MAE 3260

Term: Fall 2025

Analysis of Attitude Control Systems

Team: Angular Sass

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MAE 3260 Final Group Work: Exploring a System of Interest

Report

Title: Angular Sass

Topic of Interest: Attitude Control System

Abstract:

This project focuses on developing a foundational attitude-control model for a spacecraft that needs precision pointing. These missions exist because we need to accurately orient the satellite toward deep-space targets (like stars or planets), or toward Earth for imaging or weather observation. The objective of this work is to analyze and model the satellite's rotational dynamics, derive performance requirements for pointing accuracy and stability, and characterize the major disturbances that affect its performance such as solar pressure and drag.

Using a simplified single-axis representation of the spacecraft and its reaction wheel actuator, we will derive the transfer function and the block diagram to determine its performance metrics such as settling time and steady-state error. This project also investigates environmental disturbances such as solar radiation pressure and drag. These disturbances are taken into account to observe their effect on reaching our goal.

Students/Roles:

Student	**Task/Role**	**Portfolio**
Manuely	For the proposal, Manuely will write the abstract. For the report, Manuely will work on deriving the simplified single-axis rotational dynamics model and formulate the governing differential equations. This will focus on ODE modeling and linearization.	Will submit on Canvas Quiz by Friday
Maddie	For the proposal, Maddie will write the assigned roles. For the report, Maddie will work on designing the feedback control diagram and analyze performance metrics. This will focus on transfer functions and closed-loop systems.	Will submit on Canvas Quiz by Friday
Yussef	For the proposal, Yussef will write the interdependencies. For the report, Yussef will work on modeling the environmental disturbances, such as solar radiation pressure and drag, and integrate them into the system model. This will focus on disturbances for closed-loop systems.	Will submit on Canvas Quiz by Friday
Dylan	For the proposal, Dylan will write the list of concepts/skills. For the report, Dylan will work on implementing the controller.	Will submit on Canvas Quiz by Friday

List of MAE 3260 concepts or skills used in this group work:

- Models:
    - ODEs
    - TFs
    - Block diagrams

- Open-loop system:
    - Performance metrics
        - Steady state error, settling time, etc.

- Active control:
    - Feedback control law
    - Disturbance rejection

Background:

An attitude control system (ACS) controls a satellite's orientation in space for all three axes. It is critical for all satellite systems. An ACS can aid in aiming a camera to the correct location, angling antennas, and placing sensors in the correct orientation. Attitude control systems have two subsystems: sensors and actuators. The sensors detect the orientation of the system, and the actuators physically make any adjustments that the sensors find necessary. Examples of sensors the satellite has are sun sensors, star trackers, magnetometers, and gyroscopes. All of these sensors gather information on the environment around the satellite to determine whether adjustments are needed. Examples of actuators include reaction wheels, control moment gyroscopes, magnetorquers, and small thrusters. Many or one actuator will work at a time, depending on the task needed to reorient the satellite [1].

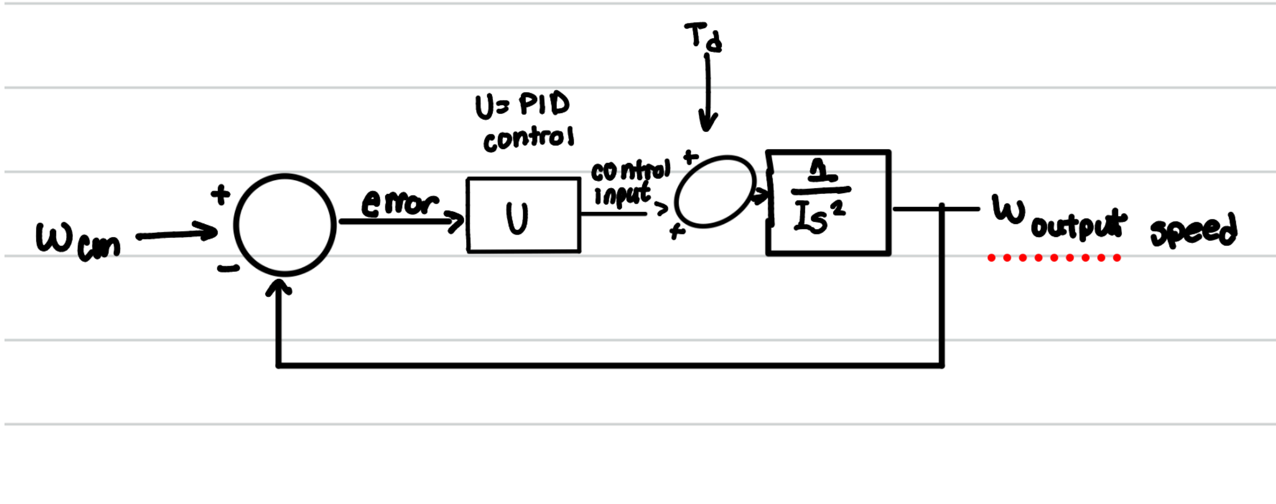
The attitude control system is a closed-loop feedback control system that senses and collects data in real time, computes whether this new data suffices for the orientation goals the satellite must meet, and finally actuates to reach the target position. Attitude control systems are important to ensure satellites are working in their optimal position for the widest possible range [1]. The satellite uses a PID control law for the system as it allows for detumbling, frame centering, and inertial pointing, all with a zero steady state approach. Additionally, the integral term leads to a smaller error [2].

Mathematical Model:

Our preliminary mathematical model is represented by the ordinary differential equation

$$I\dot{\omega} = T\ ,$$

where I is the moment of inertia, and $\dot{\omega}$ is the angular acceleration (analogous to $\ddot{\theta}$), and T is the wheel's movement, represented by an input torque into the system.



Disturbances:

Solar Radiation Disturbance

Solar radiation is a significant disturbance that affects the dynamics of satellites in space. Photons from the sun exert momentum on satellite surfaces, creating a disturbance torque that affects satellite attitude [3]. Despite its small magnitude, it is essential to account for solar radiation pressure since the disturbance it creates in space is one of the most relevant, following gravitational forces.

There are many different models for solar radiation disturbance, but a common theme between them is that they are almost always a function of distance from the sun. In our model, we will be representing the radiation pressure with NASA's estimation for radiation pressure given by the equation below [4]:

$$P_{srp} = \frac{P_0}{c}\left(\frac{R_0}{R}\right)^2 = 4.57 \times 10^{-6}\left(\frac{R_0}{R}\right)^2 \ [\mathrm{N/m^2}],$$

where c = 299,792,458 m/s is the speed of light, R_0= 149.6 million km is the sun-Earth mean distance, and R is the satellite's distance from the sun. The force that this pressure exerts on the satellite, and therefore the torque, is determined by the attitude of the satellite relative to the sun. In our 1D model, where the satellite is represented by a single, flat rectangular plate, the force due to SRP is determined by the angle between a vector pointing from the center of the satellite to the sun and a vector normal to the satellite surface. The total force caused by the SRP is the sum of 3 different forces caused by photons absorbed F_a, specular reflection F_s, and diffusive F_d reflection. According to NASA, these forces can be given by the equations below [4]:

$$\mathbf{F}_a = P_{srp} A \langle \mathbf{n}, \mathbf{r_s} \rangle \mathbf{r_s}, \qquad \mathbf{F}_s = 2 P_{srp} A \langle \mathbf{n}, \mathbf{r_s} \rangle^2 \mathbf{n}, \qquad \mathbf{F}_d = P_{srp} A \langle \mathbf{n}, \mathbf{r_s} \rangle (\mathbf{r_s} + \frac{2}{3}\mathbf{n}),$$

where A is the surface area of the satellite, n is the normal vector, and r_s is the vector between the satellite and the sun. Once these three forces are calculated, we add them up to get the total force. This net force creates a torque about the center of mass of the satellite that can represented by the equation:

$$\tau_{SRP} = (r_{CM} - r_F)F_{SRP}$$

where r_{CM} is the location of the center of mass, r_F is the average location of the net force, and F_{SRP} is the total sum of the three SRP forces.

Drag:

Drag is another one of the most significant and persistent disturbances affecting a satellite's attitude and orbital motion, especially for those in low earth orbit (LEO) [9]. Drag forces effectively act on the satellite's center of pressure, and not necessarily the center of mass. When these two centers are offset from each other, drag forces generate a disturbance torque that may cause the satellite to turn. The classic drag force equation is given by [7]:

$$F_{drag} = \frac{1}{2}\rho C_D A v^2$$

Where ρ is the atmospheric density, C_D is the drag coefficient, A is the cross-sectional area perpendicular to the fluid flow, and v is the speed of the satellite. Atmospheric density varies exponentially with altitude. Solar activity and solar storms can also affect density. A commonly used model is the US Standard Atmosphere 1976. As for the drag coefficient, it can vary with different satellite materials, but a value in the range of 2-2.2 is common for most satellites [6]. For attitude disturbances, aerodynamic torques resulting from the drag force must be taken into account. This can be modeled by discretizing the satellite geometry into a set of flat plates and computing the product of the drag force and the distance between the center of mass and center of pressure. In our model, the satellite is modeled as a singular flat plate, thus leading to the following equation:

$$\tau_{drag} = (r_{CM} - r_{CP})F_{drag}$$

Where r_{cm} is the location of the center of mass, r_{CP} is the location where the net drag force acts, and F_{drag} is the drag force.

Final Mathematical Model:

The final mathematical model for the satellite is represented by:

$$I\dot{\omega} = T + \tau_{drag} + \tau_{SRP},$$

where I is the moment of inertia of the system, $\dot{\omega}$ is the angular acceleration, T is the input torque into the system, τ_{drag} is the disturbance torque generated by drag forces, and τ_{SRP} is the disturbance torque generated by solar radiation pressure.

Pointing Accuracy:

Pointing accuracy is a performance metric used to know how accurate a satellite has to be in order to receive or read the information. For example if a satellite wants to send a signal back to Earth it would have to point its antenna at Earth's antenna tower. Therefore, pointing accuracy is how many degrees or arcseconds of error can you tolerate and still achieve the mission goals. This is due to the fact that satellites have disturbances that cause it to not perfectly be in our desired angle. As explained above by the disturbance metrics.
Pointing accuracy is determined by constants such as sensor precision, actuator performance, and control algorithm. [10] This is why we cannot have one fixed number, instead this is calculated. The key components to calculate the pointing accuracy are:

- **The Sensors:** These provide an input of what the current orientation is and the data.
- **Actuators:** These are our reaction wheels, this is what rotates the satellite.
- **Control Algorithm:** This processes the data.

Pointing accuracy is typically calculated using the Root Mean Square (RMS) error between the desired and actual orientations over a given period.

$$\theta = 2arc\cos(|\, Q_d \,.\, Q_a|)|$$

This is the equation for pointing accuracy error. Q_d is our desired orientation and Q_a is the orientation we are currently at. [10]
Pointing accuracy is usually the Root Mean Square Error of this error over multiple measurements:

$$\text{Pointing Accuracy (RMS)} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\theta_i^2}$$

The maximum allowable error depends highly on the mission. However for simple technology the maximum is a 0.5°-1° error.

PID Controller Design:

The satellite rotational dynamics are:

$$I\dot{\omega} = T + \tau_{drag} + \tau_{SRP}$$

Which is equivalent to the following:

$$I\ddot{\theta} = T + \tau_{drag} + \tau_{SRP}$$

Where $\ddot{\theta}$ is the angular acceleration.
For control design, we consider the system without disturbances:

$$I\ddot{\theta} = T_{control}$$

Taking the Laplace transform and rearranging:

$$\frac{\theta(s)}{T_{control}(s)} = \frac{1}{Is^2}$$

With PID control: $T_{control}(s) = (K_p + \frac{K_i}{s} + K_d \cdot s) \cdot E(s)$ where E(s) is the Laplace transform of the error signal e(t), such that $e(t) = \theta_r(t) - \theta(t)$ where θ_r is the reference/desired angular position and θ is the actual angular position.

The closed loop transfer function is then: $\frac{\theta_d(s)}{\theta(s)} = \frac{K_d \cdot s^2 + K_p \cdot s + K_i}{I \cdot s^3 + K_d \cdot s^2 + K_p \cdot s + K_i}$. The characteristic equation is $I \cdot s^3 + K_d \cdot s^2 + K_p \cdot s + K_i = 0$. Choosing two dominant complex poles and one real pole $s = -a\omega_n$, the characteristic equation becomes $(s + a\omega_n)(s^2 + 2\zeta\omega_n + \omega_n^2) = 0$.
Expanding this and matching coefficients with our characteristic equation, we obtain:

$$K_d = I\omega_n(a + 2\zeta)\ ;\ K_p = I\omega_n^2(2a\zeta + 1)\ ;\ K_i = Ia\omega_n^3$$

The parameters a, ζ, and ω_n can be chosen depending on design requirements, namely max input torque, overshoot, and settling time. Larger a makes integral action faster but can cause overshoot. A value between 3 and 5 would be appropriate for this application. Using the formulas $M_0 = 100 \cdot exp(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}})$ and $t_s = \frac{4}{\zeta\omega_n}$, we can determine the appropriate values for the natural frequency and damping ratio of the closed loop system to meet design requirements.

The maximum initial torque of the reaction also poses a limit on the parameters, the value of which can be determined from the initial value theorem. For the purposes of our satellite's design, a smaller settling time is preferred, with less than 10% overshoot to avoid excessive oscillations during reorientation. However, the achievable response speed is constrained by the reaction wheel torque limits, which bound the maximum control authority available for maneuvering. These requirements must be balanced with the need for disturbance rejection against environmental torques like solar radiation pressure and aerodynamic drag. The selected PID gains, derived through this pole-placement method, will therefore represent a compromise between performance specifications and the physical limitations of the satellite's actuator (a reaction wheel).

Conclusion:

This project successfully developed a comprehensive model for a single-axis satellite attitude control system, integrating rotational dynamics, environmental disturbances, and PID control design. The mathematical foundation was established through a second-order ordinary differential equation representing the satellite's rotational motion, incorporating disturbance torques from solar radiation pressure and atmospheric drag. A pole-placement methodology was employed to derive PID controller gains that satisfy performance requirements for settling time and overshoot while respecting practical actuator torque limits. This work provides a foundational framework for satellite attitude control system design, balancing theory with practical implementation considerations.

Reference Page:

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