

MAE 3260 Final Group Work: Exploring a System of Interest
Report

Title: Drilling Down Into the Dynamics: How Feedback Keeps the Bit from Having a Fit

Topic of Interest: Dissection of a corded power drill

Abstract: We decided to dissect and model a drill rather than to analyze a purely conceptual system. A physical dissection allows us to directly connect course concepts to a tangible device that a lot of us use in our work on project teams. We chose the drill specifically as it connects electrical and mechanical subsystems together, like the DC motor, gear train, clutch, etc. Within the drill system, we plan to develop an ODE model for the motor and gear train. We can then use this to derive the governing equation, then obtain the transfer functions relating a trigger input to motor speed and output torque. Additionally, we can analyze the transient response through a step change in input and/or load, and investigate the open loop response via a simple experimental set-up under no load.

Students/Roles

Student	Task/Role	Portfolio
Milo	I will analyze the open-loop behavior of the drill by estimating model parameters from experimental data and generate step and Bode plots. The skills I will focus on are system identification, steady state analysis, and frequency response interpretation. My result will be a characterized open-loop system with the parameter estimation and a description of the performance limits.	Submitted on canvas survey
Montse	I will focus on modeling and system representation, deriving a mathematical model of the drill via ODEs, the transfer function, and the state space form. I will also map each physical component to a mathematical term. The expected result is a validated model that can be used for simulation and control design.	Submitted on canvas survey
Garrett and Steven	We will take the Laplace transforms of Montse's ODEs to find both the open and closed loop transfer functions and test how our models work. We will also look into how implementing a feedback control law (P/PI/PID) might benefit the system, especially in terms of disturbance rejection.	Submitted on canvas survey

List of MAE 3260 concepts or skills used in this group work:

- Models: ODEs, TFs, State space, Transfer functions, Block diagrams, Bode plots
- Open-loop system: parameter estimation, frequency and step response, steady state behavior
- Active control: Closed-loop, Feedback control law, Command following, Disturbance rejection

Dissection

We began this exploration by dissecting the drill, with the goal of closely examining the physical components to map them to the parameters that may appear in the ODE based on the DC motor models we have looked at in class and in Lab 1 [1]. During this process, we removed half of the drill casing and pressed the trigger switch to observe how each component moved. To help us name the components and learn more about them, we followed a labeled image that looked similar to our drill [2]. Once we triggered the drill, it was



easy to identify the components that rotated, including the windings, rotor, gear train, and chuck. We also observed what the wires from the trigger switch connected to in order to understand how the electrical and mechanical components interacted with each other. After this first inspection, we took apart the individual parts to examine them more closely. We observed the brushes, windings, planetary gear systems, gear grease, and chuck. As we advanced through each component, we filled out the table below to map each component to the ODE variables we learned about in the semester, identify the type, and in what ODE it might show up. This set us up to later model the system as two coupled ODEs.

[Link to folder of dissection images and videos](#)

Variable	Name	Variable type	Relation to physical component	Description	ODE and sign
$v(t)$	Applied voltage	Input	Trigger switch (controlled by user), battery, motor	Electrical input that drive the system	Electrical (+)
L	Inductance	Physical property	Motor casing and windings	Components resist fast changes in current	Electrical (-)
R	Resistance	Physical property	Windings, motor brushes	Components cause voltage drop	Electrical (-)
$i(t)$	Current	Internal	Trigger switch, windings, magnets	Generated when voltage is applied and produces torque	Coupled
J	Rotational inertia	Physical property	Motor rotor, windings, gear train, chuck (and	The rotational components resist changes in speed	Mechanical (-)

			drill bit)		
b	Damping coefficient	Physical property	Motor brushes, gear train grease	These components resist motion, act as part of damping torque	Mechanical (-)
T_m	Torque from motor	Internal	Motor windings, magnets, motor rotor	The windings and magnetic field from the magnets produce a torque on the rotor	Mechanical
$\omega(t)$	Angular speed	Output	Motor rotor, gear train, chuck (and drill bit)	How fast the drill is spinning, related to voltage input and torques	Coupled
T_L	Torque from load	Disturbance input	Drill bit contacting material, user-applied force	**Not observed, but these would produce external opposing torques if drill was used	Mechanical (-)

Modeling

ODEs

After dissecting the drill, we proceeded to model it as a coupled electromechanical system where the electrical subsystem is composed of the trigger circuit and motor, while the mechanical subsystem includes the rotor, gear train, and chuck. To capture the behavior of the drill, we came up with two differential equations, one for each subsystem. The electrical ODE follows Kirchoff's Voltage Law while the mechanical ODE follows Newton's second law in rotation. These ODEs are coupled, showing how the input voltage v generates the motor torque T_m , and the torque yields the rotational speed ω .

$$v = L \frac{d}{dt} i + Ri + K_e \omega, \text{ where:}$$

- v is the input voltage
- $L \frac{d}{dt} i$ is the inductive effect of the motor windings (L is the inductance, $\frac{d}{dt} i$ is the current derivative)
- Ri is the voltage drop across the resistance (R is the resistance, i is the current)
- $K_e \omega$ is the back EMF generated when the motor spins and produces a voltage that opposes v (K_e is the back EMF constant)

$$T_m = J \frac{d}{dt} \omega + b\omega + T_L, \text{ where:}$$

- $T_m = K_m i$, the torque from the motor (K_m is the motor torque constant)
- $J \frac{d}{dt} \omega$ is the angular acceleration of the rotor, gears, and chuck (J is the total rotational inertia from these components, $\frac{d}{dt} \omega$ is the rotational speed derivative)
- $b\omega$ is the damping torque that results from friction (b represents the losses from friction, ω is the rotational speed)

- T_L is the torque from the load when drilling into a material ($T_L = 0$ in free space)

Rearranging in terms of the highest derivatives, the ODEs become:

$$L \frac{d}{dt} i = - R i - K_e \omega + v$$

$$J \frac{d}{dt} \omega = - b \omega + K_m i - T_L$$

The ODEs are coupled by the terms $K_m i(t)$ and $K_e \omega(t)$, accounting for how the current affects the motor torque via $K_m i(t)$ and the speed affects the electrical subsystem through $K_e \omega(t)$.

State Space

In order to track how the internal states of the drill, namely $i(t)$ and $\omega(t)$, change with time, we derived a state space model of the system. This allowed us to understand how the voltage creates current, the current creates the motor torque, and the torque accelerates the drill.

Define variables

- States: $x_1 = \omega(t)$ and $x_2 = i(t) \Rightarrow n = 2$
- Inputs: $u(t) = v(t) \Rightarrow m = 1$
- Outputs: $d(t) = T_L(t) \Rightarrow p = 1$

Isolate highest derivatives

$$\frac{d}{dt} \omega = - \frac{b}{J} \omega - \frac{1}{J} T_L + \frac{K_m}{J} i$$

$$\frac{d}{dt} i = - \frac{R}{L} i - \frac{K_e}{L} \omega + \frac{1}{L} v$$

Substitute x_1 and x_2

$$\frac{d}{dt} x_1 = - \frac{b}{J} x_1 - \frac{1}{J} d + \frac{K_m}{J} x_2$$

$$\frac{d}{dt} x_2 = - \frac{R}{L} x_2 - \frac{K_e}{L} x_1 + \frac{1}{L} u$$

Write in matrix form $\dot{x}(t) = A x(t) + B u(t)$ and $y(t) = C x(t) + D u(t)$

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K_m}{J} \\ -\frac{K_e}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u$$

$$y(t) = \omega(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Substitute variables back in

$$\dot{x}(t) = \begin{bmatrix} -\frac{b}{J} & \frac{K_m}{J} \\ -\frac{K_e}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix}$$

Data Acquisition and Analysis

To model the drill we used a high speed camera to record data of the drill ramping up to full speed. Looking back at the data we were able to use step inputs of the amount of time it took the drill to make one rotation as it ramped up to full speed. From the video, the set of times it took the drill to complete one full rotation in seconds was:

[0.0656, 0.0574, 0.0543, 0.0521, 0.0489, 0.0482, 0.0475, 0.0481, 0.0467, 0.0461, 0.0459, 0.0454, 0.0458, 0.0461, 0.0455]

It is important to note that these times were all measured by hand with a stop watch, and were originally about 8-20 seconds before being transferred into real time. From these numbers the frequency step response and bode plots above were generated. Some other experimental parameters we were able to calculate from the video are:

The rotational speed from the first rotation: 95.8 rad/s (915 RPM)

The steady state rotational speed: 138.1 rad/s (1319 RPM)

Time constant: 0.176 s

Step magnitude/gain: 42.6 rad/s (404 RPM)

Bandwidth $1/T = 5.69$ rad/s (0.91 Hz)

Performance Limits

With these values, the steady state speed ceiling of the drill of 915 RPM and the acceleration limit from the time constant of 0.176s show the max speed of the drill and how long it will take to get there. These values show how quickly the drill reaches usable speed.

The bandwidth of 0.91 Hz describes the range of input frequencies that the drill can accelerate at before the lag from the time constant begins to take effect. This value describes how quickly the drill can follow changes to input.

There is a variability of about 0.0023 s per revolution or 6.6 rad/s (65 RPM) in the steady state velocity of the drill. This means that there is either variability with the steady state velocity or with our calculations, probably some combination of both.

The fitted first-order exponential model closely follows the measured speed data, with deviations within $\pm 5\%$. This confirms that the drill's ramp-up can be approximated as a first-order system, though small discrepancies are expected due to manual timing and load fluctuations.

Overall, the drill reaches steady-state speed of 1319 RPM in under one second, with a bandwidth of 0.91 Hz and variability of ± 65 RPM. These values define the drill's performance envelope and can be used to model the drill, for controller design or for comparative analysis of similar systems.

Bode Plots

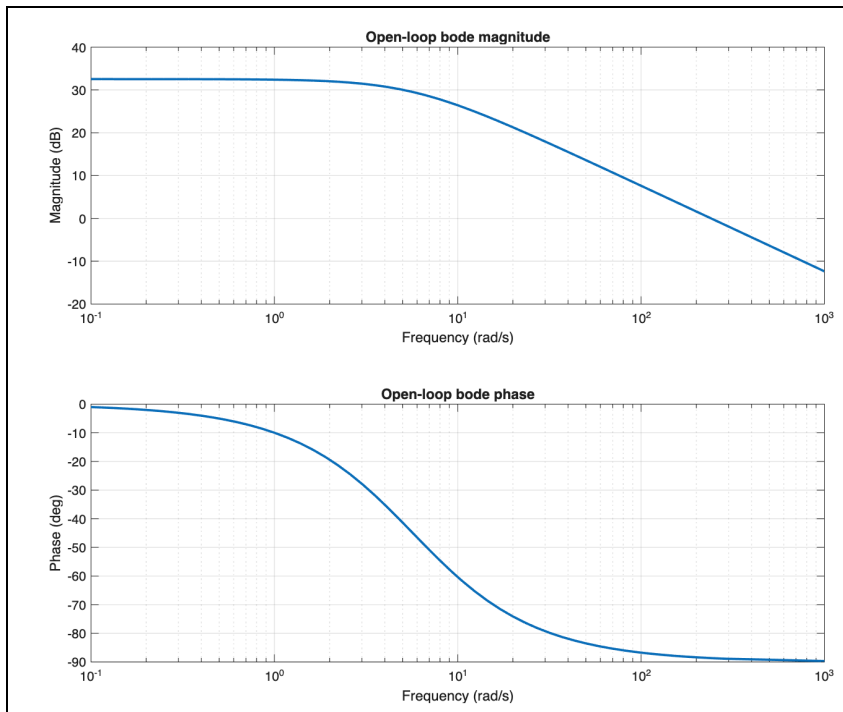


Figure 1 and 2, Bode Plot showing open loop magnitude and phase vs input frequency.

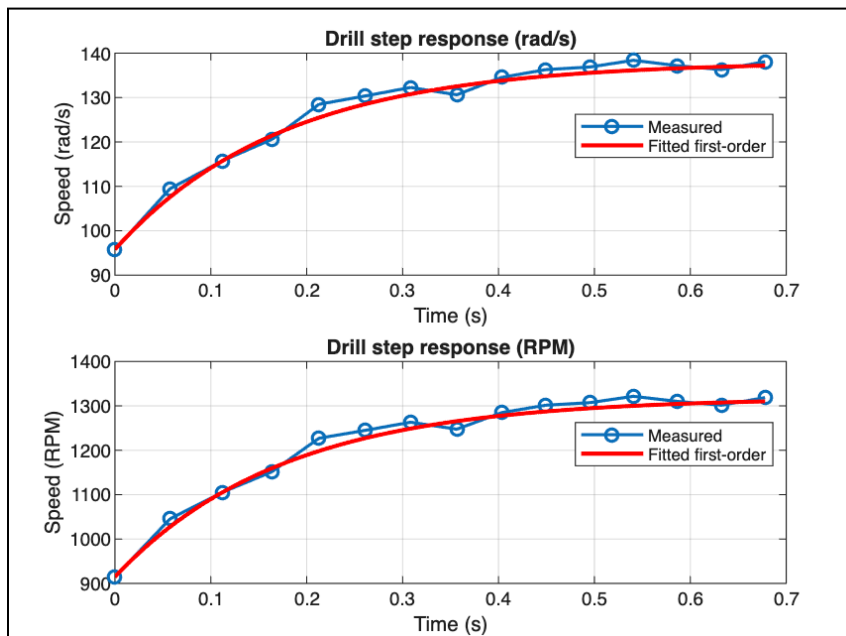


Figure 3 and 4, Frequency step response graphs in rad/s and RPM showing the speed step response, with a fitted first order curve from the calculated system parameters.

Open-Loop System

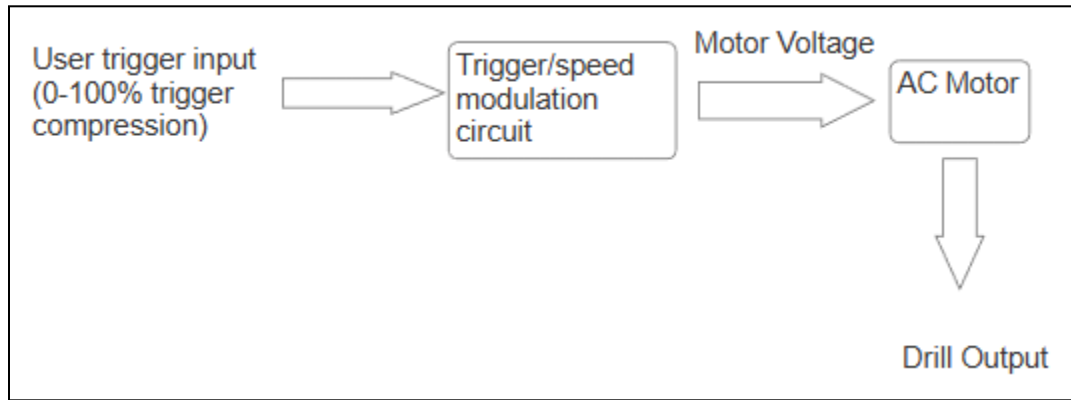


Figure 5, block diagram of open loop system.

In open loop, the drill behaves like a DC motor driven directly by the trigger voltage with no feedback controller beyond the manual control of the variable speed trigger. The trigger position directly sets the motor voltage and the drill does not internally measure any speed. We can derive a transfer function from input voltage $v(t)$ to output speed $\omega(t)$ by taking the Laplace transform of the coupled ODEs from above, where $I(s)$, $\omega(s)$, $V(s)$, $T_L(s)$ are the Laplace transforms of $i(t)$, $\omega(t)$, $v(t)$, $T_L(t)$, respectively:

$$\begin{aligned}
 L \frac{d}{dt} i &= -Ri - K_e \omega + v & LsI(s) + RI(s) + K_e \Omega(s) &= V(s) \\
 J \frac{d}{dt} \omega &= -b\omega + K_m i - T_L & Js\Omega(s) + b\Omega(s) &= K_m I(s) - T_L(s)
 \end{aligned} \Rightarrow$$

Solving these, we obtain:

$$\Omega(s) = \frac{K_m V(s) - (Ls + R)T_L(s)}{JLs^2 + (JR + Lb)s + (K_e K_m + Rb)}$$

While this is a second order system, we can approximate it as a first order system by neglecting L , as the armature inductance is typically extremely small. This equation now becomes:

$$\Omega(s) = \frac{K_m V(s) - RT_L(s)}{(JR)s + (K_e K_m + Rb)}$$

With no load disturbance torque ($T_L = 0$), the transfer function from voltage to speed is:

$$G(s) = \frac{\Omega(s)}{V(s)} = \frac{K_m}{(JR)s + (K_e K_m + Rb)}. \text{ The denominator } [(JR)s + (K_e K_m + Rb)] \text{ defines the poles of the open loop drill system.}$$

Using this open loop model, with the time constant (τ) found above and roughly estimated values for K_m , K_e , R , and b , we obtain a plot (fig. 5) comparable to figures 3 and 4 above.

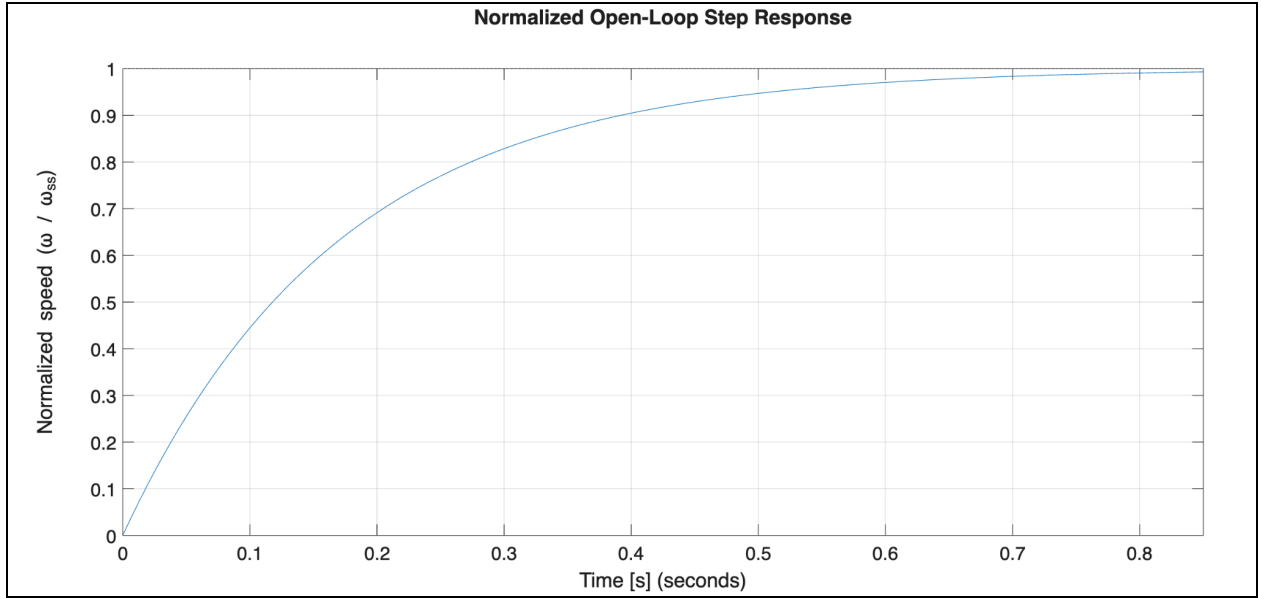


Figure 6, plot of normalized speed vs time using open loop model.

Closed-Loop System

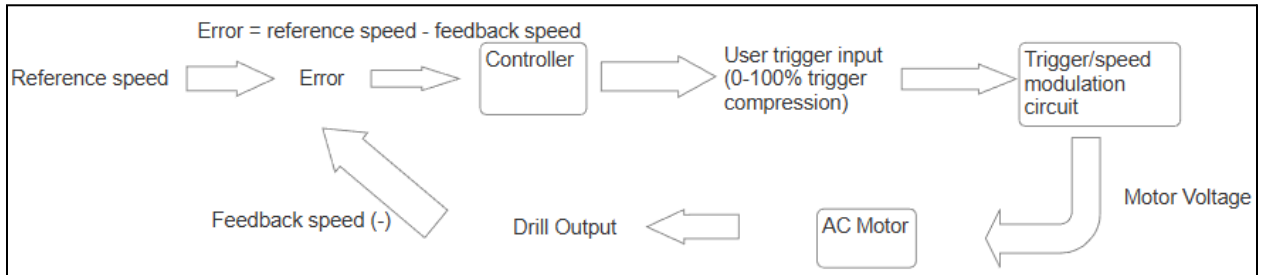


Figure 7, block diagram of closed loop system

To study feedback control, we conceptually added a speed sensor and a controller such that when the drill encounters a load disturbance $T_L(t)$, the controller will automatically adjust the applied voltage $v(t)$ to correct for the load disturbance and ensure the drill remains at the desired speed.

We let:

$r(t)$ be the step input, or speed commanded by the user

$\omega(t)$ be the measured speed

$C(s)$ be the controller transfer function

The error is then:

$$e(t) = r(t) - \omega(t)$$

The controller generates the control voltage: $V(s) = C(s)E(s) = C(s)[R(s) - \Omega(s)]$

Adding in a nonzero disturbance torque, the transfer function from load torque to speed is:

$$G_{\omega T_L} = \frac{\Omega(s)}{T_L(s)} = \frac{R}{(JR)s + (K_e K_m + Rb)}$$

From $G_{\omega v}(s) = \frac{\Omega(s)}{V(s)}$, derived above, and $G_{\omega T_L}(s)$

$$\Omega(s) = G_{\omega v}(s)C(s)[R(s) - \Omega(s)] + G_{\omega T_L}(s)T_L(s) \rightarrow \Omega(s) = \frac{C(s)G_{\omega v}(s)}{1+C(s)G_{\omega v}(s)}R(s) + \frac{G_{\omega T_L}(s)}{1+C(s)G_{\omega v}(s)}T_L(s),$$

Where $\frac{C(s)G_{\omega v}(s)}{1+C(s)G_{\omega v}(s)}$ and $\frac{G_{\omega T_L}(s)}{1+C(s)G_{\omega v}(s)}$ are the closed-loop transfer functions from user-commanded speed to drill speed ($\frac{\Omega(s)}{R(s)}$) and from disturbance torque to drill speed ($\frac{\Omega(s)}{T_L(s)}$), respectively.

If $C(s)$ is a just proportional control, the response will be sped up, and under a constant trigger input and load torque, will reduce the steady state error but not eliminate it. When the bit encounters a disturbance torque, the speed will drop and error will grow. The controller will see this error and increase the input voltage in order to pull the speed back up, however even in steady state, there will still be some error between the commanded trigger input and the drill speed. If $C(s)$ is a proportional integral control, disturbance rejection will be much better than if solely proportional control is used. Steady state error, even under load will be nearly zero, as the controller will keep winding up the integral term until the drill speed matches that of the commanded trigger input. PID control would further benefit the system, ensuring the error doesn't change too fast and thus reducing overshoot, however this feels overkill and unnecessary.

If we were to improve this drill, we would add a speed sensor and PI controller. Using the model above, with $C(s) = K_p + \frac{K_i}{s}$, we can determine the values for K_p and K_i needed to meet “better drill” design specifications.

References

- [1] M. Campbell. MAE 3260. Class Lecture, Topic: “Actuators and Sensors.” Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, NY, 2025.
- [2] K. Koehler, “Getting to Know the Parts of a Drill,” *Pro Tool Reviews*, Aug. 11, 2021.
[Online]. Available: <https://www.protoolreviews.com/getting-know-parts-of-drill/>