics of a quadrotor by breaking down how its components work together to keep it stable in flight. We chose a quadrotor because we've covered the basics of how they generate lift in fluid dynamics. This project gives us a chance to look deeper at the actual control system behind that behavior. Our main focus is the motor dynamics and the way the quadrotor responds to commands and disturbances. To do that, we'll map out the system with block diagrams and examine its control law. We will also analyze whether it operates in open loop or closed loop, how its feedback laws work, and how well it follows inputs while rejecting external disturbances.

Students/Roles:

Student	**Task/Role**	**Portfolio**
Dustin Gibian	Thrust To Vertical Position and Motor Dynamics: Analyze the quadcopters vertical dynamics and the motor dynamics to write a transfer function from thrust to vertical position and from voltage to motor speed.	https://github.com/Cornell-MAE-UG/fa25-portfolio-Dustin-Gibian
Thomas Meyer	Thrust Experiments, Measure Voltage of battery and at motor points to analyze voltage to motor rpm response.	https://cornell-mae-ug.github.io/fa25-portfolio-tmeyer2310/
Shayla Reid	Analyze how the motor voltage relates to altitude, measure the maximum weight the quadcopter can lift, and measure the voltage on the motors	https://shayla-rd.github.io/github-portfolio/
Robert Brown	Electromechanical Transduction Analysis, including applying linked first-order odes (Hadas Ritz) to our system and acknowledging shortcomings. From these ODES derive transfer functions, estimating parameters, drawing BODE plot, and validating testing.	https://cornell-mae-ug.github.io/spring-2025-portfolio-rbrown233/

List of MAE 3260 concepts or skills used in this group work:

- Transfer Functions
- Block Diagrams
- Laplace Transform
- Dynamics
- Open Loop Systems
- First Order Systems
- Second Order Systems
- Bode Plots

Pages 2-9 include two pages for each student:

- Summarize technical work, including figures
- For references, use numbers/brackets (e.g. [1]) in text with list - IEEE citation format.
- Students can combine section (i.e. 4 pages for students A and B), but will get the same grade

Notes:

- Mass: 31g
- Used spring
- Assume open loop because things do not work
- Ramp-up steady increase then jump
- Time to 50cm = 0.01 seconds
- Blade rotation: 628 RPM
- Frames per minute: 3142
- Right motor: 6.25
- Back motor: 6.5
- Front: 6V
- Left (Painted Blade): 6.4V
- Link to product: https://www.target.com/p/fao-schwarz-drone-xtreme-aero-stunt-led/-/A-90593671

Experiments and Data Gathering

(Thomas Meyer)

Motor and Battery Voltage

We disassembled the frame of the drone but kept all the electronics intact so that we could perform measurements on the electronics. Using a Voltmeter, we measured the voltage at each of the four motors as well as the battery. For the battery, we measure the voltage at rest and while spinning the motors.

Battery Voltage (at rest) = 8V
Battery Voltage (spinning motors) = 7.25V

Motor Voltages:

- Right motor: 6.25V
- Back motor: 6.5V
- Front motor: 6V
- Left motor: 6.4V

The uneven distribution of voltage to the motors is the likely cause of the drone not flying straight up when we tested it. The battery voltage drop as we spun the motor was expected as the voltage across the motor would increase when spinning the blades. Using the transfer function of motor voltage to rpm, we can use our data to calculate the rpm for each motor.

Total Thrust of Drone

To find the total thrust of the drone we tried two setups. First, we mounted the drone on spring scale with a band and accelerated the drone upward at max power (see image right). We recorded a video to see the displacement on the spring scale to record the force generated by the drone. We struggled to get consistent precise results in the experiment as the drone would not fly straight nor hold its position very well.

Next, we conducted tests where we progressively increased the weight attached to the bottom of the drone to see how much weight the drone could lift in addition to its own weight (31g). To accomplish this, we attached weights underneath the drone with a band. We found the drone could lift 12g of additional weight giving it an estimated total thrust of 0.422N.

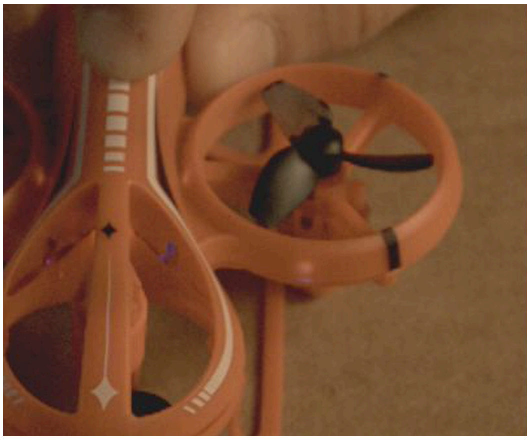
(Assuming Total thrust = Total weight)

$$(9.81 m/s^2 * (0.031 kg + 0.012 kg) = 0.422 N)$$

This is still rather approximate as the drone again struggled to fly straight up vertically and was quite unstable while carrying the weights.

Motor RPM: High Speed Camera

Using the High Speed Camera in the Taylor Studio, we tried to measure the motor speed. At first, the rotors spun much too fast for the camera to capture. However, after increasing the framerate of the camera to 3142 frames per minute, we were able to estimate the motor RPM. We found an estimated RPM of 628.

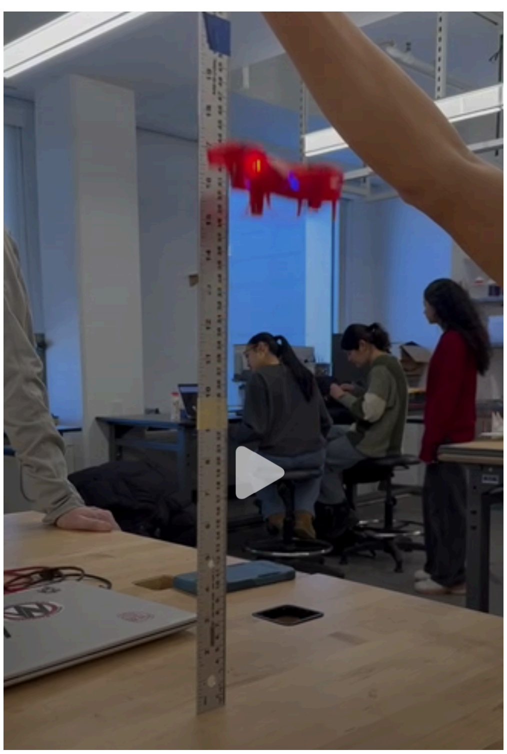


Another interesting observation we found was that the ramp up of the motor was not continuous. The motor would consistently ramp up to a first lower rpm and then have a quasi step jump up to the max rpm. This was so discernable that we could see it just through observation. It was even more evident in the high speed camera data.

To the left is a freeze frame image from the high speed camera. We colored one of the props silver to be able to measure the approximate rpm. We then measure the rotations per minute by counting the number of frames it took for the prop to travel a set number of revolutions.

Vertical acceleration

To find the vertical acceleration, we recorded videos of the drone flying upward next to a ruler. We noted the time it took for the drone to fly upward a set distance. We analyzed the video frame by frame.



Measured data:

0 cm in 0s
17cm in 0.1s
50 cm in 0.21s

$v_1 = 0$ m/s
$v_2 = 1.7$ m/s
$v_3 = 3.0$ m/s

$a_1 = 17$ m/s^2
$a_2 = 27.27$ m/s^2

$a_{avg} = 22$ m/s^2

$\Rightarrow$ 0.031kg * 22m/s^2 = 0.682N Thrust

*Similar to thrust found in weight experiments.

Thrust to Vertical Position and Motor Dynamics

(Dustin Gibian)

Our quadcopter toy has no gyroscope or sensors that we could find. Because the drone has no sensors there is no measurement of altitude, velocity, acceleration, pitch, roll, or yaw and there is no feedback stabilization. A block diagram of the system has no feedback path. Voltage commands are simply sent to motors based on inputs from the joysticks, and it is up to the user to fly the toy on their own.

The control of the system is open-loop. In an open-loop system the controller sends a command to the plant without using feedback. The controller has no knowledge of whether the desired output was achieved.

The vertical dynamics of the drone are second order and equilibrium can be written.
Solving for Z and performing a Laplace Transform with Zero initial conditions and only analyzing deviations from hover.
Defining small perturbations for Thrust and Z position.
Then plugging into the dynamics.
Yielding the transfer function for Thrust to Vertical Position.

Motor Dynamics 1st Order

$$\tau\dot{\omega} + W = KV$$

$$k_\omega : \left[\frac{rad/s}{V}\right]$$

$$\frac{d\omega}{dt} = -\frac{1}{\tau}\omega + k_\omega V$$

$$\frac{\omega(s)}{V(s)} = \frac{k_\omega}{\tau s + 1}$$

Thrust → Vertical Position

$$m\ddot{z} = T - mg$$

2nd Order

$$T_0 = mg$$

$$T(t) = T_0 + \Delta T = mg + \Delta T(t)$$

$$m\Delta\ddot{z} = (mg + \Delta T) - mg$$

$$m\Delta\ddot{z} = \Delta T$$

$$ms^2 \Delta Z(s) = \Delta T(s)$$

$$\frac{Z(s)}{T(s)} = \frac{1}{ms^2}$$

This model is a simplified representation of a quadrotor's behavior, focusing only on the vertical motion and approximating each motor as a first-order system. In reality, most drone's full dynamics are far more complex and use sensors and stabilization. The vehicle has six degrees of freedom, not just one. We are ignoring aerodynamic drag, gyroscopic effects, and other disturbances. A more complete model would include the full 6-DOF equations, the gyroscopic terms, and the sensor feedback loops required for stabilization. The simplified vertical model is useful for basic analysis. To really design a quadrotor control system the model must capture the rotational and translational dynamics as well as disturbances and stabilization.

Motor Voltage to Altitude

(Shayla Reid)

Transfer Function

Governing equation for motor: $\omega' + a\omega = Ku - T_d$ where u is the motor voltage, ω is the propeller's angular speed, and K and a come from the step response. This gives us a transfer function of $G_m(s) = \frac{\Omega(s)}{U(s)} = \frac{K}{s+a}$

The thrust from all four propellers is proportional to the rotor speed, so $T(t) = k_T\omega(t)$

As seen above, the vertical displacement could be described by $mz''(t) = T(t) - mg$

$m\Delta z''(t) = (T_0 + \Delta T(t)) - mg \quad \leftarrow$ Relates change in altitude with a change in thrust

$m\Delta z''(t) = mg + \Delta T(t) - mg \quad \leftarrow \quad T_0 = mg$ from sum of forces

$m\Delta z''(t) = \Delta T(t)$

$m\Delta z''(t) = k_T\Delta\omega(t) \quad \leftarrow \quad T(t) = k_T\omega(t)$

$ms^2Z(s) = k_T\Omega(s) \quad \leftarrow$ Laplace Transform

$G_h(s) = \frac{Z(s)}{\Omega(s)} = \frac{k_T}{ms^2} \quad \leftarrow$ Altitude Block

$G(s) = G_h(s) \cdot G_m(s) = \frac{k_T}{ms^2} \cdot \frac{K}{s+a}$

$\boxed{G(s) = \frac{K_{alt}}{s^2(s+a)}} \quad \leftarrow \quad K_{alt} = \frac{k_T K}{m}$

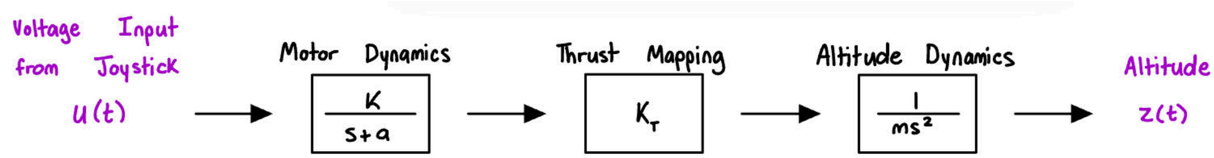
This transfer function relates the propellers' motor voltage to the altitude of the quadcopter. It shows that the motor speed does not change instantly when the motor voltage changes, which is consistent with what actually happened. When we moved the joystick all the way up to trigger the maximum velocity, the propellers had to ramp up to the maximum velocity. This transfer function was partially derived from the equations in group work 12F, since it relates input voltage to displacement.

The variable K_{alt} incorporates how strongly the voltage drives the motor (K), how the motor speed affects thrust (k_T), and the quadcopter's mass. The larger the K_{alt}, the more responsive the quadcopter is.

The $\frac{1}{s+a}$ term shows that the system behaves like a first order system with time constant $\frac{1}{a}$ and that the motor speed does not respond instantaneously to a change in voltage. The rotor likely takes time to reach the desired RPM because of inertia and friction.

The $\frac{1}{s^2}$ term comes from Newton's second law, $mz''(t) = T(t) - mg$. Since position is the second integral of acceleration, this explains why the altitude of the quadcopter responds slowly to changes in the input voltage. Since we are modeling the system as an open loop, this could bring up some problems since small mistakes can grow over time. The system is slow and difficult to control without feedback. Without internal feedback, the drone can't hold a specific altitude on its own. The user performs the stabilization, not the drone.

Block Diagram

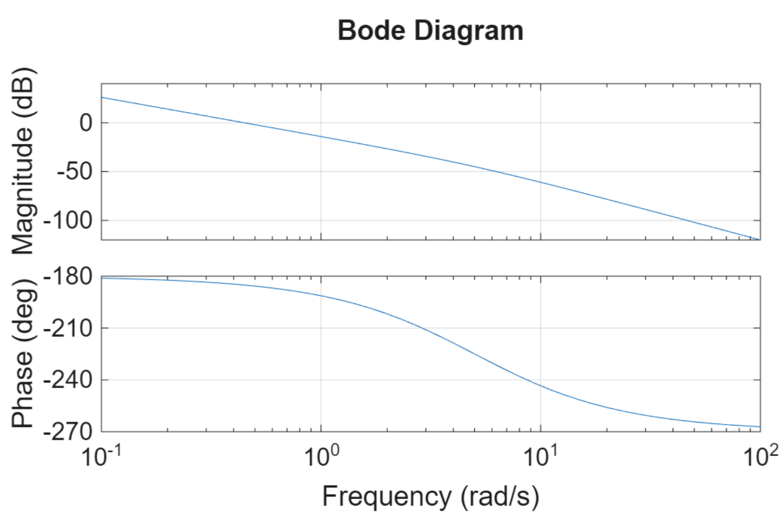


This is the block diagram of the open-loop dynamics on the quadcopter that relates input voltage to an output altitude. The joystick generates a voltage command u(t), which goes through the motor's first-order dynamics $\frac{K}{s+a}$. The rotor speed is converted to thrust via the gain k_T. The thrust drives the altitude via $\frac{1}{ms^2}$. The system does not have any sensors or feedback, so the altitude depends only on the user input and external disturbances.

System Characteristics

The quadcopter's system response is characterized by a slow rise time, overshoot, and a long settling time. All of these were observed during our experiments. When we tried to apply a sudden increase in throttle, the rotor took time to accelerate. Resulting in quadcopter having a gradual increase in altitude instead of an instantaneous rise.

Bode Plots



In the magnitude plot, the response starts at a positive gain at low frequencies then drops with slope. After the frequency reaches a pole, the slope becomes closer to -60 dB/decade. The phase is close to $-180°$. After a pole, it becomes $-270°$ which is consistent with the delayed response we noticed during flight.

Comparison of Open-Loop vs. Closed-Loop System

The quadcopter is an open loop system: the motors receive a voltage input from a joystick and it reacts based on the physics of the motors, propellers, and external forces. There are no sensors inside to measure altitude, speed, or orientation. If the same system was designed to be closed-loop, the behavior would be different. A closed-loop quadcopter would have internal sensors, like an accelerometer or gyroscope, that measure how the quadcopter moves during flight. The manual claimed the quadcopter included a gyroscope, but after dissecting and inspecting the quadcopter, we realized there was no gyroscope. Without sensors, the quadcopter can't correct errors or reject disturbances on its own.

If the system were closed-loop, the block diagram would include sensors that feed into a controller and a controller that would continually adjust the motor thrust to maintain a command altitude or speed. This would reduce steady state error, improve disturbance rejection, and decrease the response time. The quadcopter would be able to maintain a specified height rather than relying on the user.

Electromechanical Transduction Analysis

(Robert Brown)

Note: This is an in-depth analysis of this part of the system, it is simplified in other sections for analysis purposes.

A main component of the quadcopter system is the brushed dc motors that actuate the propellers to move the plane around. These take input voltage from the motors and turn it into mechanical movement, hence the electromechanical transduction name. Another important aspect of this system is that Hadas Ritz has a bunch of videos where she analyzes a system very similar to this. The first part of this analysis is heavily based on those videos. I believe it's important to acknowledge this, not only to avoid plagiarism but also to emphasize that most of the system dynamics knowledge applied is the understanding of how her model applies (and where it falls short).

This system is characterized by the two equations shown below. One is for the circuit in the motor, which includes damping and inertia via a resistor and inductor. The second is for the physical mechanics of the system. These are linked due to motor torque and back emf.

$$V_a - i_a R_a - \frac{di_a}{dt} L_a - k_b \omega = 0$$

$$I_m \omega' - k_T i_a + T_{dist} + \omega c = 0$$

Equation 1.1 (Courtesy of Hadas Ritz video)

The main analysis I did here was acknowledging how this model's assumptions apply to our system. The biggest assumption here is the ωc term in the second equation. This represents a mechanical torque due to air resistance. There are a couple big assumptions here, the biggest of all is linear air drag. This really only applies to slow moving laminar flow, but we need to assume this in order to keep our differential equations linear. There are ways to analyze non-linear differential equations, which could be an interesting thing to look into, but I decided not to dive into that.

The above equations can be used to create a State Space model (see appendix). I didn't think that analyzing the state space model would be too helpful, though. Instead I used the transfer function between angular velocity and input voltage:

$$\frac{\omega(s)}{V_a(s)} = \frac{k_t}{I_m L_a s^2 + (R_a I_a + L_a c)s + R_a c + k_t k_b}$$

Equation 1.2 (also from video)

My main interest with this transfer function was plugging in estimates for all the parameters and seeing if our measurements aligned with the model. It's pretty hard to find data on tiny brushed dc hobby motors, so I found a whole spec sheet of a large brushed motor from Moog. This might not be too applicable, as there are a lot of electrical terms like "back emf" that I don't understand fully. I also used pretty coarse estimates for moment of inertia and drag constant:

$$b = 6\pi\eta r$$

Equation 1.3 Linear Drag Coefficient for Sphere in Laminar Flow

The links to the Moog DC Motor spec sheet used for constants and inertia estimate can be found in the appendix. Note that this drag coefficient is not relevant at all to this system, but I just wanted a way to determine an order of magnitude.

I then used all the parameters I found to plug in numbers to the transfer function above. I found that some terms heavily dominated the system, and where two terms were added of more than 2 orders of magnitude difference, I dropped the smaller term.

$$\frac{\omega(s)}{V_a(s)} = \frac{2.4E-2}{(8.75E-11)s^2+(1.5E-7)s+4.56E-4}$$

Equation 1.4 Transfer function with numerical values

Looking at this transfer function, we can see in the denominator that the coefficients on the s terms are 3 or more orders of magnitude smaller than the constant term. This means that the system has largely just a constant gain Bode plot for 2 reasons:

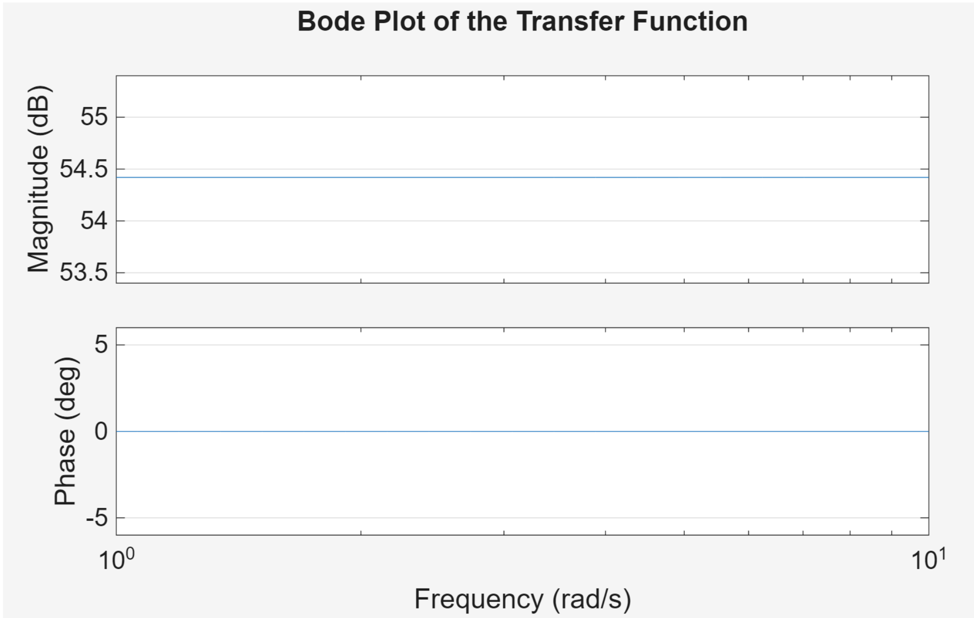
1) The drone input voltage will almost always be constant with s = 0

2) Even if there was a frequency input, the terms with the non-zero s are dominated heavily by the constant gain.

For these two reasons, we can estimate the transfer function from input voltage (Volts) to angular velocity (rad/s) as a constant gain:

$$\frac{\omega(s)}{V_a(s)} = 526$$

Since my original goal was to make a Bode plot, this was a bit disappointing, but it does make intuitive sense. There really shouldn't be any large poles or shape to the bode plot between voltage and angular velocity.



Plot 1.1 Not very interesting but it makes physical sense.

So, does this align with data we took? We can verify it with the transfer function. Note that our data collection with the high speed camera indicated that the propellers were spinning at 628 rpm. This was a great practice in data collection, but most drone quadcopter operate around 8000-9000 rpm so I'm going to just estimate 8000 rpm.

$$\omega(s) \; = 7000\, rpm \; = 837\, rad/s \; = \; 526 \, * \; V(s) \; = \; 526 \; * \; 6.35 \; = 3340$$

Equation 1.5: Validation

We are less than an order of magnitude off, which I think is actually very successful. Almost all parts of this analysis were huge estimates, from looking at a much bigger motor to using linear air drag.

Appendix

Link to estimates used in electromechanical transduction:

https://www.moog.com/literature/MCG/moc23series.pdf

https://engineering.stackexchange.com/questions/49367/how-to-calculate-the-moment-of-inertia-of-a-propeller