MAE 3260 Final Group Work: Exploring a Satellite System

By:

Mia Paz, Nate Jennewein, Hugo Larios

State Space Modelling of a Satellite

By: Mia Paz

In this project, my focus was first to develop a baseline state space model to represent how a satellite rotates about an axis. This change in orientation over time in response to internal or external torques is defined by attitude dynamics [1]. The satellite was modeled as a rigid body rotating about an arbitrary principal axis (pitch, yaw, or roll), as the general equations for any axis are structurally identical for a single-axis model at this current stage. It is also assumed that the angular motion is limited to small angles, as is typical for controlled spacecraft operations. Thus, linearization of the dynamics is valid for realistically representing satellite behavior. It is also assumed that the moment of inertia J is a constant, which we set later on to 100 kg · m^2.

Governing Physical Model:

Euler's rotation equation of motion for rigid bodies can be used to describe the satellite's rotational motion. Its general form accounts for all three rotational degrees of freedom:

$$\boldsymbol{J}\dot{\omega} + \omega \times (\boldsymbol{J}\omega) = \mathbf{T}$$

where $\boldsymbol{J}$ is the inertia matrix, ω is the angular velocity, and **T** is the applied torque. Because the satellite is rotating about one principal axis, the cross product cancels out so that the general equation reduces to a single second-order differential equation:

$$J\ddot{\theta}(t) = T(t)$$

where $\theta(t)$ represents angular displacement, $\ddot{\theta}(t)$ represents angular acceleration, T(t) is the externally applied torque input, and J is the principal-axis moment of inertia.

State Space Form:

To convert the second-order rotational equation into first-order state space form, I defined the following states:

$$x_1 = \theta(t),\ \ x_2 = \dot{\theta}$$

$$\dot{x}_1 = x_2,\ \ \dot{x}_2 = \ddot{\theta}(t) = \frac{1}{J}T(t)$$

We can then obtain the final linear state-space model in Figure 1:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = Ax + Bu \quad , \quad u = T(t)$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} u$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix}$$

Figure 1: Baseline State Space Model

The model output is the angular displacement:

$$\text{output} : \ y = \theta = x_1$$

$$\text{angle} : \ C = [1 \ \ 0] \ , \ D = 0$$

$$y = [1 \ \ 0] \, x$$

Figure 2: Model Output

Block Diagram:

The following figure illustrates a block diagram representation of how a torque input T(t) propagates through the system to produce the attitude response $\theta(t)$.

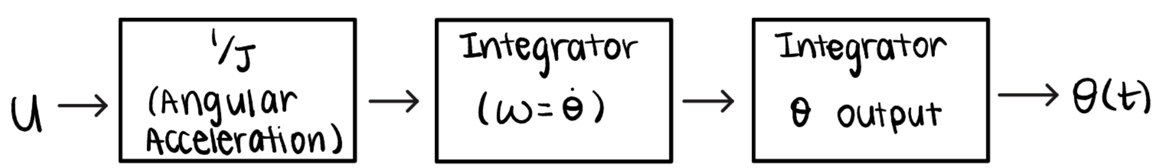

Figure 3: Block Diagram

Figure 3 demonstrates how torque generates the angular acceleration through $\frac{1}{J}$. Angular acceleration integrates into angular velocity, which then integrates into angular position. This flow matches the state-space representation and demonstrates why the satellite behaves as a double-integrator system. The $\frac{1}{J}$ block represents the input gain in the B matrix, the integrator blocks correspond to the structure of the A matrix, and the final output represents the C matrix mapping the state to the measurable output. The diagram does not contain a direct path from the input torque to the measured angle, which matches D=0 in the state-space form.

Disturbance Torques Acting on a Satellite:

By: Nate Jennewein

There can be many disturbance torques acting on a satellite when trying to control its angular position with respect to a target on earth or out in space. These come from unequal forces acting on the body of the satellite causing rotation about an axis and possibly steady state error. The main four disturbances that I identified to include in our model were solar radiation pressure, the gravity gradient, the magnetic torque, and aerodynamic drag.

Solar Radiation Pressure:

The momentum of photons is transferred to the large flat solar panels that power the satellite. Our model assumes that A perpendicular is roughly constant over the time of our adjustments. This may be a large approximation, but we needed it to fit the simplified model we are making. Therefore the disturbance torque can be modelled as $T(t) = P_sA$ where P_s is technically a function of the radius of orbit but is roughly equal to 4.5x10^-6 N/m^2 in the orbit range of satellites. Our model also chooses to ignore the radiation reflected and emitted by the Earth as it is negligible compared to that of the sun.

Gravity Gradient:

Our satellite does not orbit close enough to the Earth to assume that pull of gravity is simply a vector pointed towards the center of Earth. The gravity gradient torque expresses these differences as a function of the moments of inertia, radius of orbit, mass of the satellite, and the angle of the satellite with respect to Earth. The gravitational pull on each piece of the satellite is shown in figure 1.

$$d\underset{\rightarrow}{\boldsymbol{f}} = \frac{-GM\ dm}{r^3}\underset{\rightarrow}{\boldsymbol{r}}\ ,\quad \underset{\rightarrow}{\boldsymbol{r}} = \underset{\rightarrow}{\boldsymbol{r}}_{\oplus} + \underset{\rightarrow}{\boldsymbol{\rho}}$$

Figure 1: Differential force on each infinitesimal piece of satellite.

In the equation, M represents Earth's mass. dm is the mass of the piece of satellite. **r** is the vector from Earth's center to the satellite, and r is the scalar of **r**. Taking the cross product of this quantity with the vector of each piece from the center of mass of the satellite times the moment of inertia gives

T(t) = (GMm/r^3) * sin(q)

And using a small angle approximation T(t) = (GMm/r^3) * I *q

Magnetic Field Torque:

The magnetic field torque on a satellite is a product of the onboard electronics creating electromagnetic fields that interact with the electromagnetic fields produced by Earth. The torque is T(t) = m x b, where m is the spacecraft's net magnetic dipole and b is the geomagnetic field vector from Earth. These can all vary wildly with position, so our model must make some large assumptions to fit our simplified state space model. Figure 2 shows **b** is spherical coordinates,

$$
\begin{aligned}
b_r &= -\frac{\partial \Phi}{\partial r} &&= 2\left(\frac{R_\oplus}{r}\right)^3\left[g_1^0\cos(\phi) + \left(g_1^1\cos(\eta) + h_1^1\sin(\eta)\right)\sin(\phi)\right] \\
b_\phi &= -\frac{1}{r}\frac{\partial \Phi}{\partial \phi} &&= \left(\frac{R_\oplus}{r}\right)^3\left[g_1^0\sin(\phi) - \left(g_1^1\cos(\eta) + h_1^1\sin(\eta)\right)\cos(\phi)\right] \\
b_\eta &= -\frac{1}{r\sin(\phi)}\frac{\partial \Phi}{\partial \eta} &&= \left(\frac{R_\oplus}{r}\right)^3\left[g_1^1\sin(\eta) - h_1^1\cos(\eta)\right]
\end{aligned}
$$

Figure 2: magnetic field of earth in spherical coordinates.

with F being the field potential of Earth, R being Earth's radius, r being the height of orbit, and g_n^m and h_x^y representing constants that are obtained from tables. Assuming that our system responds quickly, the quantity in the brackets is essentially a constant that I will call B_0 (~40nT for many satellites). This leaves the disturbance torque $T(t) = mB_0(R_{Earth}/r)^3$

Aerodynamic Drag:

While aerodynamic drag is very small given how thin the atmosphere is in orbit, I believe it still has enough of an effect to include in our model. We will use the simple definition of drag:

$T_{drag} = 0.5 * r * v^2 * A_{perpindicular} * d_{cd}$

d_{cd} is the distance from the center of the drag force to the center of mass.

Adding disturbances to state space model:

These approximations let us expand our state space model so

$T(t) = T_{Gravity} + T_{Solar} + T_{Magnetic} + T_{drag}$

Figure 3 shows our updated state space model with the new disturbance included.

$$
\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{3GMm}{r^3}\frac{I}{I}\theta & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{I} & \frac{1}{I} & \frac{1}{I} \end{bmatrix}\begin{bmatrix} P_s \cdot A_\perp \\ mB_0\left(\frac{R_E}{r}\right)^3 \\ \frac{1}{2}\rho v^2 C_d A_\perp d_{cD} \end{bmatrix}
$$

$$
\begin{bmatrix} \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} P_s \cdot A_\perp \\ mB_0\left(\frac{R_e}{r}\right)^3 \\ \frac{1}{2}\rho v^2 C_d A_\perp d_{cd} \end{bmatrix}
$$

Figure 3: Updated State Space Model with Disturbance Torques

Sensors and Actuators of the Satellite:

By: Hugo Larios

Due to disturbance torques and steady state error, the use of sensors and actuators within the satellite's system is quite important. Should the system overshoot or underperform, it is necessary to have a control loop to correct the system, as any major deviations from the desired angle would provide insufficient data. The potential sensors that could be implemented are the accelerometer and star sensor. The potential actuators used to correct the system could be control moment wheel gyros or solar panels.

Sensors - Accelerometer

An accelerometer offset from the center of mass will sense the rotational accelerations of the satellite's body. A traditional accelerometer has a seismic mass that is attached to a mechanical suspension system withing a rigid frame. When the device accelerates, the mass is then displaced relative to its frame due to the inertia of the movement. It then uses this information to measure the specific force along its sensitive axis, turning this information into electrical signals (piezoelectric specifically) that it can then analyze to determine the objects movement pattern. *Tangential (linear) acceleration due to angular acceleration:*

a_{tan}=dθ_double dot (for a sensor offset distance **d** in the tangential direction).

Centripetal term: $a_{centripetal}$ = dθ_dot^2 (nonlinear)

Sensors - Star Sensor

A star sensor is an optical attitude sensor that images the viewable stars, identifies and maps them, then computes the satellite's absolute attitude in an inertial reference frame. This often creates a very high accuracy for the satellite (arcseconds to arcminutes), since it could reference its position with respect to the stars that are visible around it. This removes the drift that is associated with accelerometers or gyroscopes making it reliable long-term, but can be affected by the Sun's light, or the light reflected off the moon or the Earth. A star sensor in this system would function as an error identification, such as $e_{ss} = |\theta_{desired} - \theta_{true}|$.

Actuators - Control Moment Wheel Gyro

A control moment wheel gyro functions on the principle of conservation of angular momentum. It consists of a spinning flywheel mounted on a single or two-axis gimbal, which is ran at an angular speed by a gimbal motor. When you spin the flywheel within the satellite, the satellite is forced to rotate the opposite way in order to conserve angular momentum. This is used to further correct the attitude of the satellite, and there could be multiple gyros to make the satellite rotate in all three rotational axes (yaw, pitch, roll). The spacecraft experiences an input torque of $T_{gyro} = L * o_{gimbal}$, where L is the angular momentum of the wheel ($L = J * \theta_{dot\_wheel}$) and o_gimbal is the gimbal rate of rotation.

Actuators - Solar Panels

Solar panels don't only have to function as a power source; they can also function as an actuator to control the attitude of the satellite. Two components would allow the solar panel to act as an actuator: 1) the solar panels can orient themselves in such a way that solar radiation pressure from the Sun can provide a net torque on the satellite. 2) The motion of readjusting the solar panels can rotate the satellite through conservation of angular momentum. Because the net torque due to solar radiation pressure is already accounted as a disturbance, we can just focus on the motion of the solar panels to correct the satellite. This can be modeled using $T_{panel}=K_{\alpha}*\alpha$, where K_{α} is the local linear gain $\partial T/\partial\alpha$ evaluated about the operating point. Panel actuator dynamics (simple first order): $\alpha\_dot=-1/\tau_p * \alpha+1/\tau_p * u_p$, where u_p is the commanded panel angle.

Implementing the Sensors and Actuators to the State Space Model

The above approximations can give us the following:

$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{\alpha} \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & 1 & 0 \\ \frac{3GM_m}{r^3} & 0 & \frac{K_\alpha}{I} \\ 0 & 0 & -\frac{1}{\tau_p} \end{bmatrix}} \overset{x}{\begin{bmatrix} \theta \\ \dot{\theta} \\ \alpha \end{bmatrix}} + \overset{B}{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \frac{1}{I} & \frac{1}{I} & \frac{1}{I} & \frac{L}{I} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\tau_p} \end{bmatrix}} \begin{bmatrix} P_s A_\perp \\ mB_0\left(\frac{R_E}{r}\right)^3 \\ \frac{1}{2}\rho v^2 C_d A_\perp d_{CD} \\ L o_{gimbal} \\ u_p \end{bmatrix}$$

$$y = \begin{bmatrix} \dot{\theta} \\ \theta \\ d\ddot{\theta} \end{bmatrix} = \overset{C}{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ d\frac{3GM_m}{r^3} & 0 & d\frac{K_\alpha}{I} \end{bmatrix}} \begin{bmatrix} \theta \\ \dot{\theta} \\ \alpha \end{bmatrix} + \overset{D}{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{d}{I} & \frac{d}{I} & \frac{d}{I} & \frac{dL}{I} & 0 \end{bmatrix}} \begin{bmatrix} P_s A_\perp \\ mB_0\left(\frac{R_E}{r}\right)^3 \\ \frac{1}{2}\rho v^2 C_d A_\perp d_{CD} \\ L o_{gimbal} \\ u_p \end{bmatrix}$$

Figure 1: State Space Model with Sensors and Actuator

References

[1] FAA, “Space Vehicle Control Systems – Chapter 4.3.1,” SSRI Knowledge Base, NASA, Oct. 7, 2021. Available: https://s3vi.ndc.nasa.gov/ssri-kb/topics/28/. [Accessed: Dec. 10, 2025].

[2] Vatankhahghadim, B. *“Lecture 16: Disturbance Torques.”* AER506 — Spacecraft Dynamics and Control, Fall 2019. GitHub Pages, 2019. https://vatankhahghadim.github.io/AER506/Notes/16%20-%20Disturbance%20Torques.pdf.