

Performance metric:

$$\dot{Q} = \dot{m}_c C_p (T_{c,out} - T_{c,in})$$



\dot{Q}_{loss} : heat loss from the heat exchanger to the room

$\dot{w}_{cv} = 0$ because no work crosses the heat exchanger CV boundary

Mass balance:

$$\dot{m}_{h,in} = \dot{m}_{h,out} = \dot{m}_h$$

$$\dot{m}_{c,in} = \dot{m}_{c,out} = \dot{m}_c$$

Energy Balance:

$$\sum \dot{m}_{in} h_{in} + \dot{Q}_{cv} + \dot{w}_{cv} = \sum \dot{m}_{out} h_{out}$$

hot side:

$$\dot{Q}_{h \rightarrow c} + \dot{Q}_{loss} = \dot{m}_h C_p (T_{h,in} - T_{h,out})$$

cold side:

$$\dot{Q}_{h \rightarrow c} = \dot{m}_c C_p (T_{c,out} - T_{c,in})$$

$$\dot{m}_c C_p (T_{c,out} - T_{c,in}) = \dot{m}_h C_p (T_{h,out} - T_{h,in}) - \dot{Q}_{loss}$$

Entropy Balance:

$$\sum \dot{m}_{in} s_{in} + \sum \frac{\dot{Q}}{T_b} = \sum \dot{m}_{out} s_{out} + \dot{s}_{gen}$$

$$\dot{m}_h s_{h,in} + \dot{m}_c s_{c,in} + \frac{-\dot{Q}_{loss}}{T_{room}} = \dot{m}_h s_{h,out} + \dot{m}_c s_{c,out} + \dot{s}_{gen}$$

$$\dot{s}_{gen} = \dot{m}_h (s_{h,out} - s_{h,in}) + \dot{m}_c (s_{c,out} - s_{c,in}) + \frac{\dot{Q}_{loss}}{T_{room}}$$

assuming constant C_p :

$$s_{out} - s_{in} = C_p \ln \left(\frac{T_{out}}{T_{in}} \right)$$