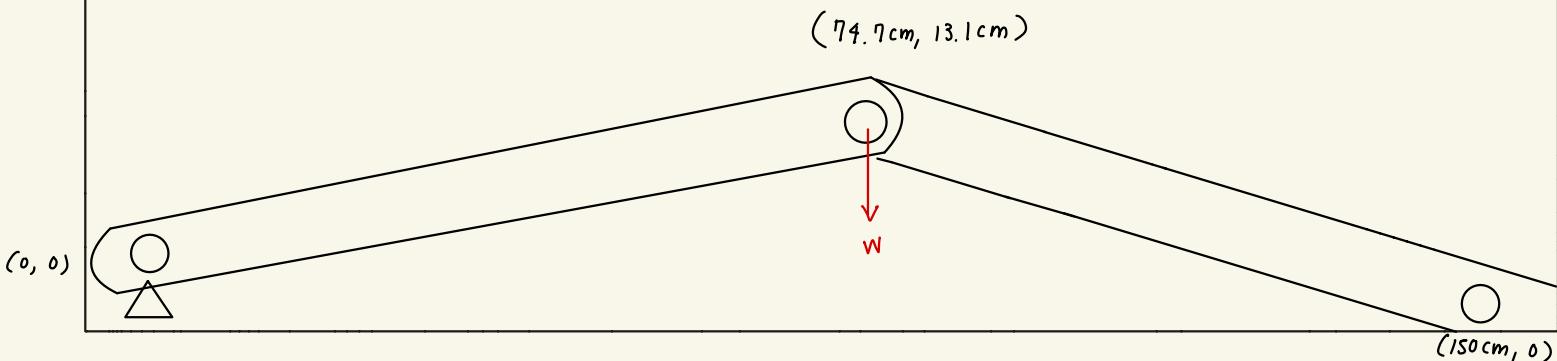


150 cm

50 cm



$$x_{\text{joint}} = \sqrt{(75 \text{ cm})^2 - (4.57 \text{ cm})^2} = 74.7 \text{ cm} \rightarrow \text{pivot pin} = (74.7, 13.14 \text{ cm})$$

$$L_{\text{beam}} = \sqrt{(74.7 \text{ cm})^2 + (13.14 \text{ cm})^2} = 75.8 \text{ cm} \rightarrow \text{Actuator length} = 76.4 \text{ cm}$$

2. a

weight lifted:

$$P = W_{\perp} + \text{Fact}_{\perp} \approx 244200 \text{ N} + 13.7 \text{ N} = 244213.7 \text{ N}$$

Bar angle:

$$\theta_{\text{bar}} = \tan^{-1} \left(\frac{13.1}{74.7} \right) = 9.9^\circ$$

$$W_{\perp} = W \cos(\theta_{\text{bar}}) = (244213.7 \text{ N}) \cos(9.9^\circ) = 242212 \text{ N} \quad W_{\perp} = 242212 \text{ N}$$

$$\text{Fact} = 8050 \text{ N}$$

Angle difference btwn actuator & bar is $\approx 0.1^\circ$

$$\text{Fact}_{\perp} = \text{Fact} \sin(0.1^\circ) = (8050 \text{ N}) \sin(0.1) = 14 \text{ N}$$

$$\text{Fact}_{\perp} \approx 13.7 \text{ N}$$

$$P = W_{\perp} + \text{Fact}_{\perp} = 242212 \text{ N} + 14 \text{ N} = 242226 \text{ N}$$

$$P = 242226 \text{ N}$$

$$\delta_{\text{max}} = \frac{PL^3}{3EI} = \frac{(2.44 \times 10^5 \text{ N})(0.758 \text{ m})^3}{3EI} \rightarrow \frac{(2.44 \times 10^5 \text{ N})(0.4355 \text{ m}^3)}{3EI} = \frac{114205 \text{ Nm}^3}{3EI}$$

2.b

shape: hollow tube

Material: aluminum

Let candidate tube be 40mm x 20mm x 2mm Wall

$$h_0 = 0.04m \quad h_i = h_0 - 2t = 0.04m - 0.004m = 0.036m$$

$$b_0 = 0.02m \quad b_i = b_0 - 2t = 0.02m - 0.004m = 0.016m$$

$$t = 0.002m$$

moment of inertia:

$$I_o = \frac{b_0 h_0^3 - b_i h_i^3}{12}$$

outer moment of inertia:

$$I_o = \frac{(0.02m) \times (0.04m)^3}{12} = 1.06 \times 10^{-7} m^4$$

inner moment of inertia:

$$I_i = \frac{(0.016m) \times (0.036m)^3}{12} = 4.2208 \times 10^{-8} m^4$$

$$I = I_o - I_i = (1.06 \times 10^{-7} m^4) - (4.2208 \times 10^{-8} m^4) = 4.444 \times 10^{-8} m^4 \quad I = 4.444 \times 10^{-8} m^4$$

$$\text{From part 2a: } \delta_{\max} = \frac{114205 \text{ Nm}^3}{3EI}$$

for aluminum: $E = 70 \text{ GPa}$

$$\delta_{\max} = \frac{114205 \text{ Nm}^3}{(70 \times 10^9 \text{ Pa})(4.444 \times 10^{-8} \text{ m}^4)} = 34.7 \text{ m}$$

$$\delta_{\text{allow}} = 0.02L = 0.02(0.78m) = 0.0156 \text{ m}$$

 $\delta_{\text{allow}} \ll \delta_{\max} \rightarrow \text{this design fails.}$

Design 2:

1st find minimum required I

$$\delta_{\max} \leq 0.02L \rightarrow \delta_{\max} = 0.02(0.78m) = 0.0156 \text{ m}$$

$$\delta_{\max} = \frac{114205 \text{ Nm}^3}{3EI}$$

$$\delta_{\max} = \delta_{\text{allow}} \rightarrow \frac{114205 \text{ Nm}^3}{3EI} = 0.0156 \text{ m}$$

$$I = \frac{114205 \text{ Nm}^3}{3E(0.0156 \text{ m})} \quad \text{let the material be aluminum} \rightarrow E = 70 \text{ GPa}$$

$$I = \frac{114205 \text{ Nm}^3}{3(70 \times 10^9 \text{ Pa})(0.0156 \text{ m})} = 3.587 \times 10^{-5} \text{ m}^4$$

candidate tube: 250mm x 150mm x 10mm

$$h_0 = 0.25m \quad h_i = h_0 - 2t = 0.23m$$

$$b_0 = 0.15m \quad b_i = b_0 - 2t = 0.13m$$

$$t = 0.01m$$

outer moment of inertia:

$$I_o = \frac{0.15m(0.25m)^3}{12} = 1.953 \times 10^{-4} m^4$$

inner moment of inertia:

$$I_i = \frac{(0.13m)(0.23m)^3}{12} = 1.138 \times 10^{-4} m^4$$

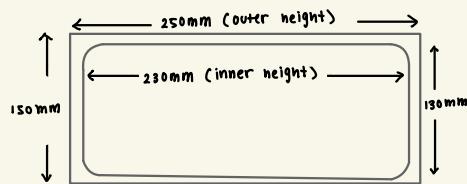
$$I = I_o - I_i = 1.953 \times 10^{-4} m^4 - 1.138 \times 10^{-4} m^4 = 0.35 \times 10^{-5} m^4$$

$$\delta_{max} = \frac{114205 \text{ Nm}^3}{3EI} = \frac{114205 \text{ Nm}^3}{3(200 \times 10^9 \text{ Pa})(0.35 \times 10^{-5} \text{ m}^4)} = 0.003 \text{ m}$$

$\delta_{allow} = 2L = 2(0.78 \text{ m}) = 0.0156 \text{ m}$ $\delta_{max} < \delta_{allow} \longrightarrow$ this beam satisfies deflection & is mass efficient for this stiffness level

Final Design choice: Hollow Aluminum tube w dimensions: 250mm x 150mm x 10mm

2.C
Final Beam Design cross-section: (Aluminum 6061-T4)



outer width: 150mm
wall thickness: 10mm
material: Aluminum 6061-T4
Young's Modulus: 70 GPa
Computed Moment of Inertia: $0.35 \times 10^{-5} \text{ m}^4$
Max deflection under load: 0.003 m