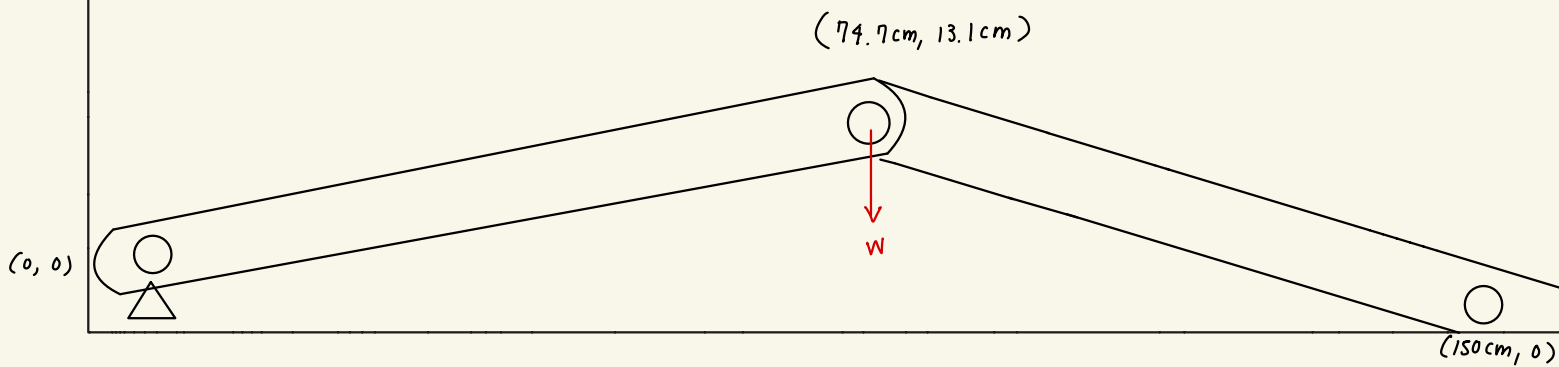


150 cm

50 cm



$$X_{\text{joint}} = \sqrt{(75 \text{ cm})^2 - (6.57 \text{ cm})^2} = 74.7 \text{ cm} \rightarrow \text{pivot pin} = (74.73, 13.14 \text{ cm})$$

$$L_{\text{beam}} = \sqrt{(74.7 \text{ cm})^2 + (13.14 \text{ cm})^2} = 75.8 \text{ cm} \rightarrow \text{Actuator length} = 76.4 \text{ cm}$$

2. a

weight lifted:

$$P = W_{\perp} + F_{\text{act}, \perp} \approx 244200 \text{ N} + 13.7 \text{ N} = 244213.7 \text{ N}$$

Bar angle:

$$\theta_{\text{bar}} = \tan^{-1} \left(\frac{13.1}{74.7} \right) = 9.9^\circ$$

$$W_{\perp} = W \cos(\theta_{\text{bar}}) = (244213.7 \text{ N}) \cos(9.9^\circ) = 242212 \text{ N}$$

$$W_{\perp} = 242212 \text{ N}$$

$$F_{\text{act}} = 8050 \text{ N}$$

Angle difference between actuator & bar is $\sim 0.1^\circ$

$$F_{\text{act}, \perp} = F_{\text{act}} \sin(0.1^\circ) = (8050 \text{ N}) \sin(0.1^\circ) = 14 \text{ N}$$

$$F_{\text{act}, \perp} \approx 13.7 \text{ N}$$

$$P = W_{\perp} + F_{\text{act}, \perp} = 242212 \text{ N} + 14 \text{ N} = 242226 \text{ N}$$

$$P = 244,214 \text{ N}$$

$$\delta_{\text{max}} = \frac{PL^3}{3EI} = \frac{(2.44 \times 10^5 \text{ N})(0.758 \text{ m})^3}{3EI} \rightarrow \frac{(2.44 \times 10^5 \text{ N})(0.4355 \text{ m}^3)}{3EI} = \frac{114205 \text{ Nm}^3}{3EI}$$

2. b

Shape: Hollow tube

Material: Aluminum

Let candidate tube be $40\text{mm} \times 20\text{mm} \times 2\text{mm}$ wall

$$h_o = 0.04\text{m} \quad h_i = h_o - 2t = 0.04\text{m} - 0.004\text{m} = 0.036\text{m}$$

$$b_o = 0.02\text{m} \quad b_i = b_o - 2t = 0.02\text{m} - 0.004\text{m} = 0.016\text{m}$$

$$t = 0.002\text{m}$$

Moment of inertia:

$$I_o = \frac{b_o h_o^3 - b_i h_i^3}{12}$$

Outer moment of inertia:

$$I_o = \frac{(0.02\text{m}) \times (0.04\text{m})^3}{12} = 1.07 \times 10^{-7} \text{m}^4$$

Inner moment of inertia:

$$I_i = \frac{(0.016\text{m}) \times (0.036\text{m})^3}{12} = 6.2208 \times 10^{-8} \text{m}^4$$

$$I = I_o - I_i = (1.07 \times 10^{-7} \text{m}^4) - (6.2208 \times 10^{-8} \text{m}^4) = 4.44 \times 10^{-8} \text{m}^4$$

$$I = 4.44 \times 10^{-8} \text{m}^4$$

From part 2a: $\delta_{\max} = \frac{114205 \text{Nm}^3}{3EI}$

for aluminum: $E = 70 \text{GPa}$

$$\hookrightarrow \delta_{\max} = \frac{114205 \text{Nm}^3}{(70 \times 10^9 \text{Pa})(4.44 \times 10^{-8} \text{m}^4)} = 36.7 \text{mm}$$

$$\delta_{\text{allow}} = 0.02L = 0.02(0.78\text{m}) = 0.0156\text{m}$$

$$\delta_{\text{allow}} \ll \delta_{\max} \longrightarrow \text{This design fails.}$$

Design 2:

1st find minimum required I

$$\delta_{\max} \leq 0.02L \longrightarrow \delta_{\max} = 0.02(0.758\text{m}) = 0.01516\text{m}$$

$$\delta_{\max} = \frac{114205 \text{Nm}^3}{3EI}$$

$$\delta_{\max} = \delta_{\text{allow}} \longrightarrow \frac{114205 \text{Nm}^3}{3EI} = 0.01516\text{m}$$

$$I = \frac{114205 \text{Nm}^3}{3E(0.01516\text{m})} \quad \text{let the material be aluminum} \longrightarrow E = 70 \text{GPa}$$

$$I = \frac{114205 \text{Nm}^3}{3(70 \times 10^9 \text{Pa})(0.01516\text{m})} = 3.587 \times 10^{-5} \text{m}^4$$

candidate tube : 250 mm x 150 mm x 10 mm

$$h_0 = 0.25 \text{ m} \quad h_i = h_0 - 2t = 0.23 \text{ m}$$

$$b_0 = 0.15 \text{ m} \quad b_i = b_0 - 2t = 0.13 \text{ m}$$

$$t = 0.01 \text{ m}$$

outer moment of inertia:

$$I_0 = \frac{0.15 \text{ m} (0.25 \text{ m})^3}{12} = 1.953 \times 10^{-4} \text{ m}^4$$

inner moment of inertia:

$$I_i = \frac{(0.13 \text{ m}) (0.23 \text{ m})^3}{12} = 1.138 \times 10^{-4} \text{ m}^4$$

$$I = I_0 - I_i = 1.953 \times 10^{-4} \text{ m}^4 - 1.138 \times 10^{-4} \text{ m}^4 = 0.815 \times 10^{-4} \text{ m}^4$$

$$\delta_{\max} = \frac{114205 \text{ Nm}^3}{3EI} = \frac{114205 \text{ Nm}^3}{3(200 \times 10^9 \text{ Pa})(0.815 \times 10^{-4} \text{ m}^4)} = 0.003 \text{ m}$$

$$\delta_{\text{allow}} = 2L = 2(0.75 \text{ m}) = 0.01514 \text{ m}$$

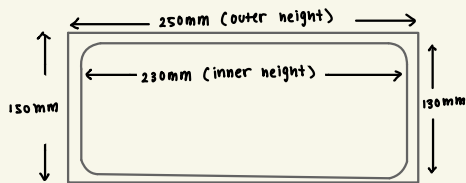
$$\delta_{\max} < \delta_{\text{allow}} \longrightarrow \text{this beam satisfies deflection}$$

& is mass efficient for this stiffness level

Final Design choice : Hollow Aluminum tube w dimensions : 250 mm x 150 mm x 10 mm

2.C

Final Beam Design cross-section: (Aluminum 6061-T4)



outer width : 150 mm

wall thickness : 10 mm

Material : Aluminum 6061-T4

Young's Modulus : 70 GPa

Computed Moment of Inertia : $0.815 \times 10^{-4} \text{ m}^4$

Max deflection under load : 0.003 m