VDP DYNAMICS & BLOCK DIAGRAM

Mathematical models of cardiac electrophysiology often rely on nonlinear oscillators to capture the periodic spike-and-recovery structure of real heartbeat signals. Among these, the modified Van der Pol (VDP) oscillator has become a standard because its nonlinear damping term naturally generates the asymmetric depolarization-repolarization waveform observed in atrial and ventricular tissue. In our project, the VDP model functions as the underlying open-loop heart dynamics, and the pacemaker behaves as an external feedback controller that senses missed or irregular beats and injects small corrective pulses.

The key idea is that the heart alone is modeled as a nonlinear second-order ODE, and the pacemaker adds an input forcing term to regulate the oscillator. Once the VDP equation is rewritten as a system of first-order equations, it becomes straightforward to construct a block diagram showing the flow of signals and a state-space representation compatible with MATLAB simulations.

We model the heart as a modified Van der Pol oscillator. The version we use follows the model used in the Yahalom & Puzanov [3] paper. It represents the pacemaker cell's voltage-like activity with a scalar variable $x(t)$:

$$x'' + a(x - v_1)(x - v_2)x' + \frac{x(x+d)(x+e)}{ed} = F(t)$$

The parameters shape different aspects of the cardiac spike:

$$a(x - v_1)(x - v_2)x'$$

produces asymmetric nonlinear damping, giving a sharp upstroke and slower recovery.

$$\frac{x(x+d)(x+e)}{ed}$$

replaces the simple linear stiffness term in the classical oscillator with a biophysically shaped restoring force, affecting contraction timing.

$F(t)$ is an external forcing term, typically modeled in heart-only simulations as:

$$F(t) = A\sin(\omega t)$$

representing background physiological simulation.

When parameters are chosen as in the literature (e.g., $a = 0.5$, $v_1 = 0.97$, $v_2 = -1$, $d = 3$, $e = 6$, $A = 2.5$, $\omega = 1.9$), the oscillator spontaneously generates chaotic or fibrillation-like rhythms, making it a suitable model of a dysfunctional heartbeat that needs pacing.

We convert the VDP ODE into state-space form. To create a dynamics block diagram or implement the system in MATLAB, the second-order differential equation is rewritten as two first-order equations:

Let

$$x_1 = x,\ x_2 = \dot{x}$$

Then our state equations are:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -a(x_1 - v_1)(x_1 - v_2)x_2 - \frac{x_1(x_1+d)(x_1+e)}{ed} + F(t) + u(t)$$

Where $F(t)$ is natural forcing and $u(t)$ is the pacemaker's electrical stimulus, which is zero unless the pacemaker fires.

The state vector becomes:

$$X(t) = \left[x_1\ x_2\right]^T$$

And the output is:

$$y(t) = x_1(t)$$

as the "measureable" electrical signal sensed by the pacemaker lead.

Because the system is strongly nonlinear, the state-space model is nonlinear rather than the linear $AX + Bu$ form. But the format above is all we need for nonlinear MATLAB solutions.

We model the single-lead pacemaker as a feedback controller. A single-lead ventricular pacemaker performs two tasks:

1. Sensing the intrinsic heartbeat signal (voltage from the ventricular lead)
2. Pacing: injecting a short stimulus pulse if a heartbeat fails to occur within the programmed escape interval T_p.

One part of the model becomes beat detection:
If

$$x_1(t) > x_{threshold}$$

and a minimum refractory time has elapsed, the pacemaker detects a natural beat.

Another part of the model becomes escape-interval logic:
If no beat is sensed for

$$t - t_{LastBeat} > T_p$$

the pacemaker fires.

The third part of the model becomes the electrical stimulus:
A pacing pulse is represented as:

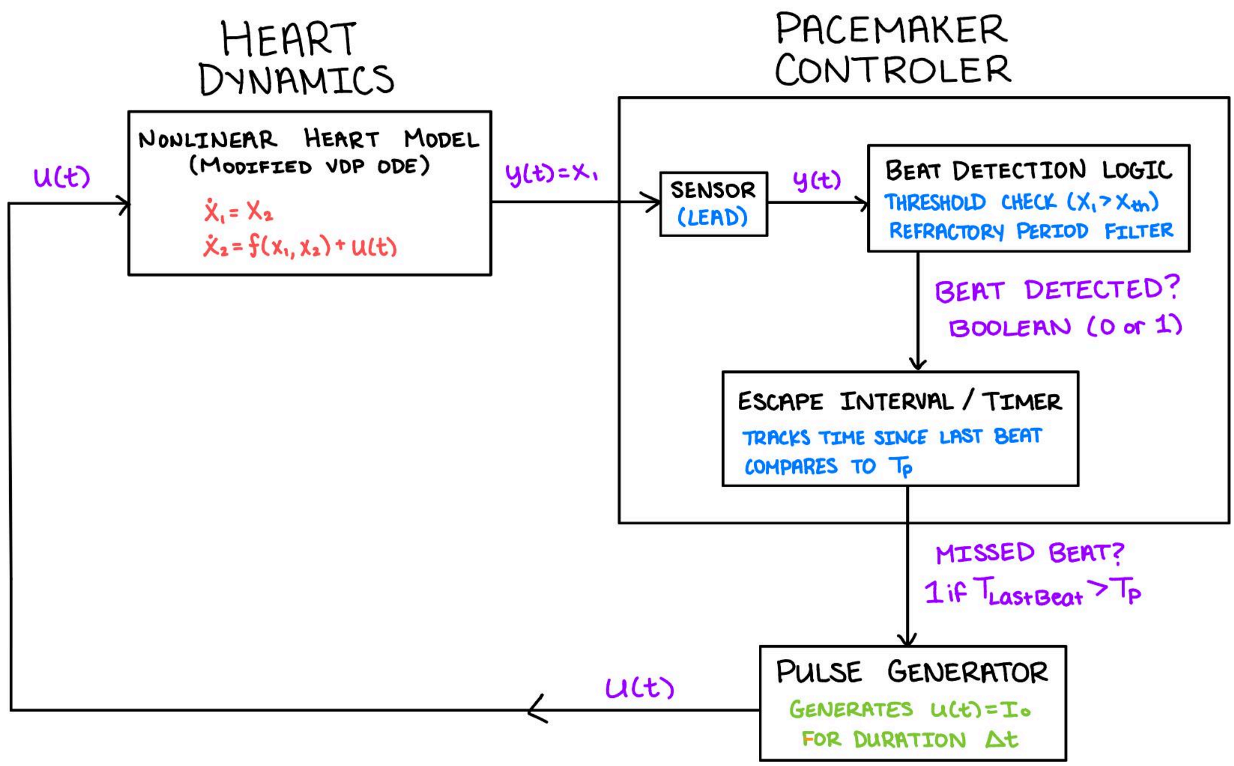
$$u(t) = I_0 \quad \text{during the pulse duration } \Delta t,$$

$$u(t) = 0 \quad \text{otherwise.}$$

This stimulus appears as an additive input in the $\dot{x}_2$ equation, matching how a real pacemaker injects current into cardiac tissue.

Because the pacemaker modifies the heart's dynamics based on measured output, it is explicitly a feedback control system, allowing us to build a complete block diagram.

We construct the block diagram. A block diagram clarifies the feedback loop and the interaction between the nonlinear heart model and the pacemaker controller:



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[3] Yahalom, Asher & Puzanov, Natalia. (2023). “Feedback stabilization applied to heart rhythm dynamics with integro-differential equations method.” *Research Square.* July 4th, 2023 0.21203/rs.3.rs-2853196/v1.