Mistake:

Identified 0 as an eigenvalue and incorrect $\sin(\sim)$ in the eigenfunction.

Why these mistakes happened / How to avoid?

I plugged $\lambda = 0$ into the wrong equation so it mistakenly showed that $\lambda = 0$ was a solution. I now know which equation to plug it into. I found the eigenvalues and function for $\lambda > 0$ but didn't correctly plug in the values to the function. I now understand better how to plug the values in and shouldn't make the same mistake in the future.

Corrected Solution:

$\lambda = 0$ $\quad y'' - 2\lambda y' + (\lambda^2 + \lambda) y = 0$ $\quad r^2 = 0$ $\quad y = C_1 + C_2 t$

ICs $\quad 0 = C_1 + C_2(0) \rightarrow C_1 = 0$ $\quad 0 = 0 + C_2(\pi) \rightarrow C_2 = 0$

$\lambda = 0$ is not an eigenvalue

$\lambda > 0$: $\quad y = C_1 e^{\lambda t}\cos(\sqrt{\lambda} t) + C_2 e^{\lambda t}\sin(\sqrt{\lambda} t)$

$y(0) = 0 \rightarrow 0 = C_1$

$y(\pi) = 0 \quad 0 = C_2 e^{\lambda \pi}\sin(\sqrt{\lambda}\pi)$

($e^{\lambda\pi}$: cannot be 0; $\sin(\sqrt{\lambda}\pi)$: $\sin(\text{multiples of } \pi) = 0$)

$\sqrt{\lambda} = n \quad 1, 2, 3 \ldots$

$\lambda = n^2$

work from prelim good up to this point

Correction for plugging eigenvalues into eigenfunction:

$y = C_2 e^{\lambda t}\sin(\sqrt{\lambda} t)$ ← left π in previously and forgot to switch it back to t

$\lambda = n^2$

$y = C_2 e^{n^2 t}\sin(\sqrt{n^2} t) \rightarrow$ $\boxed{y = C_2 e^{n^2 t}\sin(nt)}$