MAE 4060
Fall 2025
Homework 1: Things you may know but that you should look them up if you don't…

Circle the best answer:

1. (2 points) How does a spacecraft reach orbit?
   a. It rides on a launch vehicle
   b. It is assembled on the International Space Station and launches through an airlock
   c. It uses its own on-board propulsion subsystem to lift off the Earth and enter orbit.

2. (2 points) How does a spacecraft stay in Earth orbit?
   a. Its speed results in centripetal acceleration that cancels gravity until atmospheric drag or other perturbations reduce its orbital angular momentum and cause it to reenter Earth's atmosphere
   b. It constantly applies 1g worth of acceleration through thrusters until it runs out of propellant, at which point it reenters Earth's atmosphere.
   c. There is no gravity in space; so, it hovers until commanded by mission control to reenter Earth's atmosphere.

3. (2 points) Which type of heat transfer can cause an asteroid to heat up or cool down in the environment of deep space?
   a. Convective
   b. Conductive
   c. Radiative

4. (2 points) For Earth, what is the Vernal Equinox?
   a. In spring, the instant at which the sun crosses the celestial equator, and near which night and day are the same length.
   b. The time at which the sun reaches its northernmost point in the sky
   c. In northern-hemisphere winter, the day on which daylight continues without night.

5. (2 points) Which is the most common structural material for spacecraft?
   a. Aluminum
   b. Steel
   c. Carbon nanotubes

6. (2 points) Approximately what fraction of the federal budget is NASA's annual budget?
   a. 0.5%
   b. 20%
   c. 5.0%

7. Calculate by hand the cross product of ***v*** and ***w***, where ***v***=2***i***+3***j***+4***k*** and ***w***=-4***i***-6***j***-8***k***. Show your work.

$$\vec{v}\times\vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ -4 & -6 & -8 \end{vmatrix} = \hat{i}(-24+24) - \hat{j}(-16+16) + \hat{k}(-12+12) = \underline{\underline{0}}$$

$\vec{v} \parallel \vec{w}$, so it should be zero.

8. Calculate by hand the dot product of ***v*** and ***w***, where ***v***=2***i***+3***j***+4***k*** and ***w***=-2***i***+***j***. Show your work.

$$\vec{v}\cdot\vec{w} = 2(-2)+3(1)+4(0) = \underline{\underline{-1}}$$

9. Convert 6 RPM to Hz, rad/s, and deg/s

$$\frac{6\text{ rot}}{1\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} = \underline{0.1\text{ Hz}}$$

$$\frac{6\text{ rot}}{1\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rot}} = \underline{\underline{0.2\pi}}$$

$$6\,\frac{\text{rot}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{360\text{ deg}}{1\text{ rot}} = \underline{\underline{36\text{ deg/s}}}$$

10. What is the acceleration due to gravity at Earth's surface in m/s^2 to two significant figures?

$$\underline{\underline{g = 9.8\text{ m/s}}}$$

11. Convert 6 Nms to ft-lb-s

$$6\,Nms \times \frac{3.2804\,ft}{1\,m} \times \frac{0.224809\,lb}{1\,N} = \underline{\underline{4.4248\ ft\text{-}lb\text{-}s}}$$

12. Convert 6 g-cm^2 to kg-m^2.

$$6\ g\text{-}cm^2 \times \frac{1\,kg}{1000\,g} \times \frac{1\,m^2}{10000\,cm^2} = \underline{\underline{6\times10^{-7}\ kg\text{-}m^2}}$$

13. What is the inverse of the matrix A, where p and q are nonzero real numbers not equal to 7? What happens for the case of only p=7?

$$A = \begin{bmatrix} p & q \\ 7 & q \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},\quad A^{-1} = \frac{1}{det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\text{So, } A^{-1} = \frac{1}{pq-7q}\begin{bmatrix} q & -q \\ -7 & p \end{bmatrix}$$

If $p=7$, $A = \begin{bmatrix} 7 & q \\ 7 & q \end{bmatrix}$, and the columns are linearly dependent, so A^{-1} is <u>undefined</u>.

14. What is the kinetic energy of a uniformly solid ball of mass 3 kg and radius 1m spinning at 5 deg/s and traveling in a straight line at 10 m/s?

$$T_{tot} = T_{trans} + T_{rot}$$

$$= \frac{1}{2} m\vec{v}\cdot\vec{v} + \frac{1}{2} I\vec{\omega}^2$$

$$= \frac{1}{2}(3\,kg)(10\,m/s)^2 + \frac{1}{2}\left[\frac{2}{5}(3)(1)^2\right](0.0873)^2$$

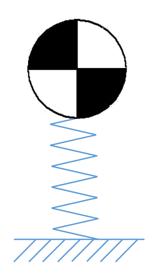
For a sphere,

$$I = \frac{2}{5}mr^2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^*\ 5\frac{deg}{s} \times \frac{\pi\,rad}{180\,deg} = 0.0873\ rad/s \qquad \underline{\underline{T_{tot} = 150\,J}}$$

15. What is the natural frequency in rad/sec of a one-dimensional massless spring of stiffness 20 N/m, fixed to ground, with a 5 kg point mass attached as shown in the figure?

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{20\,N/m}{5\,kg}} = \sqrt{4} = \underline{\underline{2\ rad/s}}$$



MAE 4060
Fall 2025
Homework 2: Vectors, Orbit Preliminaries, and a few more things you should already know (but you can look them up if you don't)

Vector Kinematics

Consider three orthonormal basis vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ that comprise coordinate system N. Let $\mathbf{a} = 3\hat{\mathbf{i}} + -14\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$.

1. What is the projection of $\boldsymbol{a}$ onto $\hat{\jmath}$?
2. Find ${}^{N}a$.
3. Calculate $\|\boldsymbol{a}\|$.
4. Show that $\boldsymbol{a} \times \boldsymbol{a} = \boldsymbol{0}$ (i.e. don't merely claim that this fact is axiomatic)

1. $\vec{a} = 3\hat{\imath} - 14\hat{\jmath} + 5\hat{k}$

projection onto $\hat{\jmath}$ is just $\vec{a}\cdot\hat{\jmath} = -14$

2. $\vec{a}$ is already written in terms of the orthonormal basis N, so

$$^{N}a = \begin{bmatrix} \vec{a}\cdot\hat{\imath} \\ \vec{a}\cdot\hat{\jmath} \\ \vec{a}\cdot\hat{k} \end{bmatrix} = \begin{bmatrix} 3 \\ -14 \\ 5 \end{bmatrix}$$

3. $\|\vec{a}\| = \sqrt{3^2 + (-14)^2 + 5^2} = 15.1658$

4.
$$\vec{a}\times\vec{a} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -14 & 5 \\ 3 & -14 & 5 \end{vmatrix}$$

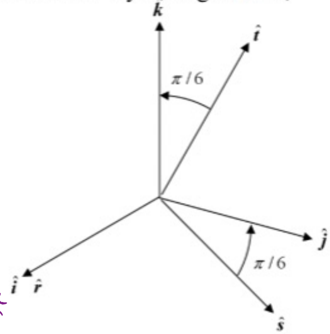
$$= \hat{\imath}\left[5(-14) - 5(-14)\right] - \hat{\jmath}\left[3(5) - 3(5)\right] + \hat{k}\left[3(-14) - 3(-14)\right]$$

$$= 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k} = \vec{0}$$

So, $\vec{a}\times\vec{a} = 0$

Consider three other orthonormal basis vectors $\hat{\boldsymbol{r}}, \hat{\boldsymbol{s}}, \hat{\boldsymbol{t}}$ that comprise coordinate system M. Note that $\hat{\mathbf{r}}$ here is simply a basis vector and does not represent any sort of position vector in an orbit-mechanics problem. N is rotated relative to M by a rotation about the axis $\hat{\boldsymbol{i}}$ by an angle $\pi/6$, as shown in the figure. Let $\boldsymbol{b} = 3\hat{\boldsymbol{r}} + -4\hat{\boldsymbol{s}} + 5\hat{\boldsymbol{t}}$

5. Write down $\hat{\boldsymbol{j}}$ in terms of $\hat{\boldsymbol{r}}, \hat{\boldsymbol{s}}, \hat{\boldsymbol{t}}$
6. Find ${}^{M}\boldsymbol{a}$
7. Calculate $\boldsymbol{a} \times \boldsymbol{b}$ and represent the answer in M axes
8. Let $\boldsymbol{c} = \boldsymbol{b} \times \boldsymbol{a} - 5\hat{\boldsymbol{t}}$. Find ${}^{N}\boldsymbol{c}$
9. Calculate $\boldsymbol{a} \cdot \boldsymbol{b}$



$\vec{a} = 3\hat{i} - 14\hat{j} + 5\hat{k}$

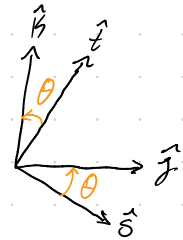
$N \triangleq \{\hat{i}, \hat{j}, \hat{k}\}$; $M \triangleq \{\hat{r}, \hat{s}, \hat{t}\}$; $\hat{i} = \hat{r}$

	$\hat{i}$	$\hat{j}$	$\hat{k}$
$\hat{r}$	1	0	0
$\hat{s}$	0	$\cos\theta$	$-\sin\theta$
$\hat{t}$	0	$\sin\theta$	$\cos\theta$

$\theta = \frac{\pi}{6}$

So reading down the $\hat{j}$ column, we see that:

$$\hat{j} = \cos\theta\,\hat{s} + \sin\theta\,\hat{t}, \quad \theta = \frac{\pi}{6}$$



$$\boxed{\hat{j} = \cos\left(\frac{\pi}{6}\right)\hat{s} + \sin\left(\frac{\pi}{6}\right)\hat{t} = \frac{\sqrt{3}}{2}\hat{s} + \frac{1}{2}\hat{t}}$$

6. Using the DCM above, we can replace $\hat{i}$, $\hat{j}$, and $\hat{k}$ to write in terms of the M basis:

$$^{M}\vec{a} = 3\hat{r} - 14(\cos\theta\,\hat{s} + \sin\theta\,\hat{t}) + 5(-\sin\theta\,\hat{s} + \cos\theta\,\hat{t})$$

$$= 3\hat{r} - (14\cos\theta + 5\sin\theta)\hat{s} + (-14\sin\theta + 5\cos\theta)\hat{t}, \quad \theta = \frac{\pi}{6}$$

So,

$$^{M}a = \begin{bmatrix} \vec{a}\cdot\hat{r} \\ \vec{a}\cdot\hat{s} \\ \vec{a}\cdot\hat{t} \end{bmatrix} = \begin{bmatrix} 3 \\ -14\cos\left(\frac{\pi}{6}\right) - 5\sin\left(\frac{\pi}{6}\right) \\ -14\sin\left(\frac{\pi}{6}\right) + 5\cos\left(\frac{\pi}{6}\right) \end{bmatrix} = \begin{bmatrix} 3 \\ -\frac{14\sqrt{3}+5}{2} \\ \frac{-14+5\sqrt{3}}{2} \end{bmatrix}$$

7.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{r} & \hat{s} & \hat{t} \\ 3 & -14\cos\left(\frac{\pi}{6}\right)-5\sin\left(\frac{\pi}{6}\right) & -14\sin\left(\frac{\pi}{6}\right)+5\cos\left(\frac{\pi}{6}\right) \\ 3 & -4 & 5 \end{vmatrix}$$

$$=\hat{r}\left[5(-14)\cos\left(\tfrac{\pi}{6}\right)-5(5)\sin\left(\tfrac{\pi}{6}\right)-(4)(14)\sin\left(\tfrac{\pi}{6}\right)+5(4)\cos\left(\tfrac{\pi}{6}\right)\right]$$

$$-\hat{s}\left[15+(3)(14)\sin\left(\tfrac{\pi}{6}\right)-3(5)\cos\left(\tfrac{\pi}{6}\right)\right]+\hat{t}\left[-12+(3)(14)\cos\left(\tfrac{\pi}{6}\right)+3(5)\sin\left(\tfrac{\pi}{6}\right)\right]$$

$$=\left[-50\cos\left(\tfrac{\pi}{6}\right)-81\sin\left(\tfrac{\pi}{6}\right)\right]\hat{r}-\left[15+42\sin\left(\tfrac{\pi}{6}\right)-15\cos\left(\tfrac{\pi}{6}\right)\right]\hat{s}+\left[-12+42\cos\left(\tfrac{\pi}{6}\right)+15\sin\left(\tfrac{\pi}{6}\right)\right]\hat{t}$$

$$\boxed{{}^{M}\vec{a}\times{}^{M}\vec{b} = {}^{M}\begin{bmatrix} -50\cos\left(\frac{\pi}{6}\right)-81\sin\left(\frac{\pi}{6}\right) \\ 15\cos\left(\frac{\pi}{6}\right)-42\sin\left(\frac{\pi}{6}\right)-15 \\ 42\cos\left(\frac{\pi}{6}\right)+15\sin\left(\frac{\pi}{6}\right)-12 \end{bmatrix}}$$

8. $\vec{b}\times\vec{a} = -\vec{a}\times\vec{b}$;

$$\vec{b}\times\vec{a}-5\hat{t} = {}^{M}\begin{bmatrix} 50\cos\left(\frac{\pi}{6}\right)+81\sin\left(\frac{\pi}{6}\right) \\ 15+42\sin\left(\frac{\pi}{6}\right)-15\cos\left(\frac{\pi}{6}\right) \\ 7-42\cos\left(\frac{\pi}{6}\right)-15\sin\left(\frac{\pi}{6}\right) \end{bmatrix}$$

$\hat{r} = \hat{i}$

$\hat{s} = \cos\left(\frac{\pi}{6}\right)\hat{j}-\sin\left(\frac{\pi}{6}\right)\hat{k}$

$\hat{t} = \sin\left(\frac{\pi}{6}\right)\hat{j}+\cos\left(\frac{\pi}{6}\right)\hat{k}$

Convert to the N basis:

$${}^{N}\vec{b}\times{}^{N}\vec{a}-5\hat{t} = \left[50\cos\left(\tfrac{\pi}{6}\right)+81\sin\left(\tfrac{\pi}{6}\right)\right]\hat{i}+\left[15+42\sin\left(\tfrac{\pi}{6}\right)-15\cos\left(\tfrac{\pi}{6}\right)\right]\left[\cos\left(\tfrac{\pi}{6}\right)\hat{j}-\sin\left(\tfrac{\pi}{6}\right)\hat{k}\right]$$

$$+\left[7-42\cos\left(\tfrac{\pi}{6}\right)-15\sin\left(\tfrac{\pi}{6}\right)\right]\left[\sin\left(\tfrac{\pi}{6}\right)\hat{j}+\cos\left(\tfrac{\pi}{6}\right)\hat{k}\right]$$

$${}^{N}\vec{b}\times{}^{N}\vec{a}-5\hat{t} = {}^{N}\begin{bmatrix} 81\sin\left(\frac{\pi}{6}\right)+50\cos\left(\frac{\pi}{6}\right) \\ -15+15\cos\left(\frac{\pi}{6}\right)+7\sin\left(\frac{\pi}{6}\right) \\ -42-15\sin\left(\frac{\pi}{6}\right)+7\cos\left(\frac{\pi}{6}\right) \end{bmatrix}$$

9.

$$\boxed{\vec{a}\cdot\vec{b} = 9 - 31\cos\left(\frac{\pi}{6}\right) + 90\sin\left(\frac{\pi}{6}\right)}$$

Orbit Mechanics Preliminaries

10. Calculate the magnitude of the angular momentum of the Earth about the sun. Report the answer in SI mechanical-engineering units, i.e. Nms. Find the relevant constants in any trustworthy reference(s) and include citation(s) with your answer.
11. What is the answer to problem 10 using the conventional units for orbit mechanics?
12. Calculate the acceleration due to gravity on an object 6378.137 km above the Earth's mass center.
13. Calculate the acceleration due to gravity on an object 42,000 km above the Earth's mass center.
14. What is the potential energy of an object 42,000 km above the Earth's mass center in units of m^2/s^2?
15. What is the total energy of an object 42,000 km above the Earth's mass center in units of m^2/s^2?

10.

$M_E = 5.9722e24$ kg

$\|\vec{v}_E\|_{max} = 30.29$ km/s

$\|\vec{r}$ @ max $v\| = 147.095e6$ km

$\vec{h} = \vec{r} \times M_E \vec{v}$

$$\boxed{|\vec{h}| = 2.66092e40 \text{ kg}\cdot\text{m/s}}$$

Citation: NASA Earth Fact Sheet

11. Mass normalize: $\boxed{|h| = 4.4555e15}$

12. $F_g = -\frac{\mu m_B}{r^2} = ma \Longrightarrow a = -\frac{\mu}{r^2}$ $\quad \mu_E = 3.986e14 \text{ m}^3/\text{s}^2$, $r = 6378.137$ km $\quad \boxed{a = -9.7983 \text{ m/s}^2}$

13. $a = \frac{\mu_E}{r^2}$; $r = 42000$ km ; $\boxed{a = -0.22596 \text{ m/s}^2}$

14. $PE = -\frac{G m_A m_B}{r}$; Mass normalized: $PE = -\frac{\mu}{r}$; $\boxed{PE = 9.4905e6 \text{ m}^2/\text{s}^2}$

15. $E = \frac{1}{2}v^2 - \frac{\mu}{r} \Longrightarrow \boxed{E = \left(\frac{1}{2}v^2 - 9.4905e6\right) \text{m}^2/\text{s}^2}$

Not specified

Orbit Topics

Consider a circular solar orbit at a distance of 1,000,000 km.

1. Calculate the orbit's eccentricity.
2. Calculate its specific energy.
3. Calculate its period.

Consider an Earth orbit of a=7000km and e=0.2

4. Calculate the mean motion.
5. Calculate the radius at perigee.
6. Calculate the velocity at apogee.

Solar Orbit @ $r = 1\times10^9\,m$

1. Eccentricity

$$\boxed{e = 0,\ \text{circular}}$$

2. Specific Energy

$$V_{circ} = \sqrt{\frac{\mu_{sun}}{r}} = \sqrt{\frac{GM_{sun}}{r}} = \sqrt{\frac{1.32\times10^{20}}{1\times10^9}} = \underline{3.64\times10^5\ m/s}$$

$$\frac{1}{2}v^2 - \frac{\mu}{r} = \boxed{E = -6.64\times10^{10}\ m^2/s^2}$$

3. Period

$$T^2 = (2\pi)^2\frac{r^3}{\mu} = 17248\ s \times \frac{1\ min}{60\ s}\times\frac{1\ hr}{60\ min}\times\frac{1\ day}{24\ hr} = \boxed{0.200\ days = T}$$

Earth Orbit $a = 7000$ km and $e = 0.2$

4. Mean motion, n

$$n^2a^3 = \mu \Longrightarrow n = \sqrt{\frac{\mu}{a^3}} = \boxed{0.001078\ rad/s = n}$$

5. r_p

$$r_p = a - c \qquad e = \frac{c}{a} \rightsquigarrow c = ea = 1.4\times10^6\,m$$

$$r_p = 7000\times10^3\,m - 1.4\times10^6\,m = 5.6\times10^6\,m = \boxed{r_p = 5600\ km}$$

6. Velocity @ apogee

$$r_a = 2a - r_p = \underline{8.4\times10^6\,m}$$

$$V_a = \sqrt{\mu\left(\frac{2}{r_a} - \frac{1}{a}\right)}$$

$$\boxed{V_a = 6161\ m/s}$$

7. Calculate the velocity at perigee of a spacecraft in a parabolic Jovian (i.e. Jupiter) orbit whose closest approach to the planet's center is 100,000 km.

8. Calculate a, e, E, n, and the orbital period for a spacecraft in an equatorial Mars orbit that remains above the same point on the surface, like a geostationary Earth-orbiting spacecraft.

7.
$e = 1$ in parabolic orbit

$$a = \frac{r_p}{1-e} \rightarrow \infty, \text{ so } V_p = \sqrt{\mu\left(\frac{2}{r_p} - 0\right)} = \boxed{50339 \text{ m/s}}$$

8. Find $a, e, E, n,$ and T for a spacecraft in equatorial Mars orbit

$\mu = 4.28 \times 10^{13} \text{ m}^3/\text{s}^2$; $\boxed{e = 0}$ for geostationary orbit

$\boxed{T = 88642 \text{ s}}$

$$a = \sqrt[3]{\frac{\mu T^2}{4\pi^2}} = \boxed{20428 \text{ km} = a}$$

$$n = \sqrt{\frac{\mu}{a^3}} = \boxed{7.09 \times 10^{-5} \text{ rad/s}}$$

$$E = -\frac{\mu}{2a} = \boxed{-1.048 \times 10^6 \text{ m}^2/\text{s}^2}$$

9. Discuss why such an orbit is difficult or impossible to achieve for a spacecraft in orbit around Earth's moon.

The sphere of influence of the moon in an Earth-Moon system is:

$$R_{SOI} \simeq a_{Moon}\left(\frac{\mu_{Moon}}{\mu_{Earth}}\right)^{2/5} = \underline{66194 \text{ km}}$$

The radius of the orbit is larger than the SOI of the moon, so Earth's gravity will dominate.

The rest of the questions assume an Earth orbit.

10. What are the semimajor axis and eccentricity of an orbit whose radius at perigee is 6700 km and whose radius at apogee is 7000 km?
11. For an orbit with e=0.5 and a=42,000 km, at what values of true anomaly is the radial distance of the spacecraft from Earth's center equal to 54,000 km? What is the value of r when true anomaly is 90°?
12. What is the inclination of a prograde polar orbit?
13. What is the specific energy of an orbit whose radius at perigee is 7,000 km and whose eccentricity is 0.25?
14. Consider two orbits with a=10,000 km and e=0.2. What is the difference in specific energy (which we have written as curly E) between such an orbit with i=0.3 rad and another with i=0.1 rad?
15. While working at APL over the summer, an MAE 4060 student observes a piece of space debris traveling toward the Earth and calculates its position and velocity in ECI (SMAD's **x**, **y**, **z** coordinate system) to be **r**=34205.52**x**+-21532.42**y**+-14872.82**z** km and **v**=2.970126**x**-0.220070**y** -1.291448**z** km/sec. Does the debris hit the Earth?
16. Compute the six classical orbital elements (as defined in SMAD) for the space debris described in problem 6 at the time of the observation.

10.

$$a = \frac{r_p + r_a}{2} = \boxed{6850 \text{ km}}$$

$$e = \frac{r_a - r_p}{r_a + r_p} = \boxed{0.0219}$$

11. $$r = \frac{a(1-e^2)}{1+e\cos\phi} \Rightarrow \cos\phi = \frac{\left(\frac{a(1-e^2)}{r} - 1\right)}{e} \Rightarrow \boxed{\phi = 146.44°}$$

$$\phi = 90° \Rightarrow r = \frac{a(1-e^2)}{1+e\cos\phi} \Rightarrow \boxed{r = 31500 \text{ km}}$$

12. $\boxed{i = 90° \text{ for a prograde orbit}}$

13. $r_p = 7000$ km ; $e = 0.25$

$$r_p = a(1-e) \Rightarrow a = \frac{r_p}{1-e} = \boxed{9333 \text{ km} = a}$$

$$E = -\frac{\mu}{2a} = \boxed{2.135e7 \ m^2/s^2}$$

14. No difference b/c specific energy isn't dependent upon inclination.

15 and 16

```
mu = 3.986e14;
r = [34205.52e3;-21532.42e3;-14872.82e3];
v = [2.970126e3;-0.220070e3;-1.291448e3];
k = [0;0;1];
i = [1;0;0];
j = [0;1;0];

E = (((norm(v))^2)/2) - (mu/(norm(r)));

h = cross(r,v);

a = -mu/(2*E)
```

```
a =
4.9998e+07
```

```
e_vector = ((cross(v,h))/mu) - (r/norm(r))
```

```
e_vector = 3×1
   -0.8254
    0.0000
    0.3589
```

```
norm_e = norm(e_vector)
```

```
norm_e =
0.9000
```

```
n = cross(k,h);

i_rad = acos(dot(k,h)/(norm(h)));
i_deg = (180/pi)*i_rad
```

```
i_deg =
23.5000
```

```
omega = acos(dot(n,e)/(norm(n)*norm_e));
omega_deg = (180/pi)*omega
```

```
omega_deg =
89.9980
```

```
u_omega = acos(dot(i,n)/norm(n));
u_omega_deg = (180/pi)*u_omega
```

```
u_omega_deg =
90.0012
```

```
phi = acos(dot(e_vector,r)/(norm_e*norm(r)));
phi_deg = (180/pi)*phi
```

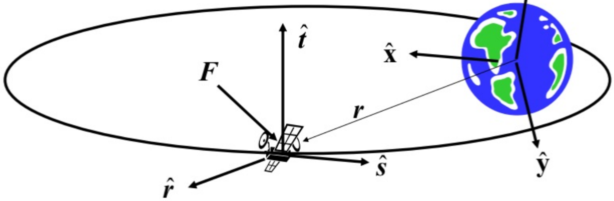
```
phi_deg =
150.0034
```

```
rp = a*(1-norm_e)
```

```
rp =
4.9989e+06
```

```
% rp < rEarth, so it WILL collide with Earth
% Orbit is elliptical bc e < 1
```

MAE 4060
Fall 2025
Homework 4: Perturbations



Simple Perturbations for R2BP orbits

1. Which orbital elements change as a result of radial force, i.e. A_r in the notation of Lecture 6?
2. Which orbital elements change as a result of tangential force, i.e. A_s in the notation of Lecture 6?
3. Which orbital elements change as a result of normal force, i.e. A_t in the notation of Lecture 6?
4. Consider a circular, equatorial, Earth orbit with r=a=20,000 km. A 10N thruster fires for 10 seconds in the $\hat{\mathbf{s}}$ direction. Use the appropriate perturbation equation to estimate the change in semimajor axis for a 100 kg spacecraft that results from this pulse (in meters)
5. Consider the same circular, equatorial, Earth orbit, i.e. with r=a=20,000 km. The same10N thruster fires for 10 seconds but in the $\hat{\mathbf{t}}$ direction. Use the appropriate perturbation equation to estimate the change in inclination for a 100 kg spacecraft that results from this pulse (in degrees).

1.

a, e, ω

2.

a, e, ω

3.

i, Ω, ω

4. $e=0$; $a=20000\,km$ $\quad \vec{F}\cdot\hat{s}=10N$

$m=100\,kg$, so $A_s = \frac{\vec{F}}{m}\cdot\hat{s}$

$A_s = 0.1\,N/kg$, $\mu_E = 3.986\times10^{14}\,m^3/s^2$

$$\dot{a} = 2\left[\frac{a^3}{\mu(1-\cancel{e^2}^{\,0})}\right]^{1/2}\left[\cancel{A_r e \sin\theta}^{\,0} + A_s(1+\cancel{e\sin\theta}^{\,0})\right],\quad e=0$$

$$\dot{a} = 2A_s\sqrt{\frac{a^3}{\mu}} \qquad \dot{a} = 895.99\,m/s\,(10\,s) = \boxed{8959.9\,m}$$

5. Now, $A_t = 0.1\,N/kg$

$$\dot{i} = \frac{r\cos(\omega+\theta)}{(\mu a(1-e^2))^{1/2}}A_t = A_t\sqrt{\frac{a}{\mu}} = 2.240e\text{-}5\,rad/s \times \frac{180^\circ}{\pi\,rad}\times 10s$$

$$\boxed{0.0128^\circ}$$

More Interesting Perturbations for R2BP Orbits

1. (2 points) Cornell's Alpha Cubesat spacecraft weighs approximately 1 kg. Assume its drag coefficient is 2.2 and that it is initially in a 600 km altitude, circular, equatorial orbit. For this point in the solar cycle and at this altitude, use $\rho = 1.04 * 10^{-13}$ kg/m3. What is the rate of change in its orbit's semimajor axis and eccentricity due to atmospheric drag?
2. (2 points) Use the tables in SMAD to estimate how long the Alpha Cubesat will stay in orbit (i.e. how long before it reenters due to drag). These tables are posted as a PDF alongside this homework assignment on Canvas.
3. (2 points) Consider a 1000 kg, square solar sail 1 km on each side and in a circular, heliocentric orbit the at a radius of 1 A.U. With the sail's surface facing the sun, what is the instantaneous rate of change in the spacecraft's semimajor axis and eccentricity due to solar radiation pressure?
4. (2 points) Redo problem (3) with the only difference that the spacecraft is orbiting the sun at Mercury's mean radial distance.

5. (2 points) At a particular time, a 10 kg spacecraft in Earth orbit is located at $\mathbf{r} = 28494803\hat{\mathbf{x}} + 14247401\hat{\mathbf{y}}$ m (in ECI). Its velocity is $\mathbf{v} = 1099.397951\hat{\mathbf{x}} - 2221.00596\hat{\mathbf{y}} + 222.100596\hat{\mathbf{z}}$ m/s. It is transmitting a 10 kW radio-frequency signal in a narrow beam toward a receiver located at $\mathbf{d} = 5698961\hat{\mathbf{x}} + 2849480\hat{\mathbf{y}} + 284948\hat{\mathbf{z}}$ m. At this instant, what is the rate of change in each of the first five classical orbital elements due to this transmission?

6. (2 points) Consider the same spacecraft as in Problem 5 but in an elliptical Mars orbit with the RF transmitter pointed always at the center of Mars. The orbit is equatorial, with periapsis at 1.000 km altitude and apoapsis at 10,000 km altitude. What is the instantaneous rate of change of the semimajor axis due to its RF photon force at the instant when ϕ=90 deg?

1. $m = 1\ \text{kg}$; $C_d = 2.2$; $e = 0$; $alt = 600\ \text{km}$

$a = alt + r_E \quad a = 6978\ \text{km}$

$\vec{F}_d = -\frac{1}{2}\frac{\rho C_d A}{m} v^2 \hat{s} \quad v = \sqrt{\frac{\mu}{a}}$

$A = 0.01\ m^2$

$\vec{F}_d = -6.535\times10^{-8}\ \hat{s}\ N$

So, $A_s = -6.535\times10^{-8}\ N/kg$

$\dot{a} = 2A_s\sqrt{\frac{a^3}{\mu}} = \boxed{-1.207\times10^{-4}\ m/s}$

$\dot{e} = \sqrt{\frac{r}{\mu}}\left[\cancel{A_r \sin\phi}^{\,0} + 2A_s\cos\phi\right]$

ϕ undefined for circular orbit. But we define as $\phi = 180°$ $\boxed{\dot{e} = 1.729\times10^{11}\ 1/s}$

2. Need $BC = \frac{m}{C_d A} = 45.45\ kg/m^2$ use 50 in table.

For 600 km, our orbit life is 1613 days @ Solar min and 941 days @ Solar max

$\boxed{T_{avg} = 1277\ \text{days}}$

3. $m = 1000\ kg$

$A = 1.0\times10^6\ m$

$r = 1\ AU$

$e = 0$

Solar Sail

$\dot{a}$ and $\dot{e}$

$P = \frac{W}{c} \quad W_{1AU} = 1366\ Watts/m^2$

$P = \frac{W}{c} = 4.5\times10^{-6}\ Pa$

$\vec{F}_{solar} = 2P_{solar}A\hat{r} = 9.11\ N\ \hat{r}$

$A_r = \frac{\vec{F}\cdot\hat{r}}{m} = 0.00911 = A_r \quad A_s = 0$, so $\boxed{\dot{a} = 0}$

$\dot{e} = \sqrt{\frac{r}{\mu}}\left[A_r\sin\phi + \cancel{2A_s\cos\phi}^{\,0}\right]$

$\dot{e} = \sqrt{\frac{r}{\mu}}A_r = \boxed{3.06\times10^{-7}\ /s}$

4. $r = 0.39\,AU$, $W = \frac{L}{4\pi r^2}$ $\quad L = 3.828e26$ $\quad W = 20637$, $P = \frac{W}{c} = 60.6\times10^{-6}\,N/m^2$

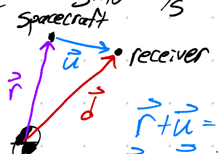
$\vec{F}_{solar} = 2PA = 121.2\,\hat{r}\,N$ $\quad$ Again $A_s = 0$, so $\boxed{\dot{a} = 0}$

$$\dot{e} = \sqrt{\frac{r}{\mu}}\,A_r = \boxed{2.541\times10^{-6}\,/s}$$

5. $^{E}\vec{r} = {}^{E}\begin{bmatrix} 28494803 \\ 14247401 \\ 0 \end{bmatrix} m$ $\quad$ $^{E}\vec{v} = {}^{E}\begin{bmatrix} 1099.39795 \\ -2221.00596 \\ 222.100596 \end{bmatrix} m/s$ $\quad$ $P = 10\,kW$ radio sig. toward a receiver located @:

$$^{E}\vec{d} = {}^{E}\begin{bmatrix} 5698961 \\ 2849480 \\ 284948 \end{bmatrix} m$$

$$F = \frac{T}{c} = \frac{10\times10^3\,W}{3\times10^8\,m/s} = 3.33\times10^{-5}\,N$$



$\vec{r} + \vec{u} = \vec{d}$

$\vec{u} = \vec{d} - \vec{r}$

$$^{E}\vec{u} = {}^{E}\begin{bmatrix} -22795842 \\ -11397921 \\ 284948 \end{bmatrix} m \qquad ^{E}\hat{u} = \begin{bmatrix} -0.89437 \\ -0.447185 \\ 0.0111796 \end{bmatrix}$$

$$^{BST}\hat{F} = {}^{BST}\begin{bmatrix} -0.9999 \\ -2.912e-3 \\ 1.1114e-2 \end{bmatrix}$$

$$^{BST}\vec{F} = \underbrace{\|F\|}_{3.33e-5}\,{}^{BST}\hat{F}$$

$$\boxed{\begin{aligned} \dot{a} &= -1.844e-6\,m/s \\ \dot{e} &= 1.269e-12\,1/s \\ \dot{i} &= 1.491e-11\,rad/s \\ \dot{\omega} &= 1.313e-9\,rad/s \\ \dot{\Omega} &= -1.065e-12\,rad/s \end{aligned}}$$

6. $r_{Mars} = 3396\,km$, $\mu_{Mars} = 4.283e13\,m^3/s^2$

$$a = \frac{r_A + r_p}{2} = 8896\,km;\quad e = \frac{r_a - r_p}{r_a + r_p} = 0.5058$$

$$\vec{F} = 3.33e-5\,N\,\hat{r}$$

$$\boxed{\dot{a} = 0.158\,m/s}$$

Rocket Equation

HW 5

1. Assume that the gun-style launch of a spacecraft described in Jules Verne's *From the Earth to the Moon* involves an impulsive ΔV that results in a parabolic orbit. Assume that the spacecraft is launched in a prograde orbit from earth's equator. Neglect the so-called "gravity losses" during this virtually instantaneous launch, and neglect atmospheric drag. How much ΔV is imparted?
2. Further assume that the impulsive force on the satellite-as-bullet is imparted during a 2 second explosion and that the force is of constant magnitude during those 2 seconds. What acceleration (m/s^2) is felt by the occupants of Verne's launch vehicle?
3. Assume that the fuel for this ΔV is initially contained within the launch vehicle. In fact, assume that 85% of the total vehicle mass is initially propellant. If the launch into parabolic orbit uses all of this fuel, what is the required propellant exhaust velocity?
4. A lunar-exploration spacecraft carrying two astronauts blasts off from its landing site on the moon's equator into an 1850 km radius, circular, lunar orbit. Note that the moon spins with a period of 27.32 days. The spacecraft's mass at liftoff is 4500 kg. What is the propellant mass required if the spacecraft uses Aerozine-50, a propellant with approximately I_{sp}=300 sec?

1. ΔV to get $e=1$ $\quad V_{esc} = \sqrt{\frac{2\mu}{r}} = 11154\ m/s$

↳ Final V needed

V_0 is rotation of E

$V_0 = \omega_E r_E = 73.8\ m/s$ $\quad \boxed{\Delta V = 1.108e4\ m/s}$

2. $t = 2\ s$

$a = \frac{\Delta V}{\Delta t} = \boxed{5540\ m/s^2}$

3. $0.85 m_0 = m_p$

$\Delta V = V_e \ln\left(\frac{m_0}{0.15 m_0}\right)$ $\quad \boxed{V_e = 5.65e3}$

4. $T = 27.32\ \text{days}\left(\frac{24\ \text{hours}}{1\ \text{day}}\right)\left(\frac{60\ \text{min}}{1\ \text{hr}}\right)\left(\frac{60\ s}{1\ \text{min}}\right) = 2360448\ s$

$V_0 = \omega_{moon} r_{moon} = 4.62\ m/s$ $\quad \Delta V_1 = \sqrt{\mu\left(\frac{2}{r_{moon}} - \frac{1}{(r_m + r_{orb})/2}\right)} - \omega_{moon} r_{moon} = 1.702e3\ m/s$

$\Delta V_2 = \sqrt{\frac{\mu_m}{r_{orb}}} - \sqrt{\mu\left(\frac{2}{r_{orb}} - \frac{1}{(r_m + r_{orb})/2}\right)} = 25.758\ m/s$ $\quad \Delta V_{tot} = \Delta V_1 + \Delta V_2 = 1.7275e3$

$m_p = m_0\left[1 - \exp\left(-\frac{\Delta V}{g_0 I_{sp}}\right)\right]$ $\quad \boxed{m_p = 1998\ kg}$

Maneuvers

Questions 5-10 assume Earth orbit.

5. How much ΔV does a spacecraft require to transition from a GTO orbit (r_p=6700 km, r_a=geostationary altitude) into a circular geostationary orbit in the same plane, using only one instantaneous burn?
6. What is the minimum ΔV required for a spacecraft to transition from a LEO orbit (r_a= r_p=6700 km) to a geosynchronous orbit in the same plane?
7. What is the minimum ΔV required for a spacecraft to escape from a GTO orbit (r_p=6700 km, r_a=42164km)? At what point in the orbit should this maneuver occur to minimize the ΔV?
8. How much ΔV does a spacecraft in an a=8,000, e=0.1 equatorial orbit require to change its inclination to i=28°? At what point in the orbit should this maneuver occur to minimize the ΔV? Note that there is no requirement on the change in other orbital elements.
9. How much ΔV is available to a spacecraft with a dry mass of 1000 kg and carrying 9000 kg of beryllium-oxygen/hydrogen tripropellant, which is alleged to have an I_{sp} of about 705 seconds?
10. Consider a 10 kg CubeSat (spacecraft + propellant) on the surface of the asteroid Ceres. It has a novel type of propulsion, one that uses the surface regolith as propellant, impelling it through a nozzle with an electric motor. What mass-flow rate is required for the spacecraft's on-board rocket engine to cause the spacecraft to lift off the surface—just barely, without accelerating significantly—for the case of Isp=25 seconds?

5.

$r_p = 6700\text{ km};\ r_a = 42164\text{ km}$

$$\Delta V = \sqrt{\frac{\mu_E}{r_{GEO}}} - \underbrace{\sqrt{\mu_E\left(\frac{2}{r_{GEO}} - \frac{1}{a}\right)}}_{v_i}$$

$v_f = v_{GEO}$

$$a_{GTO} = \frac{6700\text{ km} + 42164\text{ km}}{2} = 24432\text{ km}$$

$$\boxed{\Delta V = 1465\ m/s}$$

6. $r_{LEO} = 6700$ km

$$\Delta V_1 = \sqrt{\mu\left(\frac{2}{r_{LEO}} - \frac{1}{(r_{LEO}+r_{GEO})/2}\right)} - \sqrt{\frac{\mu}{r_{LEO}}} = 2419\ m/s$$

$$\Delta V_2 = \sqrt{\frac{\mu}{r_{GEO}}} - \sqrt{\mu\left(\frac{2}{r_{GEO}} - \frac{1}{(r_{GEO}+r_{LEO})/2}\right)} = 1465\ m/s$$

$$\Delta V = \Delta V_1 + \Delta V_2 = \underline{3884\ m/s} = \Delta V$$

For one burn, apply @ periapsis. $\Delta V = \sqrt{\mu\left(\frac{2}{r_{LEO}} - \frac{1}{a_{GEO}}\right)} - \sqrt{\frac{\mu}{r_{LEO}}} = \boxed{3753\ m/s}$

7. $V_{esc} = \sqrt{\frac{2\mu}{r_p}} - \sqrt{\mu\left(\frac{2}{r_p} - \frac{1}{a_{GTO}}\right)} = \boxed{775.39\ m/s}$ Do @ periapsis to minimize ΔV

8. $i_{eq} = 0°$, $i_f = 28°$, $\Delta i = 28°$ Like a torque, so apply @ largest radius, r_a

$r_a = a(1+e) = 8800\text{ km};\ V_a = \sqrt{\mu\left(\frac{2}{r_a} - \frac{1}{a}\right)} = 6385\ m/s$

$$\Delta V^2 = 2V_a^2 - 2V_a^2\cos(28°) = \boxed{3039\ m/s}$$

9. $M_p = 9000$ kg
$M_o = 10{,}000$ kg
$I_{sp} = 705$ s

$$\Delta V = g_o I_{sp} \ln\left(\frac{10000}{1000}\right) \quad \boxed{\Delta V = 15908\ m/s}$$

10.

$M_{tot} = 10\ kg$

$\dot{m} = ?$

$I_{sp} = 25\ s$

$F = \dot{m} g_0 I_{sp}$ $\quad$ $F = \frac{\mu M}{r^2}$

$\boxed{\dot{m} = 0.0114\ kg/s}$

Interplanetary Trajectories, Flyby Maneuvers, and Spheres of Influence HW 6

1. Calculate the Sphere of Influence of a 1,000,000 kg asteroid at a point where it is 1,000 km from the Earth's surface. Perform the same calculation at a point 100,000 km from the Earth's surface. Ignore the presence of the sun.

2. The stars Alpha Centauri A and Alpha Centauri B form a binary pair. The average distance between them is 11 AU. A is about 1.1 times the mass of our sun. B is about 0.91 times the mass of our sun. At what point between these two stars would a spacecraft feel equal gravity from both (i.e. perfectly balanced gravity)? Ignore the effects of Proxima Centauri.

1. $M_E = 5.97e24$ kg

$r = h + r_E = 7738$ km

$R_{SOI} = r\left(\frac{m_A}{M_E}\right)^{2/5}$

$\boxed{R_{SOI} = 2.278e{-4} \text{ km}}$

For $r = 106378$ km, $\boxed{R_{SOI} = 3.284e{-3} \text{ km}}$

2. $r_{avg} = 11$ AU, $m_A = 1.1 m_s$, $m_B = 0.91 m_s$

$$\frac{\cancel{G}m_A}{r^2} = \frac{\cancel{G}m_B}{(11AU - r)^2} \Longrightarrow \frac{1.1}{r^2} = \frac{0.91}{(11AU - r)^2} \Longrightarrow 1.1(11AU - r)^2 - 0.91r^2 = 0 \quad \boxed{r = 5.76 \text{ AU from A}}$$

Consider a spacecraft in an orbit around a planet identical to Earth. Its orbit is 500 km altitude, circular, and equatorial. This planet has a moon identical to Earth's in every way except that it is in a circular orbit with the orbital radius equal to the semimajor axis of Earth's lunar orbit. Also, this planet's moon is in an equatorial orbit.

3. Calculate the details of a Hohmann transfer (two delta-V maneuvers) that sends this spacecraft to the moon's orbit. Neglect the gravity of the moon and its position—just calculate what it would take to reach a point on that circular lunar orbit.

4. Now, consider a case in which the spacecraft enters the moon's sphere of influence when the moon is exactly along the line of apsides of the Hohmann ellipse. Starting with the parameters of this Hohmann transfer, calculate the orbital elements of the spacecraft's lunar orbit at this point (i.e. before the second Hohmann burn, where the spacecraft's position relative to the moon is r_{SOI} from the moon's center.) The figure shows this point as a red dot.

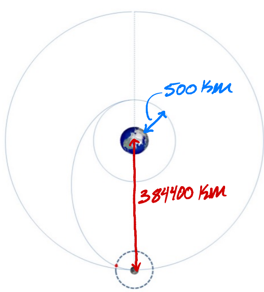


Figure 1. Problem 4

3.

$V_1 = \sqrt{\frac{\mu}{r_1}}$; $V_1 = 7617$ m/s

$V_2 = \sqrt{\mu\left(\frac{2}{r_p} - \frac{1}{a}\right)}$ $\quad e = 0.9649$

So $a = 195630$ km

$V_2 = 10676$ m/s

$\boxed{\Delta V_1 = 3059 \text{ m/s}}$

$V_3 = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)} = 190.7$ m/s $\quad V_4 = \sqrt{\frac{\mu}{r_{tot}}} = 1018.3$ m/s $\quad \boxed{\Delta V_2 = 827.6 \text{ m/s}}$

So, $\Delta V_{tot} = \Delta V_1 + \Delta V_2$ $\quad \boxed{\Delta V_{tot} = 11.504 \text{ km/s}}$

4. To find $\vec{r}$ and $\vec{v}$... $(x, y) = (a\cos t, b\sin t)$, $b = a\sqrt{1-e^2}$

$(x_{moon}, y_{moon}) = (-a, 0) \Longrightarrow \sqrt{(a\cos t + a)^2 + (b\sin t - 0)^2} = R_{SOI} \Longrightarrow t = 135.7°$

$\vec{r} = a\cos t\,\hat{i} + b\sin t\,\hat{j} = -1.4e8\,\hat{i} + 3.587e7\,\hat{j}$

$\vec{r} - \vec{r}_{moon} = 5.548e7\,\hat{i} + 3.587e7\,\hat{j}$ $\quad r = \frac{a(1-e^2)}{1 + e\cos\theta} = 3.305e7$ m

$\theta = 2\arctan\left(\sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)\right) = 173.8°$ $\quad V = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} = 610$ m/s

$$\hat{V} = \frac{-a\sin t}{\sqrt{a^2(1-e^2)\cos^2 t + a^2\sin^2 t}}\hat{i} + \frac{b\cos t}{\sqrt{a^2(1-e^2)\cos^2 t + a^2\sin^2 t}}\hat{j} = -0.9655\hat{i} + 0.2605\hat{j}$$

$$\vec{V} - \vec{V}_{moon} = -5895\hat{i} + 1178\hat{j}$$

So we use these to solve for the 6 orbital elements.

$\boxed{i = 0} \Rightarrow$ Equatorial

$\boxed{a = -3.092e-6\ m}$ $\boxed{\Omega = \text{Und.}}$

$\boxed{e = 22.23}$ $\boxed{\omega = 0^\circ}$ $\boxed{\theta = 302.9^\circ}$

<u>Launch Vehicles</u>

THIS MATERIAL WILL NOT BE ON THE COMING QUIZ. IT IS MEANT TO REFINE YOUR UNDERSTANDING OF THIS WEEK'S LECTURES.

Circle the letter corresponding to the correct answer

5. After a spacecraft separates from the launch-vehicle upper stage using a transverse-spin tip-off,
   a. its angular momentum is equal in magnitude and opposite in direction to the launch vehicle's angular momentum
   b. its angular velocity is equal in magnitude and opposite in direction to the launch vehicle's angular velocity
   c. its angular velocity is less than 30 RPM in magnitude
   d. its angular velocity is zero
6. A launch-vehicle fairing is generally designed to
   a. separate from the launch vehicle at the same time as the spacecraft
   b. protect the spacecraft from incident solar heating until spacecraft separation
   c. maintain a constant-pressure environment for the spacecraft until spacecraft separation
   d. separate from the launch vehicle without contacting the spacecraft
7. Coupled-loads analysis is
   a. A finite-element analysis of the loads on the spacecraft due to the coupled effect of gravity acting downward on the launch vehicle and constant thrust acting upward on the spacecraft.
   b. a way to estimate the pressure forces due to venting through the fairing as the rocket leaves the atmosphere, using finite-difference methods.
   c. an analysis of the thermal loads on a spacecraft due to rocket-induced vibrations.
   d. A finite-element analysis of loads experienced by both the spacecraft and the launch vehicle due to rocket-induced vibration.
8. Which of the following goals cannot be accomplished by staging?
   a. Increasing the mass of payload to orbit.
   b. Improving the reliability of the launch vehicle.
   c. Reducing the dry mass of the launch vehicle's final stage.
   d. Creating an SSTO launch vehicle.

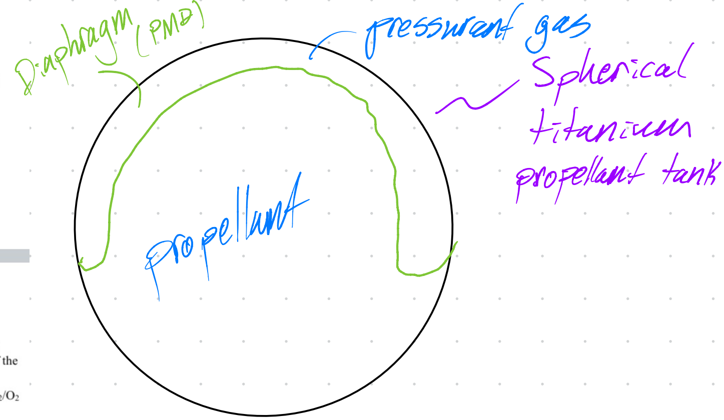
Big-Picture Questions

THIS MATERIAL WILL NOT BE ON THE COMING QUIZ. IT IS MEANT TO REFINE YOUR UNDERSTANDING OF THIS WEEK'S LECTURES.

9. Choose one of the following three NASA spacecraft: Dawn, Orion, or JIMO. Write one paragraph defending its choice of propulsion technology on the basis of its operational environment and the technology's performance (thrust, system mass, volume, and risk/reliability).

10. Write one paragraph explaining what type of propellant tanks would be appropriate on a high-precision Earth-observation spacecraft in LEO. Defend your choice on the basis of the considerations offered in SMAD (or elsewhere).

11. Write one paragraph explaining the trade-offs between cold-gas propulsion and liquid H_2/O_2 propulsion for ~ 200 m/s worth of delta-V maneuvers in lunar orbit.



9

I chose NASA's Dawn spacecraft. The Dawn spacecraft was designed for a deep space mission and for long durations, meaning a high delta-V was needed but low thrust could be tolerated. Thus, ion propulsion was used with Xenon propellant. The Xenon is ionized and accelerated, providing an extremely low thrust (on the order of mN). However, Xenon's specific impulse is high, which allows it to impart a very large delta-V over long durations, fitting in with the mission's goals. The propellant mass required for Dawn's mission was significantly lower than a chemical alternative, and the majority of the propellant system mass comes from power processing electronics rather than propellant tanks. Since the thrusters themselves are rather small, the spacecraft has a good volume efficiency for the mission. Ion engines are reliable over long missions as they have a relatively simple, low-risk ignition and stable performance.

10

One appropriate propellant tank for a high-precision Earth-observation spacecraft in LEO is a spherical titanium tank. A spherical tank eliminates very large stress concentrations that arise at sharp corners in pressure vessels, and titanium has a high strength relative to its weight. Therefore, the walls of the tank can be thin, which reduces the overall mass. Because of the spherical symmetry, fitting the tank into the spacecraft will not create any mass offset issues that could disturb the high-precision requirements for this spacecraft. A diaphragm or vane type propellant management device (PMD) should be included to manage propellant location during attitude maneuvers, increasing the overall precision of the spacecraft. Using a bladder also reduces center-of-mass slosh, limiting attitude disturbance even further. If hydrazine or another non-cryogenic monopropellant is used, no specialized insulation or cryogenic hardware is required.

11

Since cold gas has a very low specific impulse, the rocket equation requires a very large propellant mass for a delta-V of 200 m/s, while liquid H2/O2 has a very high specific impulse and thus requires significantly less propellant mass. Although H2/O2's high specific impulse doesn't require as much mass, it introduces major system complexity due to the cryogenic nature of both propellants, as cryogenic valves, insulation, high tank pressures, and boil-off management are now required. Cryogenic systems are also of significantly more risk, as these engines must be ignited in orbit, which is not an easy task.