

Droning On: Analysis of the BAE/Malloy T-150 Heavy Lift UAS Yaw Control System Dynamics

MAE 3260—System Dynamics

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Abstract:

In this analysis, we are studying the yaw control of the BAE/Malloy T-150 heavy-lift unmanned aerial system (UAS). Since flight dynamics are naturally nonlinear, we first linearized the equations of motion for steady, standard, flight conditions. We aimed to maximize the stability around a desired yaw angle, while maintaining constant altitude above the ground and rejecting disturbances. We studied the dynamics surrounding this system based on the inertial measurement unit (IMU), developed the relevant ODEs and thus the transfer functions, generated the block diagram for our system, and then implemented PD control. After the controller was implemented, we used Bode plot analysis in order to further understand the system behavior of different configurations of payloads. Our analysis does not consider the GPS navigation system, pitot-static system and other pressure-based instruments, and other electronic sensors.

Student Roles:

Lucas Libshutz	Modeled yaw dynamics, developed ODEs, and designed a PD controller to meet system response requirements, e.g., overshoot and rise time
Pranv Shankar	Modeled yaw dynamics, and created and validated physical models to characterize system properties, e.g., moment of inertia, maximum torque
Rochelle Gao	Modeled system dynamics using transfer functions and block diagrams. Performed Bode plot analysis to verify the PD controller performance
Carmen Lin	Modeled system dynamics using transfer functions and block diagrams. Performed Bode plot analysis to verify the PD controller performance

Concepts:

- Models:
 - ODEs
 - Transfer Functions
 - State-space
 - Bode plots
 - Block diagrams
- Open-loop system:
 - Parameter estimation
 - Step or frequency response
- Active control:
 - Feedback control law
 - Command following

1 Physics of Quadcopter Flight

1.1 How a Quadcopter Flies

A quadcopter uses the thrust from four rotors on arms held up by arms suspended from the central structure in the shape of an “X.” The downwash of air through the rotors creates thrust according to Newton’s Third Law; the exact relationship between the rotors and the amount of thrust generated is explained by Blade Element Theory and aerodynamics. To ensure the net angular momentum is zero, two of the rotors spin clockwise and two counterclockwise during steady hover. If all the rotors spun in the same direction, conservation of angular momentum dictates that the drone would spin in the opposite direction with an equal magnitude of angular momentum. Given the undesirability of such uncontrolled spin, the two pairs of rotors spin in opposite directions to maintain the orientation of the drone in yaw.

To ascend, the thrust of all four rotors is increased; to descend, the thrust of all four rotors is decreased. To pitch forward ($\theta > 0$), the thrust of the back rotors is increased and the front rotors’ thrust is decreased to tilt the quadcopter forward. To roll ($\phi > 0$) right, the thrust of the left rotors is increased and the right rotors’ thrust is decreased to tilt the quadrotor right. To yaw in the clockwise direction ($\psi > 0$), the thrust of the counterclockwise rotors are increased and the clockwise rotors’ thrust is decreased, creating a reaction torque that rotates the quadcopter in the clockwise direction around the axis of rotation.



Figure 1. Image of the BAE/Malloy Aerospace T-150 Heavy Lift UAS

For the following analysis of the BAE/Malloy T-150, the physics models below make several key assumptions, namely:

- The quadcopter is in a steady hover, i.e., its linear velocity is zero or negligible.
- Drag forces on the body of the quadcopter are negligible, i.e., inviscid flow
- The coaxial rotor pair on each arm can be modeled as a single rotor with combined thrust and torque output, neglecting wake effects between them.

- The T-150 behaves as a rigid body; the total mass and moment of inertia remain constant for a given phase of flight.

1.2 Moment of Inertia of the BAE/Malloy T-150

The moment of inertia of the T-150 is a key parameter in determining how the applied torque relates to the change in angular velocity and, subsequently, the yaw control. Due to the T-150's defense applications, there are no publically available models of the UAS. In order to characterize the moment of inertia, the moment of inertia of a similarly shaped but smaller drone, the DJI Phantom 3, was determined from the publicly available CAD model [1]. Then, the moment of inertia is scaled according to the dimensional relationship between the masses and lengths of the Phantom 3 and T-150.

In this report, we will be primarily concerned with yaw control. Thus, only the moment of inertia around the yaw axis, I_{zz} , is relevant to the analysis. The moment of inertia of the Phantom 3 is $0.02572 \text{ kg}\cdot\text{m}^2$ corresponding to a mass of 1.652716 kg and dimensions of $15.55 \times 15.54 \times 7.95 \text{ inches}$. The T-150 has a mass of 55 kg and can carry a payload of 68 kg with dimensions in meters of $2.65 \times 2.05 \times 0.71$ per BAE's specifications.

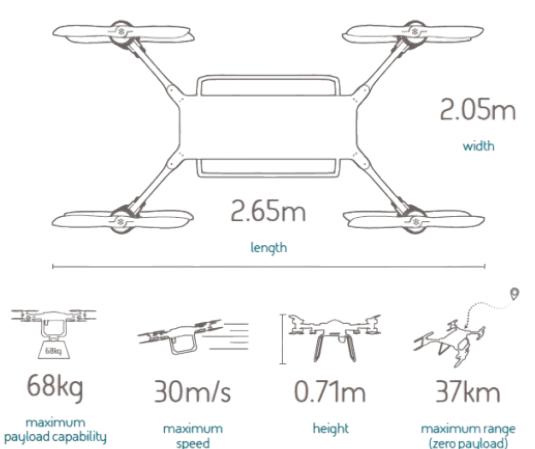


Figure 2. Provided Technical Specifications of the T-150 (source: BAE Systems T-Series Brochure) [2]

The moment of inertia scales proportionally with mass and with the square of length, i.e.,

$$I_z \propto m L^2$$

The scaling factor between the moment of inertia of the DJI Phantom 3 and the BAE/Malloy T-150 can be given as

$$k = \frac{m_{\text{BAE}}}{m_{\text{DJI}}} \frac{L_{e,\text{BAE}}^2}{L_{e,\text{DJI}}^2}$$

The effective length, $L_e = L_x^2 + L_y^2$, since z is the rotation axis. If we convert the dimensions of the Phantom 3 into meters, we can plug in the values for the scaling factor and then calculate the moments of inertia when the T-150 is and is not carrying the payload:

- Empty: $k_{\text{empty}} \approx 1.20 \times 10^3$, $J_{z,\text{empty}} \approx k_{\text{empty}} J_{z,\text{DJI}} \approx 30.8 \text{ kg} \cdot \text{m}^2$
- Full $k_{\text{full}} \approx 2.68 \times 10^3$, $J_{z,\text{full}} \approx k_{\text{full}} J_{z,\text{DJI}} \approx 68.9 \text{ kg} \cdot \text{m}^2$

1.3 Conversion from Torque to Power Change

Assume that the torque required to produce and maintain a given yaw is calculated; the derivation of the equations of motions that specify ψ and τ_z are included later in the document. The actual control input will be an increase and decrease to the motors' power.

In our configuration, we assume steady hover conditions, thus mandating that the total upward thrust of the drone does not change when the yaw control is applied. Mathematically, this is expressed as

$$T_1 + T_2 + T_3 + T_4 = W$$

The thrust of a motor is given by

$$T_i = C_T \rho A r^2 \omega_i^2 \implies T_i = k_T \omega_i^2 [3]$$

Thus, the angular rate must be increased and decreased by equal amounts for the clockwise and counterclockwise to maintain the total thrust. This corresponds to

$$\omega_{CW} = (\omega_h + \Delta\omega) \text{ and } \omega_{CCW} = (\omega_h - \Delta\omega)$$

Note that for quadrotors, positive yaw is defined as clockwise as the e_3 unit direction points down.

In steady hover, assume that the motors are spinning with rate $\omega_{\text{hover}} = \omega_h$. We want to apply a change to the angular speed, $\Delta\omega$, to each of the motors to generate the torque. The reaction torque of a motor is given by

$$Q_i = C_Q \rho A r^3 \omega_i |\omega_i| e_3 \implies Q_i = k_Q \omega_i^2 [3]$$

Therefore, the net torque on the system is given by

$$\tau_z = 2Q_{CW} - 2Q_{CCW} = 2k_Q(\omega_h + \Delta\omega)^2 - 2k_Q(\omega_h - \Delta\omega)^2$$

If we expand the terms, we find that all of the ω_h^2 and $\Delta\omega^2$ terms cancel and we are left with

$$\tau_z = 8k_Q \omega_h \Delta\omega \implies \Delta\omega = \frac{\tau_z}{8k_Q \omega_h}$$

Given the formula for power $P = E/t$, we can derive that

$$P = Q\omega = k_Q \omega^2 \cdot \omega = k_Q \omega^3$$

From that, we can determine the increase in the power as ΔP :

$$\Delta P = P_{\text{new}} - P_{\text{old}} = k_Q(\omega_h + \Delta\omega)^3 - k_Q \omega_h^3$$

Substituting $\Delta\omega$, expanding, and solving, we get that ΔP is

$$\Delta P = \left(\frac{3}{8} \omega_h \tau_z \right) + \left(\frac{3\tau_z^2}{64k_Q \omega_h} \right) + \left(\frac{\tau_z^3}{512k_Q^2 \omega_h^2} \right)$$

We can assume that $\tau_z/\omega_h \ll 1$ because $\omega_h \gg 1$ and τ_z is relatively small. Thus, we get that

$$\Delta P \approx \frac{3}{8} \omega_h \tau_z$$

In order to maintain altitude, the same ΔP will be applied to both the clockwise and counterclockwise rotating rotors. Given the linear relationship between τ_z and ΔP , the outputs for the system dynamics models will be τ_z . It may be assumed that the flight computer can easily translate from the control output to the required motor dynamics.

1.4 Maximum Total Torque

Again, because the BAE/Malloy T-150 has military applications, specifications for its motors are not available. For the purpose of the control analysis, we will need the maximum torque that the motors can apply. The drag torque is approximately 1–2% of the total torque computed as the product of total thrust and the rotor diameter [4]. The maximum thrust can be approximated as twice the weight of the quadrotor, i.e., $2m_{\text{full}}g \approx 2400$ N. The rotor diameter is approximately two feet in length [5]. Thus, the maximum torque is approximately 14.6–29.3 Nm. If we further assume that the motors are canted, i.e., tilted so the thrust vector contributes to yaw, by as little as 5–10 degrees, which is common in heavy-lift applications, the maximum motor torque will be 25–50 Nm. We can, thus, make a conservative estimate of the maximum torque to be around 34 Nm.

2 Derivation of Linearized ODEs for Quadcopter Yaw

2.1 Finding generalized equations of motion

In order to consider all linear and rotational dynamics simultaneously, we use the rigid body dynamics notation adapted from Featherstone, 2008 [6]. We start with the general “screw” theory formulation:

$$\begin{bmatrix} \mathbf{f}^W \\ \boldsymbol{\tau}^B \end{bmatrix} = \begin{bmatrix} m\mathbf{I}_3 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\mathcal{J}} \end{bmatrix} \begin{bmatrix} \mathbf{a}^W \\ \boldsymbol{\alpha}^B \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\omega}^B \times \boldsymbol{\mathcal{J}}\boldsymbol{\omega}^B \end{bmatrix}$$

If we model the thrust force with some coefficient of friction c_f , then we can write the thrust force as:

$$\mathbf{T}_i^{P_i} = c_f w_i |w_i| \hat{\mathbf{e}}_3$$

This describes the thrust force on the i -th propeller, in the $\hat{\mathbf{e}}_3$ direction, which we define as upwards, aligned with gravity. From this, we can formulate the total thrust force on the body B as:

$$\mathbf{f}_{thrust}^B = \sum_{i=1}^4 \mathbf{R}_{P_i}^B \mathbf{T}_i^{P_i}$$

We define $\mathbf{R}_{P_i}^B$ as the rotation matrix from the i -th propeller to the body's center of mass. We also define $\mathcal{J} \in \mathbb{R}^{3 \times 3}$ as the generalized inertia tensor of the quadcopter, along with the “hat” operator being equivalent to the skew symmetric operator in \mathbb{R}^3 , such that:

$$[\boldsymbol{\omega}]_\wedge = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

For torques on our body, there are only two to consider: arm torque, and drag torque. We model the arm torque per propeller arm as $\tau_i^{\text{arm}} = \rho_i^B \times \mathbf{f}_i^B$, and $\tau_i^{\text{drag}} = c_d \omega_i |\omega_i| \hat{\mathbf{e}}_3$, with c_d as the corresponding coefficient of drag. With these, we can now write the sum of forces and torques, on the body:

$$\mathbf{f}^B = \sum_{i=1}^4 \mathbf{f}_i^B, \quad \boldsymbol{\tau}^B = \sum_{i=1}^4 (\tau_i^{\text{arm}} + \tau_i^{\text{drag}})$$

If we want to implement full per-propeller control in the future, we would use the derivation from Section 1.3. However, for our purposes, we only consider per-axis torque control. With these force formulations, we can now write Newton’s second law for linear motion for the quadcopter:

$$m\mathbf{a}^w = \sum \mathbf{f} = -mg\hat{\mathbf{e}}_3 + \mathbf{R}_B^w \mathbf{f}^B$$

Where \mathbf{R}_B^w is the rotation matrix from the drone body to the world frame. If we define the angular momentum of the system as $\mathbf{H} = \mathcal{J}\boldsymbol{\omega}^B$, then we can write Newton’s second law for angular momentum:

$$\mathcal{I} \frac{d}{dt} \mathbf{H} = \mathcal{B} \frac{d}{dt} (\mathcal{J}\boldsymbol{\omega}^B) + \boldsymbol{\omega}^B \times (\mathcal{J}\boldsymbol{\omega}^B)$$

Which, after equating to the sum of body torques that we just defined above, we can write this as:

$$\mathcal{J}\dot{\boldsymbol{\omega}}^B = -\boldsymbol{\omega}^B \times (\mathcal{J}\boldsymbol{\omega}^B) + \boldsymbol{\tau}^B$$

Finally, we also define our control inputs. We define $\mathbf{u} \in \mathbb{R}^4$, such that $u_1 = T$ (the thrust force), and with $(u_2, u_3, u_4) = (\tau_x, \tau_y, \tau_z)$ respectively. This means that we can now define $\mathbf{f}^B = u_1 \hat{\mathbf{e}}_3$, and $\boldsymbol{\tau}^B = u_2 \hat{\mathbf{e}}_1 + u_3 \hat{\mathbf{e}}_2 + u_4 \hat{\mathbf{e}}_4$. From all of these quantities, we can finally write our full rigid body formulation:

$$\begin{aligned}\dot{\mathbf{p}}^w &= \mathbf{v}^w \\ m\dot{\mathbf{v}}^w &= -mg\hat{\mathbf{e}}_3 + \mathbf{R}_B^w \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix} \\ \dot{\mathbf{R}}_B^w &= \mathbf{R}_B^w [\boldsymbol{\omega}^B]_\wedge \\ \mathcal{J}\dot{\boldsymbol{\omega}}^B &= -\boldsymbol{\omega}^B \times (\mathcal{J}\boldsymbol{\omega}^B) + \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix}\end{aligned}$$

2.2 Simplifying equations of motion to only yaw control

With our full equations of motion, we now add the following assumptions for our only yaw-controlled system:

- The quadcopter will have a steady hover, which means a constant position in the world frame, and $z = 0$ for all time.
- The quadcopter will maintain constant rotation angles for roll and pitch, such that $\dot{\phi} = 0$ and $\dot{\theta} = 0$.
- The total thrust must balance any weight, so $u_1 = T = mg$.
- No roll or pitch torques are being applied, so $u_2 = u_3 = 0$.
- Only $u_4 = \tau_z \neq 0$ for rotation control.

Even though we have a non-zero control input in u_1 , this is assumed to be in steady state, so we do not consider it here. Because our rotation will be purely about the z axis, this means that $\mathbf{R}^{Bw} = R_z(\psi)$, which also means that our angular velocity will only have a component in z .

This means that we can write the $\mathcal{J}\dot{\boldsymbol{\omega}}^B$ term as follows:

$$\mathcal{J}\dot{\boldsymbol{\omega}}^B = \begin{bmatrix} J_x \dot{\phi} \\ J_y \dot{\theta} \\ J_z \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ J_z \ddot{\psi} \end{bmatrix}$$

This also means that the original $-\boldsymbol{\omega}^B \times (-\mathcal{J}\boldsymbol{\omega}^B)$ must be zero, as both vectors must be parallel to one another, making the cross product evaluate to zero. This means, for our *rotational* equation of motion, we formulate:

$$J_z \ddot{\psi} = \tau_z + \tau_d(t)$$

To express the yaw dynamics in standard state-space form, we define the state vector as

$$\mathbf{x} = \begin{bmatrix} \psi \\ r \end{bmatrix},$$

where ψ is the yaw angle and r is the yaw rate. The inputs to the system consist of the control yaw torque τ_z and an external disturbance torque τ_d , collected as

$$\mathbf{u} = \begin{bmatrix} \tau_z \\ \tau_d \end{bmatrix}.$$

Using these definitions and the linearized yaw equations

$$\dot{\psi} = r \text{ and } J_z \dot{r} = \tau_z + \tau_d,$$

the system, expressed in the linear form of $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$, is given by

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\psi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} r \\ \frac{1}{J_z}(\tau_z + \tau_d) \end{bmatrix}.$$

The system matrices are

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ \frac{1}{J_z} & \frac{1}{J_z} \end{bmatrix}.$$

As the yaw angle and yaw rate are measured outputs, the output equation is given by $y = Cx + Du$ with

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

3 Transfer Functions and Block Diagrams

To develop a reliable yaw-control model for the quadcopter, we begin by examining its fundamental rotational dynamics. The yaw motion is governed by the moment of inertia J_z , which relates motor-generated torque to angular acceleration. Because yaw rate is the integral of angular acceleration and yaw angle is the integral of yaw rate, the plant behaves as a double integrator. When first designing the controller, a full PID controller was considered; however, the combination of the plant's inherent integral behavior and the additional integral term in the controller introduced significant difficulties. The system became more difficult to stabilize, accumulated excessive phase lag, and exhibited large overshoot. These issues resulted in us utilizing a PD controller instead, which retains the proportional correction and provides needed

damping through the derivative term without further destabilizing the dynamics. The resulting block diagram reflects this more stable and practical control approach.

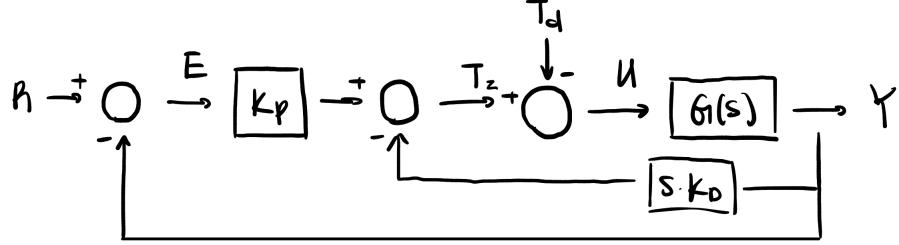


Figure 3. Block Diagram

$$G(s) = \frac{1}{J_z s^2}$$

$$\frac{Y(s)}{R(s)} = \frac{K_p}{J_z s^2 + K_D s + K_p}$$

The two state variables ψ and r describe the quadcopter's yaw orientation and its yaw rate, respectively. These states change according to the torque applied by the motors, which serves as the control input. The transfer function $G(s) = \frac{1}{J_z s^2}$ reflects the relationship between this input torque and the resulting yaw motion: applying torque changes angular acceleration, which affects yaw rate, and ultimately alters the yaw angle. Within the PD controller, the reference yaw angle is continuously compared with the current yaw angle, and the resulting error determines the corrective torque. The proportional term addresses immediate angle error, while the derivative term reacts to the yaw rate, adding damping that helps reduce overshoot and stabilize the response. This structure ensures that the quadcopter can accurately follow yaw commands while remaining robust to disturbances and variations in operating conditions.

4 Active Control

As outlined above, we decided to use a PD control structure. We started with the dynamics in the Laplace domain, as:

$$J_z s^2 \Psi(s) = T_z(s)$$

From the dynamics above, we implemented our PD control law, as

$T_z(s) = K_P(\Psi_{\text{ref}}(s) - \Psi(s)) - K_D s \Psi(s)$. We only have a $\Psi(s)$ term on K_D , as our desired velocity is zero. We are able to rearrange our original dynamics equation above to find our closed loop transfer function:

$$H(s) = \frac{\Psi(s)}{\Psi_{\text{ref}}(s)} = \frac{K_P}{J_z s^2 + K_D s + K_P}$$

Now, we know that the maximum torque for this system is 34 Nm, and with a desired step size from 0 to π , we know that our K_P value to achieve this must be:

$$\tau_{\text{initial}} = K_P e_{\max} \implies K_P = \frac{34 \text{Nm}}{\pi \text{ rad}} \approx 11$$

To find K_D , we first need to find the system's natural frequency. Since the poles of this system are in general second order form, we can rewrite the K_P term as ω_n^2 . This means that after normalizing, we find that:

$$\omega_n = \sqrt{\frac{K_P}{J_z}} \implies \omega_n = \sqrt{\frac{11}{69}} \approx 0.40 \text{ rad/s}$$

Now, we can rewrite the $K_D s$ term as the second order analogue of $2\zeta\omega_n$. We want our system to be critically damped ($\zeta = 1$), so writing out the parameter in normalized form:

$$\frac{K_D}{J_z} = 2\zeta\omega_n \implies K_D = 2J_z\zeta\omega_n \implies K_D \approx 55$$

This means that for the loaded case, we get $K_P = 11$ and $K_D = 55$. For the unloaded case, we can repeat the same derivation process with the unloaded inertia value of $J_z = 31 \text{ kg m}^2$, and get $K_P = 11$ and $K_D = 37$.

5 Bode Plot Analysis

To better understand how the yaw-control system behaves in real flight conditions, Bode plots were generated for both the loaded and unloaded configurations of the aircraft. Because the yaw dynamics behave like a double integrator, the system's performance is largely shaped by a pair of dominant poles associated with the vehicle's rotational inertia. These poles determine how quickly the vehicle can respond and how much phase lag it accumulates as frequency increases. The following will analyze each plot individually and compare how payload-induced changes in inertia influence overall stability and responsiveness.

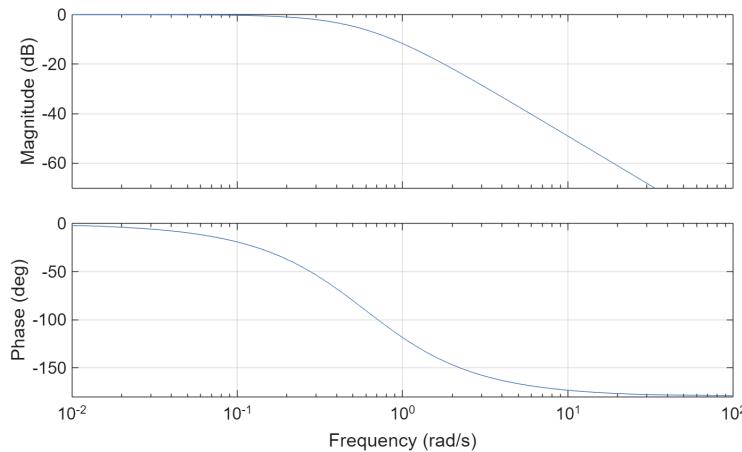


Figure 4. BAE/Malloy T150 Heavy Lift UAS Yaw Control Bode Plot (Unloaded)

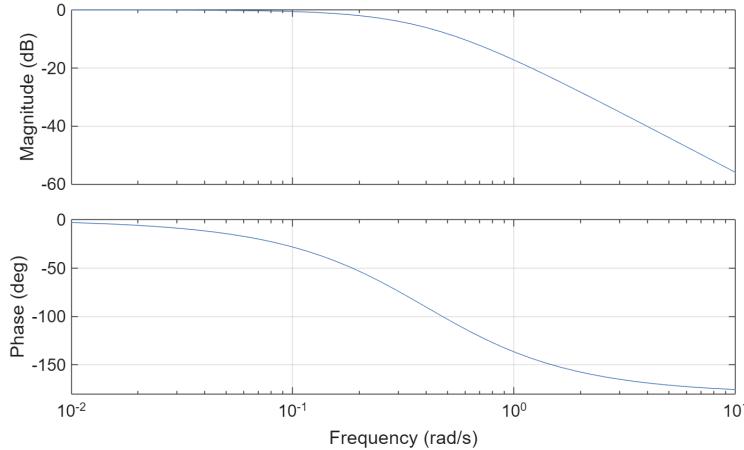


Figure 5. BAE/Malloy T150 Heavy Lift UAS Yaw Control Bode Plot (Loaded)

A comparison of the unloaded and loaded yaw-control responses shown in Figures 3 and 4 demonstrates how increasing the rotational inertia slows the dynamic response and reduces the bandwidth. In the unloaded case (Figure 4), the aircraft has a lower rotational inertia of $J_z = 31\text{kgm}^3$, combined with $K_p = 11$ and a derivative gain of $K_d = 37$. The system achieves the same magnitude at a higher frequency and a more gradual magnitude roll-off in comparison to the loaded case, indicating the yaw control responds more quickly and has a wider bandwidth. This occurs because the lower inertia allows the dominant poles of the double-integrator plant to remain at higher frequencies, enabling faster yaw response and more gradual phase decay.

In contrast, the loaded configuration (Figure 5) increases the inertia to $J_z = 69\text{kgm}^3$, requiring a larger derivative gain of $K_d = 55$ to counteract the greater phase lag introduced by the added payload. Both the magnitude and phase curves are shifted leftward, showing that the system begins losing gain and phase margin at lower frequencies. Even with a higher K_d value, the loaded response exhibits an earlier magnitude roll off and the phase approaches -180° at a lower frequency, indicating a narrower bandwidth and slower dynamic response.

Overall, Figures 4 and 5 show that the unloaded aircraft is naturally more agile, while the loaded configuration trades some responsiveness for stability under higher inertia.

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