

## Maximum Deflection Calculations

Deflection at tip by superposition (standard beam formula):

$$\delta_{tip} = \frac{WL^3}{3EI} + \frac{F_{\perp} a^2(3L-a)}{6EI}$$

Rearrange to compute the required second moment of area I for a target maximum deflection  $\delta_{max}$ :

$$I_{req} = \frac{\frac{WL^3}{3} + \frac{F_{\perp} a^2(3L-a)}{6}}{E\delta_{max}}$$

Substitute numbers:

- $L = 1.20 \text{ m}$
- $a = 1.00 \text{ m}$
- $W = 31,630 \text{ N}$
- $F_{\perp} = 34,986.38 \text{ N}$
- $E = 200 \times 10^9 \text{ N/m}^2$
- $\delta_{max} = 0.02L = 0.02 \times 1.20 = 0.024 \text{ m}$

Compute numerator:

- $\frac{WL^3}{3} = \frac{31,630 \times (1.20)^3}{3} = 18,216.5546667$
- $\frac{F_{\perp} a^2(3L-a)}{6} = \frac{34,986.3823 \times 1.00^2 \times (3 \times 1.20 - 1.00)}{6} = 15,163.0909216$
- Numerator total =  $18,216.5546667 + 15,163.0909216 = 33,379.6455883$

Compute  $I_{req}$ :

$$I_{req} = \frac{33,379.6455883}{200 \times 10^9 \times 0.024} = \frac{33,379.6455883}{4.8 \times 10^9} \approx 6.95409 \times 10^{-6} \text{ m}^4$$

Conclusion (part a):

- Required second moment of area to meet 2% deflection limit:

$$I_{req} \approx 6.95 \times 10^{-6} \text{ m}^4$$