MAE 3260 Final Group Work Report

Outline:

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Title: Climate change, but only in my living room

Topic of Interest: Room Temperature Control

Abstract: In this project, we will look at how to control the temperature inside a room, since it's a system everyone deals with and it's simple enough to model, but still interesting from a dynamics and control perspective. We'll build a basic thermal model using ODEs and then represent it with transfer functions and state-space forms so we can study how the room responds to heating and environmental changes. Using MATLAB, we plan to simulate the open-loop behavior, look at steady-state and transient responses, and estimate key parameters like thermal resistance and capacitance. After that, we'll design a feedback controller to help the system follow temperature commands and reject disturbances, and compare how passive design choices and active control each affect the overall performance.

Students/Roles:

Student	Task/Role
Langston Johnson	MATLAB: Simulate Plant and Controller, Determine Steady State Error and Saturation of the controller.
Cion Kim	Models: Create the state-space representation of the room temperature system, including identifying states, inputs, outputs, and system parameters. Construct the block diagrams and overall system layout that capture the thermal dynamics and interactions within the model. These models will be used to develop both the open-loop and active control
Laura Ren	Active control: Design a feedback controller to help the system follow temperature commands and reject disturbances, and compare how passive design choices and active control each affect the overall performance.
Christina Ge	Open loop control: Estimate the thermal parameters of a room and use them to design an open-loop strategy for heating using parameter estimation, first-order system modeling, and step-response analysis.

List of MAE 3260 concepts or skills used in this group work:

- Modeling: Formulating energy balance equations, Deriving first-order ODE models, State-space representation, Block diagram construction
- Open loop: steady state behavior, time constant, frequency response, passive design
- Close loop: transfer function, PI control, PI fine-tune, system requirements, disturbance rejection

Modeling

The model for our change in temperature in a room can be explained by the following.

Chosen Variables

Output

- $T_R(t)$ = Room air temperature

Inputs

- T_h = heater temp
- T_{out} = outside temp
- ṁ = mass flow rate from heater (air)
- c = specific heat of air
- C = heat compacting of the room
- M_{air} = mass of air in room
- R = thermal resistance of room (considering heat loss through window)(also includes thermal conductivity/insulation of house walls)

Equations

Heat flow from heather

$$\dot{Q}_{heater} = (T_h - T_r)\,\dot{m}\,c$$

Heat flow to outside

$$\dot{Q}_{loss} = (T_h - T_{out})\,/R$$

Energy Balance (ODE)

Energy stored in room

$$C\frac{dTr}{dt} = \dot{Q}_{heater} - \dot{Q}_{loss}$$

Substitute flows as

$$C\frac{dTr}{dt} = (T_h - T_r)\,\dot{m}\,c - \frac{(T_r - T_{out})}{R_{eq}}$$

→ becomes

$$\frac{dTr}{dt} = (T_h - T_r)\frac{\dot{m}\,c}{C} - \frac{(T_r - T_{out})}{C\,R_{eq}}$$

$$\frac{dTr}{dt} = \left(-\frac{\dot{m}\,c}{C} - \frac{1}{C\,R_{eq}}\right)T_r + \frac{\dot{m}\,c}{C}T_h + \frac{1}{C\,R_{eq}}T_{out}$$

State-space form

State x = T_r

Input u = $[x1] = [\ T_h\ ]$ = heater outlet temperature

$[x2] = [\ T_{out}\ ]$ = outside temperature

The final form we use would be

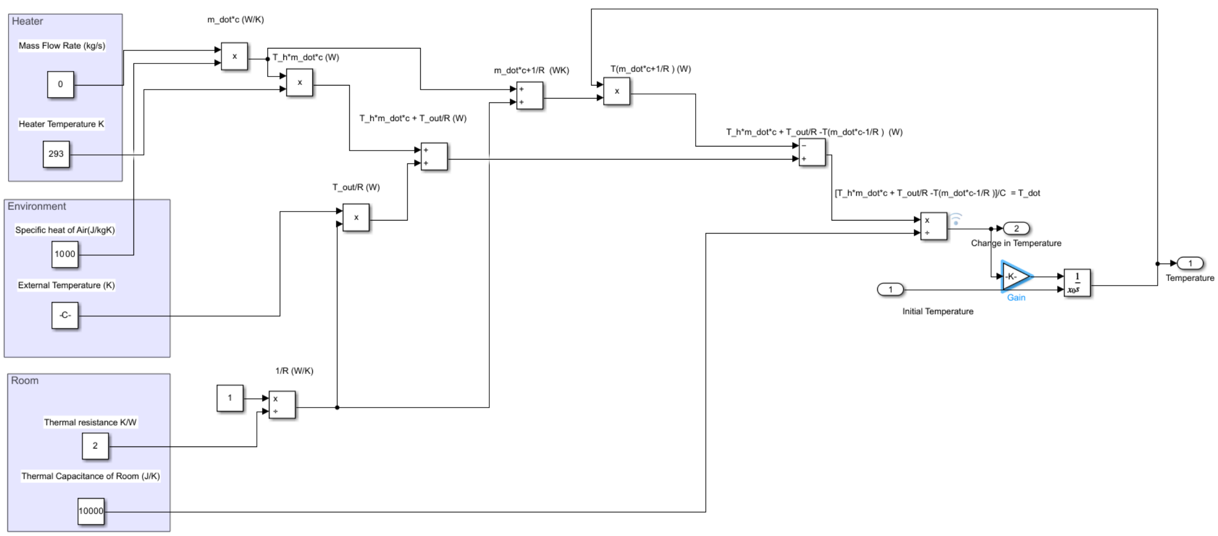
$$\dot{x} = [a]\, x + [b_1\ b_2]\, u,\ y = [1]\, x$$

$$a = \left(-\frac{\dot{m}\,c}{C} - \frac{1}{C\,R_{eq}}\right),\ \ b_1 = \frac{\dot{m}\,c}{C},\ \ b_2 = \frac{1}{C\,R_{eq}}$$

The initial conditions we can use to calculate values would be :

$$\text{Initial condition: } x(0) = T_r(0)$$

Block Diagram



Design Considerations

Included all the main input measurements for the system: the Temperature of the house, of the outside, and of the room. What the system wants to detect are mainly two kinds of heat transfer. Heat that the room is gaining from the heater, and that is being lost outside from the room. The system measures the net heat transfer to get a change in room temperature.

Conclusion

The model gives a meaningful and mathematically tractable first-order system that tells up how the room temperature changes due to heater input and environmental heat loss. Representing the system using an ODE, transfer functions, and state-space form provides a unified framework for both open-loop and closed-loop analysis. The block diagram shows the structure of heat addition, heat loss, and thermal storage. This model would be the base system for the open-loop characterization, parameter estimation, and feedback controller design of the later sections of the project.

Open Loop System

An open-loop thermal system can be modeled by considering a room heated by a constant flow heater while losing heat to the outside environment through the building envelope. The heater delivers warm air at a fixed temperature and flow rate, and the room loses heat through an equivalent thermal resistance representing walls and windows. No sensing or feedback is used to adjust the heater output, so the room temperature is governed entirely by the physical parameters of the system and the constant heater input. This configuration produces predictable first-order thermal behavior, with the temperature evolution determined by the balance between heat addition and passive heat losses.

Chosen Theoretical Values

The following values are selected for simulation of the open-loop model:

Parameter	Symbol	Value
Specific heat of air	c	$1000\ \mathrm{J/kg \cdot K}$
Air mass flow rate	$\dot{m}$	$0.10\ \mathrm{kg/s}$
Thermal capacitance of room	C	$10,000\ \mathrm{J/K}$
Thermal resistance	R	$2\ \mathrm{K/W}$
Outside temperature	T_{out}	$0°\mathrm{C}$
Chosen heater outlet temperature	T_h	$20.1°\mathrm{C}$
Initial room temperature	$T_r(0)$	$10°\mathrm{C}$

Steady State Behavior

To study steady state behavior, the energy balance equation is used to model the system:

$$C\frac{dT_r}{dt} = (T_h - T_r)\,\dot{m}c - \frac{(T_r - T_{\text{out}})}{R}.$$

In steady state, the change in temperature in the room is 0, which means the energy leaving the room by convection is the same as the energy entering the room by the heater in the long run:

$$0 = (T_h - T_{\text{ss}})\,\dot{m}c - \frac{(T_{\text{ss}} - T_{\text{out}})}{R}.$$

Solving for steady state temperature, the equation becomes:

$$T_{\text{ss}} = \frac{\dot{m}cRT_h + T_{\text{out}}}{\dot{m}cR + 1}.$$

After plugging in the numbers chosen from earlier in the report, we get a steady state temperature that is approximately 20 °C.

Time Constant Calculations

To calculate the time constant, the model can be rewritten as a first-order system:

$$\frac{dT_r}{dt} = -\left(\frac{\dot{m}c}{C} + \frac{1}{RC}\right)T_r + \left(\frac{\dot{m}c}{C}T_h + \frac{1}{RC}T_{\text{out}}\right).$$

Where the time constant is

$$\tau = \left(\frac{\dot{m}c}{C} + \frac{1}{RC} \right)^{-1}.$$

After substituting in the values from above, the time constant is equal to 99.5 seconds, which is the time required for the system to complete about 63% of the total response toward its steady state. The step response from the initial temperature is:

$$T_r(t) = T_{\text{ss}} + [T_r(0) - T_{\text{ss}}]\, e^{-t/\tau}.$$

Frequency Response

Using the Laplace transform of the linearized model leads to the transfer function from heater outlet temperature to room temperature:

$$G(s) = \frac{T_r(s)}{T_h(s)} = \frac{\dot{m}cR}{\dot{m}cR + 1} \cdot \frac{1}{\tau s + 1}.$$

This transfer function corre to sponds to a first-order low-pass system. At low frequencies, there are slow variations in heater temperature or outside temperature that are transmitted almost directly to the room, scaled by the steady-state gain. At high frequencies, there are Rapid fluctuations in heater temperature that are heavily attenuated because the thermal mass C prevents the room temperature from changing quickly.

Passive Design Considerations

The system's behavior depends strongly on passive thermal parameters:

- Increasing R reduces heat loss to the environment, resulting in a higher steady-state temperature for the same heater input.
- Increasing C adds thermal mass to the system, slowing temperature changes and increasing the time constant.
- Increasing the mass flow rate enhances heat transfer from the heater, producing a faster response and reducing the time constant.
- Adjusting the heater outlet temperature directly determines the steady-state temperature in an open-loop configuration, since no feedback regulation is present.

Conclusion

The open-loop room heating model behaves as a first-order thermal system in which the final temperature and transient response are determined entirely by physical parameters and fixed heater settings. With appropriate parameter selection, the model predicts a steady-state temperature of approximately 20 °C and a time constant near 100 seconds. The system filters high-frequency disturbances and responds exponentially to step inputs.

Close Loop

Introduction

In the close loop system, we model the heater as a system with constant air mass flow rate, and a variable system input of chosen heater outlet temperature and a disturbance input of outside temperature. Transfer functions are derived for both inputs, and a PI controller is selected to regulate room temperature due to its simplicity and suitability for slow thermal systems, avoiding the noise amplification associated with derivative action in PID control. The controller gains are tuned via pole-placement to meet design specifications on settling time, overshoot, and steady-state error.

System Model and Transfer Functions

The room is modeled as a single thermal mass with heat capacity C. Warm air at temperature T_h enters at a constant mass flow rate $\dot{m}$ and specific heat c. Heat is lost to the outside through an equivalent thermal resistance R_{eq}. The energy balance is:

$$C\dot{T}_r = \dot{m}c(T_h - T_r) - \frac{T_r - T_{out}}{R_{eq}}$$

Rewriting in linear state-space form with $x=T_r$, control input $u_1=T_h$, and disturbance $u_2=T_{out}$,

$$\dot{x} = ax + b_1u_1 + b_2u_2, \quad where \quad a = -\frac{\dot{m}c + 1/R_{eq}}{C}, b_1 = \frac{\dot{m}c}{C}, b_2 = \frac{1}{CR_{eq}}$$

The transfer functions from heater input and outdoor temperature to the indoor temperature are thus:

$$G_h(s) = \frac{b_1}{s-a}, \qquad G_{out}(s) = \frac{b_2}{s-a}.$$

PI Controller

We choose to use PI control instead of PID control because the heating system is a slow system that does not require fast rise time, and noisy derivative terms or large fluctuations may be worse to the system. Therefore, using a PI controller or tiny K_D for a PID controller is usually a better choice.

The temperature error is $e(t) = T_{ref}(t) - T_r(t)$, thus the heater temperature command is:

$$T_h(s) = C(s)\left(T_{ref}(s) - T_r(s)\right),$$

where $C(s) = K_P + \frac{K_I}{s}$ is the PI controller.

Combining the T_h equation from both the system model and PI control with the transfer functions, we obtain the equation for T_h:

$$T_r(s) = G_h(s)C(s)\big(T_{ref}(s) - T_r(s)\big) + G_{out}(s)T_{out}(s)$$

Thus the disturbance–to–room transfer function is:

$$\frac{T_r(s)}{T_{out}(s)} = \frac{G_{out}(s)}{1 + G_h(s)C(s)}$$

Substituting in the transfer functions gives:

$$\frac{T_r(s)}{T_{out}(s)} = \frac{b_2 s}{s^2 + s(-a + b_1 K_p) + b_1 K_i}$$

thus the characteristic equation that could be matched to a 2nd order model

$$s^2 + s(-a + b_1 K_p) + b_1 K_i = 0 \; ; \qquad s^2 + 2\zeta\omega_n s + \omega_n^2$$

By equating the coefficients, we could relate the proportional/integral gain in the PI controller with the system performance as $K_I = \frac{\omega_n^2}{b_1}$, $K_P = \frac{2\zeta\omega_n + a}{b_1}$.

PI Controller Fine-Tune

	Parameter	Symbol	Value
System requirements	2% Settling time	*2%* t_s	< 15min
	Maximum overshoot	M_O	< 5%
	Steady state error	e_{ss}	< 0.2%
Theoretical system constants	Specific heat of air	c	1000 J/kgK
	Air mass flow rate	$\dot{m}$	0.10 kg/s
	Thermal capacitance of room	C	10000 J/K
	Thermo resistance	R	2 K/W

The numerical state-space model constants are:

$$a = \left(-\frac{\dot{m}\,c}{C} - \frac{1}{C\,R_{eq}}\right) = -\ 0.01005\ s^{-1} \; ; b_1 = \frac{\dot{m}\,c}{C} = 0.01\ s^{-1}$$

In order to fulfil the system requirements listed in the table above, we can pick our ζ and Ω:

$$\zeta \approx 0.6901 \text{ for } M_O = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \approx 5\%$$

$$\omega_n = \frac{4}{\zeta\, t_s} = 0.0074310\ s^{-1} \text{ for } t_s \approx 13min$$

Therefore, for this particular system, the fine-tuned proportional and integral gain is:

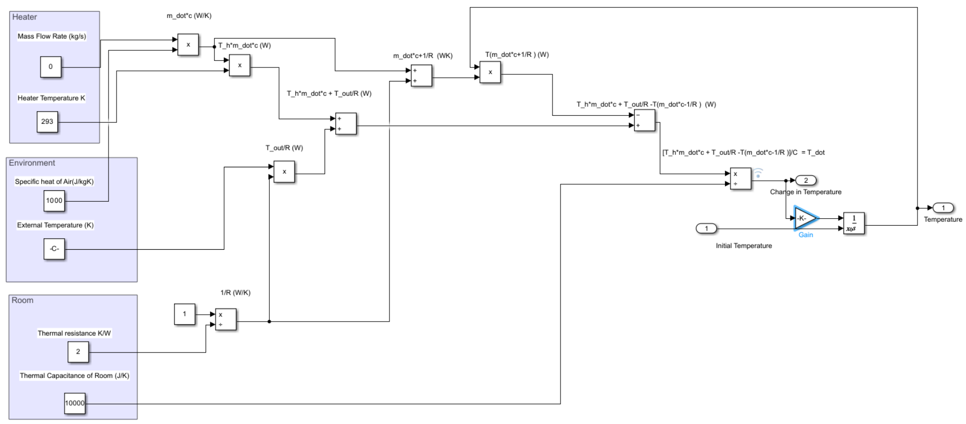
$$K_I = \frac{\omega_n^2}{b_1} = \frac{0.0074310^2}{0.01} = 0.005522\ ,\ K_P = \frac{2\zeta\omega_n + a}{b_1} = \frac{2*0.6901*0.0074310-0.01005}{0.01} = 0.02064$$

Conclusion:

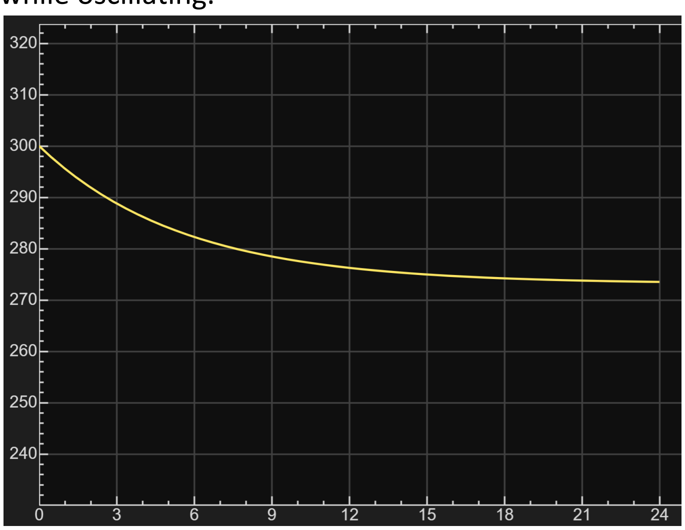
Tested with a series of carefully selected system requirements and variables, the resulting close loop control system of the heater ensures that the system meets the required performance criteria, providing fast disturbance rejection, minimal overshoot, and negligible steady-state error with a PI controller of $K_I = 0.005522$ and $K_P = 0.02064$.

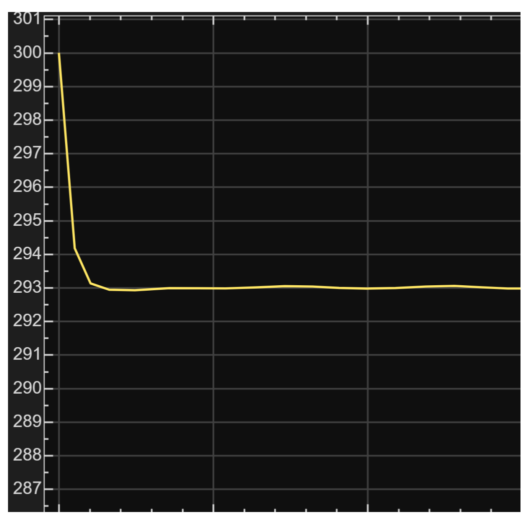
Simulation

Introduction: We will be using Simulink to simulate both our open loop and closed loop system under a variety of conditions. For the Open Loop System the Block Diagram, we have an effectively first order system.Based on the equation for the open loop system we can isolate for dT and using an integrator block in simulink to define the system plant.

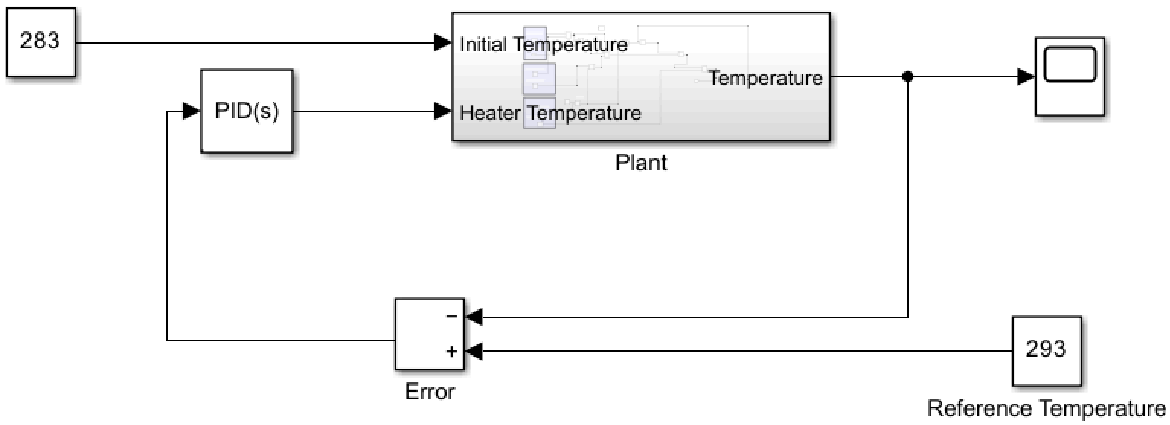


Because the room has such a large thermal capacitance (10000 J/K) it takes a very long time for the temperature to change. Take for example a room that is 26.3 Kelvin warmer than the ambient temperature. The rate at which temperature is lost to the environment is initially 13.1 J/s. In order to cool the room by 1 K 10 kJ of energy must be lost. The system takes 24 hours to reach the outside temperature if T(0) = 300K. With the heater turned on the room actually cools faster to increase mass flow rate and then reaches a steady state temperature of around 293K while oscillating.

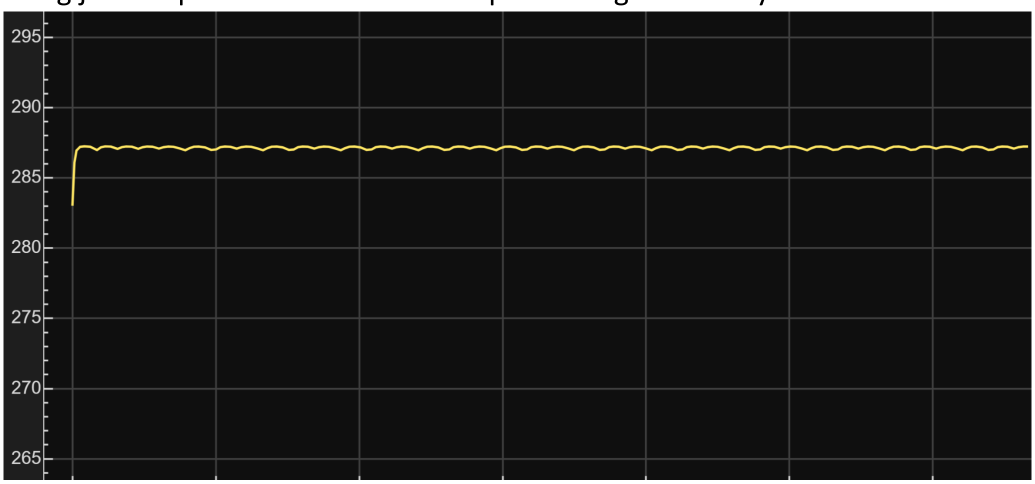




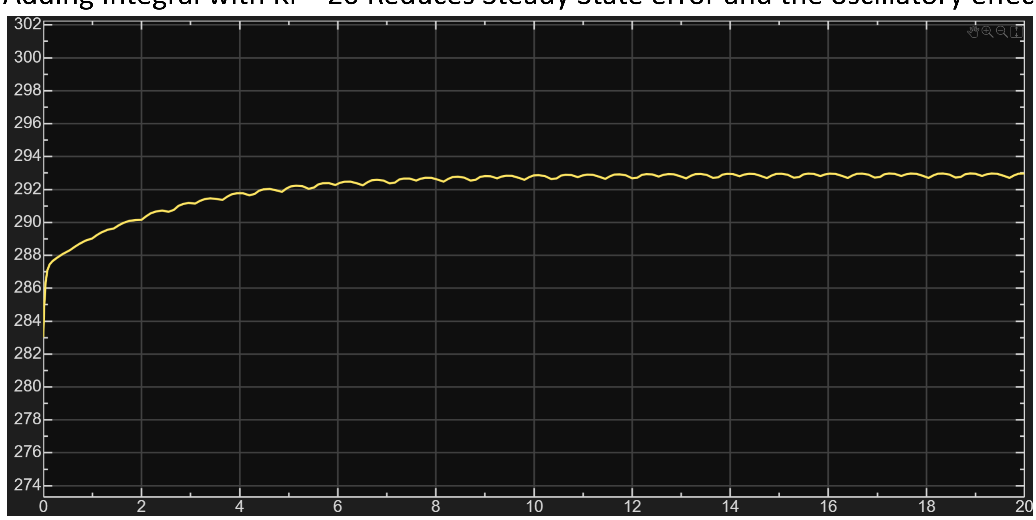
For the closed loop system we implement the PI controller defined in the earlier section
First we design the closed loop system.



Using just Proportional Control with Kp=50 we get a steady state error of 3K.



Adding Integral with Ki = 20 Reduces Steady State error and the oscillatory effects.



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