Portfolio

150 cm

50 cm

145 cm

pivot

linear actuator

pivot

pivot

(projected)
Path of pivot of bar + actuator:

θ $\frac{L}{2}$ θ L F_{LA}
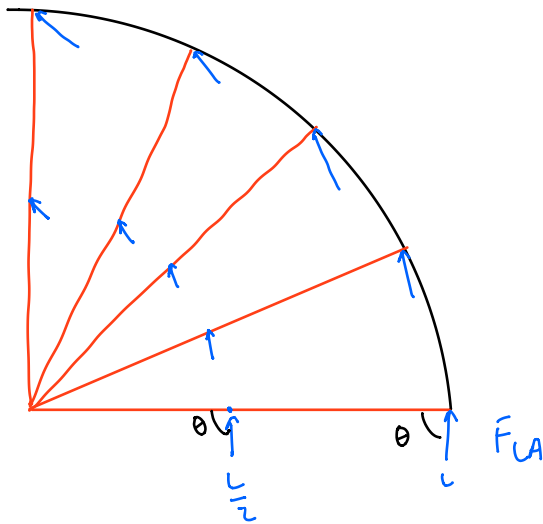
$M = rF\sin\theta$

$\sin\theta$ decreases proportionately as bar moves upward, independent of r

$\therefore M \propto r$ and F

$\therefore$ placing linear Actuator at L provides most torque, maximizes upwards force of L.A.

Selected: RSX because it has the greatest stroke length and force

Assumption: Speed, screw/nut type, and duration capacity are not considered s/c they are not specified in the question.

Length: 145cm - A little leeway (as opposed to a max 150cm length and hence max torque) to provide for size of actuator, size of pivot, size of load - essentially a little leeway.

Bending Analysis

Maximum deflection?

Due to setup with the actuator being attached to the beam by a pin, it can be concluded that the linear actuator constantly applies a "pinned joint" to the beam as the lever's angle with the ground increases, allowing us to treat the bar's bending situation as pinned-pinned.

But, as the lever's height increases, the weight of the load is still straight down, meaning the force it applies directly to the bar is no longer decreases.
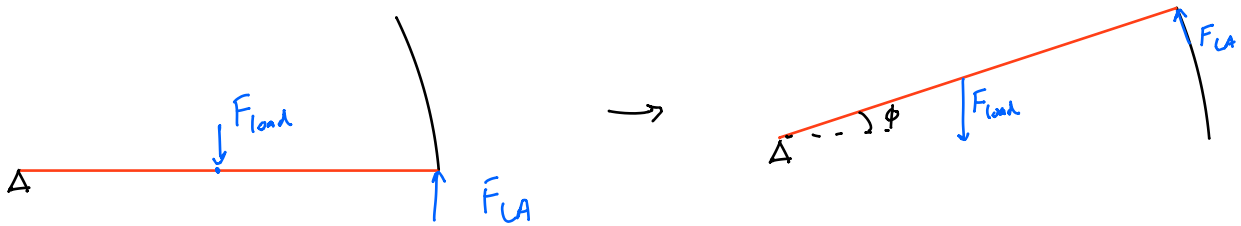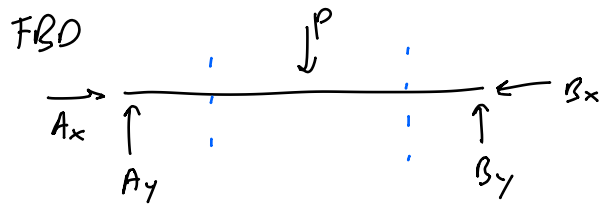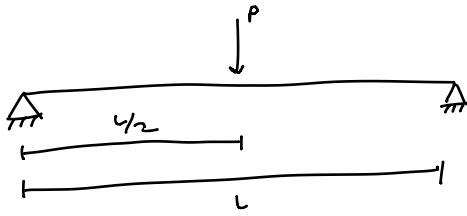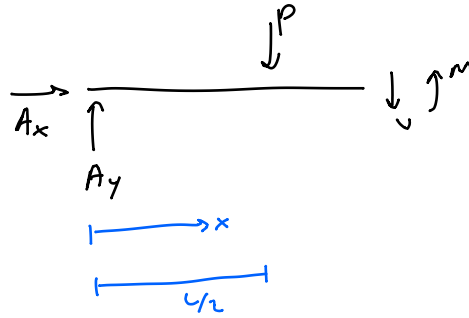

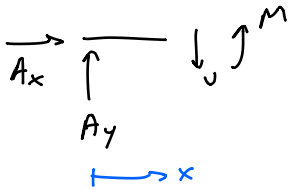
In terms of torque...

$$T_{load} = rF\cos\phi$$

when $\phi = 0$, T_{load} is at maximum

Hence, to analyze beam bending, I consider a horizontal loading scenario. When horizontal, the beam is pinned-pinned and the maximum deflection occurs at $\frac{L}{2}$. The linear actuator applies no force.

FBD

By symmetry $R_y = A_y = B_y = \frac{P}{2}$

$M(x)$:

for $0 \le x \le \frac{L}{2}$: $-R_y x + M = 0$

$$M = R_y x = \frac{P}{2} x$$

for $\frac{L}{2} \le x \le L$: $-R_y x + (x - \frac{L}{2})P + M = 0$

$$M = R_y x - Px + \frac{PL}{2}$$
$$= \frac{P}{2} x - Px + \frac{PL}{2}$$
$$= -\frac{P}{2} x + \frac{PL}{2}$$

$EI y'' = M(x)$

$$EI y' = \int M(x)\,dx = \frac{P}{4} x^2 + C_1$$

$$EI y = \int\int M(x)\,dx = \frac{P}{12} x^3 + C_1 x + \cancel{C_2}$$

$$EI y' = \int M(x)\,dx = -\frac{P}{4} x^2 + \frac{PL}{2} x + D_1$$

$$EI y = \int\int M(x)\,dx = -\frac{P}{12} x^3 + \frac{PL}{4} x^2 + D_1 x + D_2$$

BCs: $y(0) = 0$ $y(L) = 0$ $y(\frac{L}{2}^-) = y(\frac{L}{2}^+)$ $y'(\frac{L}{2}^-) = y'(\frac{L}{2}^+)$

$y(0) = 0$: $C_2 = 0$

$y(L) = 0$: $0 = -\frac{P}{12} L^3 + \frac{PL^3}{4} + L D_1 + D_2$ $\rightarrow$ $0 = \frac{PL^3}{6} + L D_1 + D_2$

$y(\frac{L}{2}^-) = y(\frac{L}{2}^+)$: $\frac{P}{12}(\frac{L}{2})^3 + C_1(\frac{L}{2}) = -\frac{P}{12}(\frac{L}{2})^3 + \frac{PL}{4}(\frac{L}{2})^2 + D_1 \frac{L}{2} + D_2$ $\rightarrow$ $-\frac{PL^3}{24} + \frac{L C_1}{2} = \frac{L D_1}{2} + D_2$

$y'(\frac{L}{2}^-) = y'(\frac{L}{2}^+)$: $\frac{P}{4}(\frac{L}{2})^2 + C_1 = -\frac{P}{4}(\frac{L}{2})^2 + \frac{PL}{2}(\frac{L}{2}) + D_1$ $\rightarrow$ $-\frac{PL^2}{8} + C_1 = D_1$

System

$C_1 = D_1 + \frac{PL^2}{8}$

$D_2 = -L D_1 - \frac{PL^3}{6}$

$\rightarrow$

$-\frac{PL^3}{24} + \frac{L}{2}(D_1 + \frac{PL^2}{8}) = \frac{L}{2} D_1 + (-L D_1 - \frac{PL^3}{6})$

$-\frac{PL^3}{24} + \frac{L}{2}\cancel{D_1} + \frac{PL^3}{16} = \frac{L}{2}\cancel{D_1} - L D_1 - \frac{PL^3}{6}$

$\frac{3}{16} PL^2 = -L D_1$

$$D_1 = -\frac{3}{16} PL^2$$

$$D_2 = -L\left(-\frac{3}{16} PL^2\right) - \frac{PL^3}{6}$$

$$= \frac{1}{48} PL^3$$

$$C_1 = -\frac{3}{16} PL^2 + \frac{PL^2}{8}$$

$$= \frac{-1}{16} PL^2$$

$$y(x) = \frac{1}{EI} \begin{cases} \frac{P}{12}x^3 - \frac{1}{16} PL^2 x & , \quad 0 \leq x \leq \frac{L}{2} \\[2mm] -\frac{P}{12}x^3 + \frac{PL}{4}x^2 - \frac{3}{16} PL^2 x + \frac{1}{48} PL^3 & , \quad \frac{L}{2} \leq x \leq X \end{cases}$$

$$y_{max} \text{ at } y\left(\frac{L}{2}\right) = \frac{1}{EI} \cdot \left[\frac{P}{12}\left(\frac{L}{2}\right)^3 - \frac{1}{16} \cdot PL^2\left(\frac{L}{2}\right) \right]$$

$$y_{max} = -\frac{PL^3}{48EI}$$

A mass efficient design would be a wide flanged I-beam where mass is distributed to the edges of the shape, increasing the I value.

Want vertical deflection $\leq 2\%$ of its length:

- length $= 145$ cm $= 1.45$ m

- $y = .02(1.45) = 0.029$ m

- My chosen RSX linear actuator can lift a maximum load of 294 kN

$$\sum M_{pivot} : -W_{load} \cdot \frac{L}{2} + L(294 \times 10^3) = 0$$

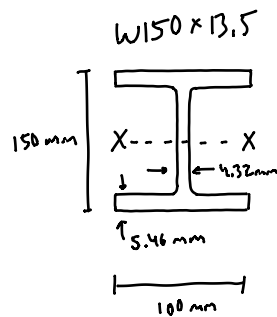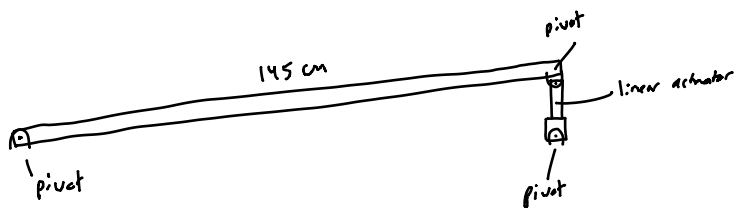$$W_{load\,max} = 588 \times 10^3 \text{ N}$$

- Choose rolled steel I-beam $\to E \approx 200$ GPa

$$y = \frac{PL^3}{48EI} \quad \to \quad I = \frac{(588 \times 10^3)(1.45)^3}{48(200 \times 10^9)(0.029)} = 6.439 \times 10^{-6} \text{ m}^4 = 6.439 \times 10^6 \text{ mm}^4$$

Considering the x-x axis of bending for the cross-section, I think that W150×13.5 (little leeway, $I = 6.83×10^6$ mm⁴) or W130×23.8 (more leeway, $I = 8.91×10^6$) would be efficient and stable choices.

Final Design: