Executive Summary

1. The function of my design should be a wing sail that can be utilized on future boats of the CUSail Project Team.
2. The sail must not cause the boat to capsize and must remain within the geometry limits imposed by our annual competition.
3. The aim of this sail is to achieve a thrust coefficient higher than that of our existing sail. Additionally, the sail must not structurally fail in any of our expected operating conditions.
4. The final design iterated over the design space of wing sails. Different airfoil shapes, wing spans, and aspect ratios were all considered for the design.
5. Matlab scripts, structural hand calculations, FEA, and CFD were all utilized to refine the final design.
6. The final design was tested against a baseline used for traditional sails used in prior research papers on this topic.
7. ENGRD 2020 - Loading of cantilever beam, Free body diagram analysis
MAE 3270 - Combined loading, Ansys Static Structural simulation
MAE 3240 - Ansys fluent simulation
MAE 4272 - Airfoil optimization
8. Compared to our existing sail options the design poses great improvement. Prior wing sails from the team have numerous failure modes and design flaws that are solved in this design. Other wing sails may be more optimized for particular conditions but lack the versatility offered in thai design.
9. The design provides the project team with a source of propulsion that is far more durable than existing options, allowing for greater research potential over a longer period in harsh conditions. Ocean and lake research has the potential to benefit their respective communities and global scientific understanding.
10. The format of the design is a complete set of CAD drawings, both fluid and structural simulations, and a MATLAB script. The design process given in the report can be followed to construct a design optimized for different environmental conditions.
11. Individual Project.

Introduction

A stated goal of the Cornell University Autonomous Sailboat project team (CUSail) is to complete a trans-Atlantic crossing. To survive the harsh environment of the Atlantic, the boat must be as durable as possible. A key failure point for any sailboat is its form of propulsion, the sail. In a traditional sail, a single tear can lead to complete destruction and loss of power. However, a wingsail eliminates this failure mode. Even in cases where the surface of the wing sail is fabric, a single tear can be isolated to a small region and large portions of the sail are still operable. This key benefit, alongside potential performance improvements, have stemmed the development of a wingsail for CUSail's most recent boat.

However, CUSail's boat still needs to succeed in the International Robotic Sailing (Sailbot) Regatta, and thus will be optimized to perform under these conditions rather than the Atlantic Ocean. The aim of this project is to provide CUSail with a competitive wingsail in competition alongside a parameterized design process that can be reused to design a wingsail for an Atlantic crossing. This design project will draw inspiration from previous studies, alongside fluid and mechanical simulations to design an optimal wingsail.

General Structure

In the past, CUSail has developed several wingsails for their autonomous boats, an example of which is shown in Fig 1. Almost all, unfortunately have suffered from reliability issues or caused excessive heeling. Excessive heeling forces are problematic because they require ballast that encumbers performance. Prior wingsails were driven by a geartrain at the base of the mast and utilized a tail sail to act as an "air rudder". Due to the high torsional forces produced on the mainsail the gear train frequently failed. Additionally, the air rudder was ineffective at turning the boat. This is reflected in industry, where no largely produced autonomous sailboats use this technique. Instead, the tail sail can be used to turn a freely rotating mainsail by producing lift that counteracts the turning moment in the mainsail. This utilizes the lever arm of the tail sail. Thus the mainsail is free to rotate. A FBD of this general structure is shown below.

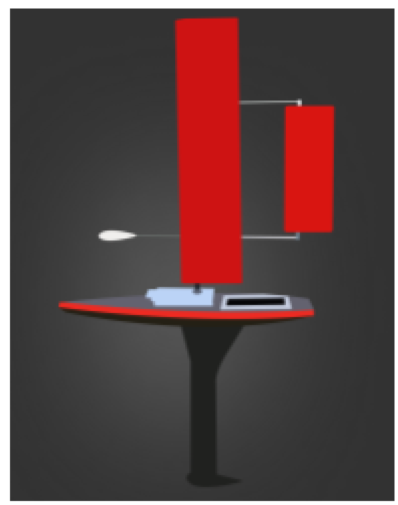


Fig. 1: Picture of SailVane II, 2017

Force Model

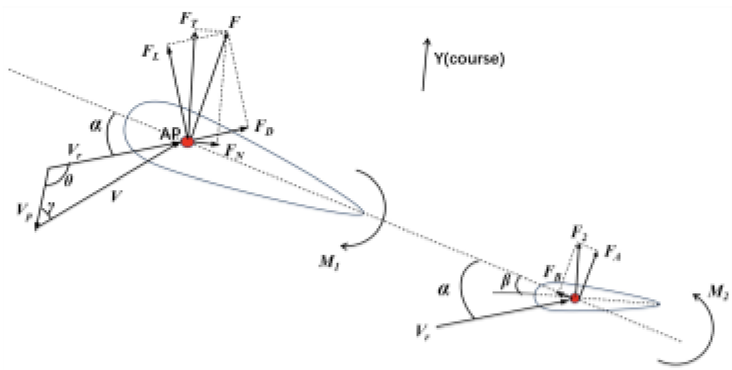


Fig. 2: FBD of wingsail, reference [1]

General		Main Sail		Tail Sail	
V	Environmental wind	α	Angle of Attack	β	Tail Deflection
V_p	Wind from movement	F	Combined force	F_2	Combined force
V_r	Relative Wind	F_L	Lift Force	F_B	F_2 component along chord
γ	Wind angle	F_D	Drag Force	F_A	F_2 component ⊥ F_B
θ	Relative wind angle	F_T	Thrust Force	M_2	Pitching Moment (Clockwise)
		F_N	Side Force		
		M_1	Pitching moment (clockwise)		

Geometry

The design of our rigid sail will utilize the parameters shown in figure 3 below.

General		Main Sail		Tail Sail	
AP	Aerodynamic Center of mainsail	B_1	Span	B_2	Span
H	Height of AP	C_1	Chord	C_2	Chord
L	Distance between AP of main and tail sail	C_n	Tip Chord		
		C_m	Root Chord		

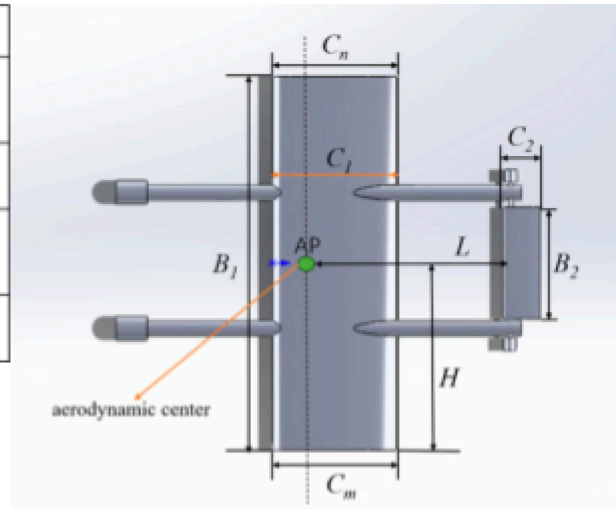


Fig. 3: Structure of wingsail, reference [1]

Note: The model seen able is purely for geometric parameters
and not reflective of the complete design

Metrics, Operating Conditions, and Constraints

Metrics

To analyze performance improvements in the wingsail over our current sail, key parameters such as lift, drag, thrust, and side thrust coefficients are used and defined on the right.

$$C_L = \frac{F_L}{12\rho V_r^2 S_A}$$

$$C_D = \frac{F_D}{12\rho V_r^2 S_A}$$

$$C_T = C_L sin\theta \; - C_D cos\theta$$

$$C_N = C_L cos\theta + \; C_D sin\theta$$

*Theta equal to relative wind angle

Operating Conditions

To establish operating conditions of the wingsail, we will look into the most recent Sailbot regatta. The following quote is provided on the Sailbot website regarding last competition:

"The week provided… sailable breezes (4-11 kts) most times Mon-Wed… Thursday proved more challenging with 5-12 kts sustained but with occasional 20 kt gusts." [2]

From these results, we can identify an average wind speed of 8 kts, equal to 4.12 mps. Additionally, we can determine that the sail must survive wind speeds of 20 kts = 10.29 mps. This is a structural challenge on the wing, but also at these wind speeds the wing must not cause the boat to excessively heel and capsize.

Constraints

To quantify how much heeling moment the boat can withstand a simple test was performed. A digital luggage scale was fastened to the mast of our current boat. Then the scale was pulled perpendicular to the mast until the deck of the boat reached the water line. The digital scale was attached 35 in above the roll axis of the boat, and the digital scale read 3.25 lbs. This produced a maximum heeling moment of 113.75 in-lbs, equal to 12.85 N-m. It should be noted that this test was conducted with no ballast. Ballast was not included as the hull performed best in competition last year without any ballast. Furthermore, this choice implements conservatism into our design as more ballast could be added easily.

Our competition rules regulate total overall height from the lowest underwater point to the highest point on the largest rig to not exceed 5 meters. Sensors and their mountings are not included in the height measurement [2]. This is realistically a non-issue, as our keel only extends 1 meter downwards and the wing sail is not going to exceed 2 meters in span.

Preliminary Design

Airfoil Choice

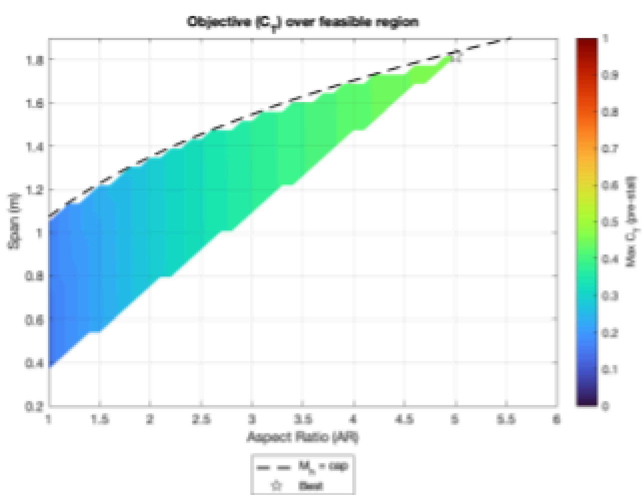
To pick an airfoil that optimizes the thrust coefficient, one must first calculate the Reynolds number: $Re = \frac{\rho v L}{\mu} = \frac{1.21*4.12*0.4}{1.81*10^{-5}} = 110,000$. With the Reynolds number, airfoiltools.com was used to investigate which airfoils would provide an optimal $\frac{C_L}{C_D}$. The results of this investigation are shown in the table below.

Airfoil	Peak $\left\|\frac{C_L}{C_D}\right\|$	AoA at Peak $\left\|\frac{C_L}{C_D}\right\|$	Stall Angle
NACA 0016	20	3°	11°
NACA 0018	38	6.5°	14°
NACA 0024	35	8°	14.5°

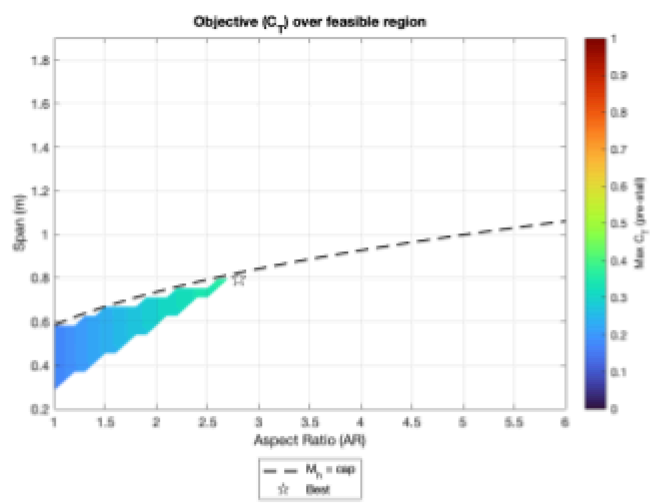
The short range of airfoils shown is largely due to the limited amount of airfoils airfoiltools.com has data on. However, the choice of airfoil shape for this project will be based on maximizing Cl/Cd and stall angle. Thus, the NACA 0018 airfoil was selected for the mainsail. The tailsail has a lower chord length, and thus smaller Reynolds number. Because of this, a NACA 0016 was selected for the tail sail.

Mainsail Span and AR Selection

To choose an appropriate Span and AR, Matlab scripts were utilized as they can be written and iterated upon much faster than other forms of simulation. The Matlab script utilized is named "OptimizeSailCalc" and is provided alongside an explanation in Appendix A. The script provides a mainsail AR and span to maximize the thrust coefficient, and provides a heat map that excludes options that exceed the allowable heeling force. The heat map output and input parameters are shown below. Two conditions were tested, the normal wind speed of 4.12 m/s and our maximum wind speed of 10.28 m/s.



Heatmap from Input 1



Heatmap from Input 2

	Input 1	Input 2
Wind Speed (m/s)	4.12	10.28
Center of Mass Height (fraction of span)	0.5	0.5
Airfoil (NACA)	0018	0018

Figure 4: Matlab heat maps and input table

The gap between these two inputs is quite large and reflects that it will be difficult to have one sail that is capable of performing well in common wind speeds and not cause capsizing in gusts. Thus, a modular approach is opted for, so that the sail area can be increased or decreased depending on the environmental wind speed. For this modular design an AR of 4 is used with a span of 1.6 meters. However, the 1.6 meter sail will be assembled from two 0.8 meter long subsections. In high wind conditions, the span can be reduced to 0.8 meters. At the highest wind speeds this design could still be at risk of capsizing, but more ballast can be inserted to further stabilize the boat. Alternatively, one could instead manufacture and design two separate sails to solve this problem. This approach has been done in the past, but would require at least 50% more material, more design work, additional CFD simulations for different geometries, and a longer manufacturing timeline. By choosing a modular design, some peripheral connections are complicated, but the overall project is simplified.

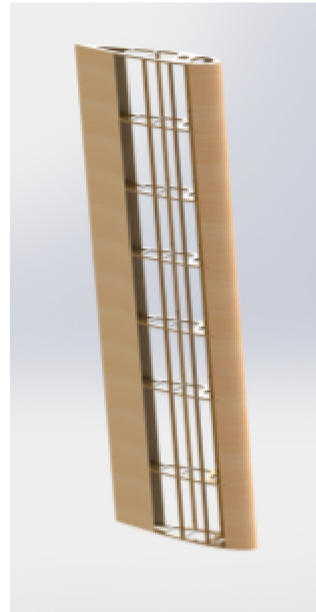


Figure 5: Prior wingsail skeleton

Wing Structural Design

To create the wings needed in this project, two designs were considered at the start of this project. The wingsails could be constructed as a composite structure with a foam core and carbon fiber shell. Alternatively, several cross sections of the airfoil could be wrapped in monokote to create the wing. The composite structure has the potential to be more dimensionally accurate and significantly stronger. However, the costs and equipment associated with that method are unfeasible for CUSail. Thus, the wing will be constructed from several balsa cross sections with a spar to align them. This approach has been used by CUSail for their previous wingsails and performed well. The rendered model of one of these previous wingsails is shown to the right in figure 5. Note that the panels on the trailing and leading edges of the sail ensure the monokote strictly adheres to the airfoil contour as the monokote tends to sink in those regions without additional support.

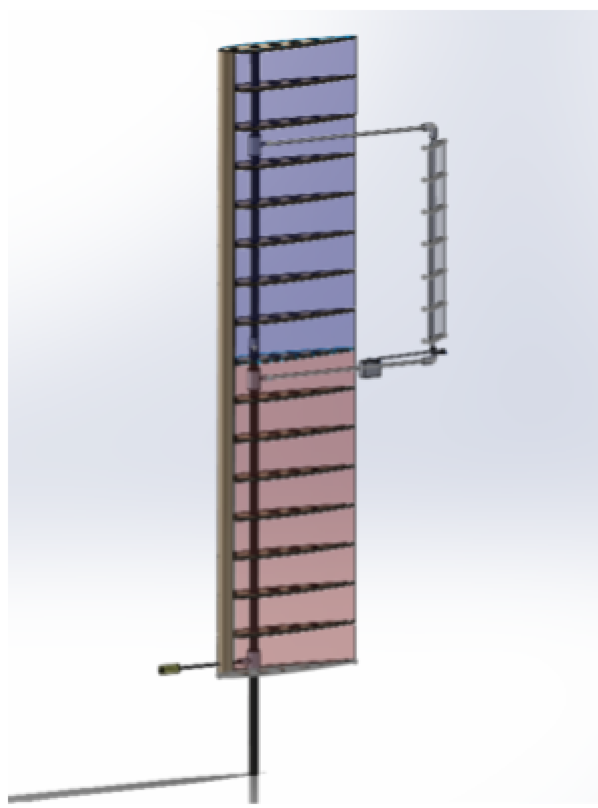


Figure 6: Complete CAD assembly

CAD design

The overall CAD design for this project is shown in figure 6. The two modular sections are highlighted in blue and red, and are connected via a central mast connector. Also on this mast, are connectors that attach the horizontal tubes which hold up the tailsail. Several of these connectors will be glued in place in predetermined positions for proper tail sail positioning in both the full and shortened format of the mainsail. The tubes connect into these connectors by screwing into them with aluminum male threads that are bonded onto the carbon fiber rods as shown in figure 7. In areas where the boom enters the trailing edge, the balsa will need to be cut out. The cuts can also include reliefs to allow the wiring of the servo mechanism to run through the mainsail.

Alongside the movement of the tailsail in different configurations, the servo steering mechanism must also shift as well. This is accomplished simply by attaching the servo to the side of the bottom boom. Using a linkage mechanism, the servo is then capable of manipulating the AoA of the tail sail.

There is a mass counterweight suspended from the front of the mainsail, this is to counteract the weight of the tailsail shifting the COM far from the mast and is sized to compensate for both the weight of the servo and tailsail. Setting the counterweight 230mm in front of the mast, the calculation for the weight is as follows:

$x(230) + 0.05(-300) + 0.04(-500) = 0$, $x = 0.152$ kg

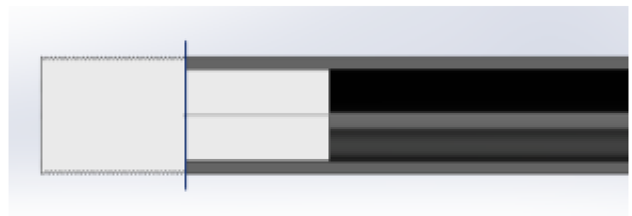


Figure 7: Threads bonded into horizontal connectors

This weight will be machined out of a bronze cylinder for its high density, inexpensive cost, and corrosion resistance. The actual weight will likely need to be fine tuned during manufacturing but an example is shown in the CAD model.

Tailsail Servo mechanism

To direct the tailsail's AoA, a simple servo linkage is utilized as seen in Figure 8. This mechanism was chosen due to its flexibility, as integrating a servo into the mainsail would not be compatible with the modularity requirement.

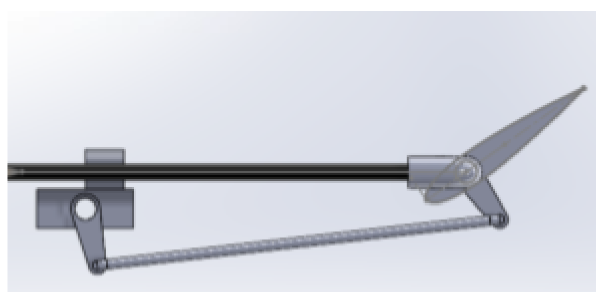


Figure 8: Tailsail Servo Linkage

The servo sized for this is a 40 Kg hobby servo that is readily available and should provide reliable service. The torque recruitment, found in the CFD section below is at a maximum of 2.82 N-m. A 40 Kg servo is rated to sustain a 3.92 N-m load and thus is sufficient for our purpose.

Structural Sizing

Several components on the CAD are significant stressed members that require structural calculations to guide optimal sizing. For this project, we will focus on the structural integrity of the mast, carbon connecting tubes to the tail sail, and balsa cross sections.

Mast:

The mast is modeled as a cantilever beam with hollow circular cross section and a point load at end equal to drag force when perpendicular to wind at 10 mps. A safety factor (SF) of 5 will be used for the mast sizing since it is a critical component of the assembly. To closely meet the desired safety factor, a carbon tube with an 0.878" outer diameter (OD) and 0.750" inner diameter (ID) will be used. The calculations of the maximum bending stress are shown on the right.

$F_D = \frac{1}{2}C_D\rho AV^2$, C_D = 1.28 (Flat plate), ρ = 1.21

A = 1.6 * 0.4 = 0.64 m^2

$F_D \approx 50$ N

F =F_D

$$\sigma_{bending,\,max} = \frac{FLR_o}{\frac{\pi}{64}(D_o^4 - D_i^4)}$$

$$R_o = 0.011\,m,\ D_o = 0.022\,m,\ D_i = 0.019\,m$$

$$\frac{50*1.6*0.0011}{0.049(0.022^4 - 0.019^4)} = \frac{0.088}{5.09*10^{-9}} = 172.8\,MPa$$

The maximum tensile strength of the tubes is 861 MPa, giving us a safety factor of 4.98. Of the available tubes on McMaster Carr, this meets the desired safety factor best by far.

Horizontal Tubes:

The connecting tubes are modeled as a cantilever beam with hollow circular cross section. However, there is a dual loading of the weight of the tail sail, weight of the servo system, and the drag force from the tail sail. All of these loadings are treated as a point load at the end of the beam. In this case, the smallest available sized tube with a .32” OD and ¼“ ID provided a safety factor of 11.3 and thus was chosen.The calculations for maximum principle stress are shown on the right.

$$F_D = \frac{1}{2}C_D\rho AV^2$$

$$A = 0.55 * 0.12 = 0.066$$

F_D = 5.11 N

$W_{tailsail} \approx$ 0.04 kg

$$W_{servo} \approx 0.05\,kg$$

$$W_{total} = 0.09\,kg$$

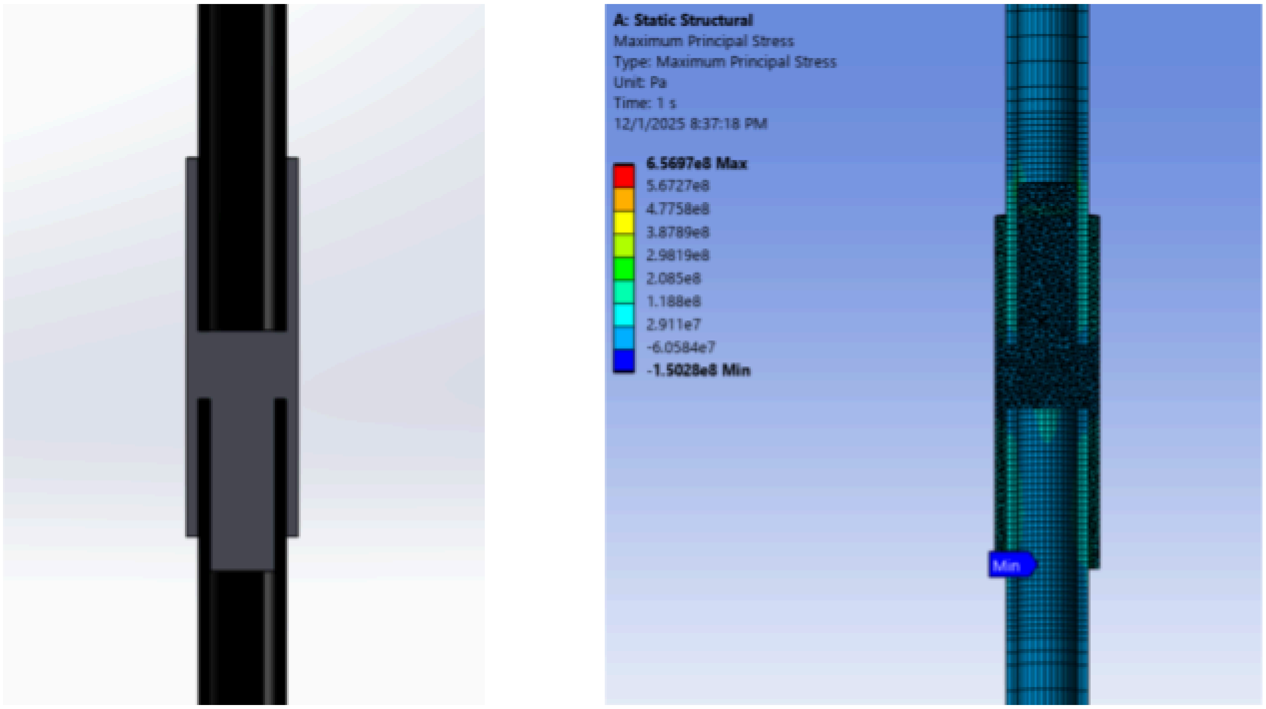
$$M = L * \sqrt{M_w^2 + M_d^2} = 2.52\,Nm$$

$$\sigma_{bending} = \frac{Mc}{I} = 76.1\,MPa$$

$$SF = \frac{862}{76.1} = 11.3$$

FEA

The interface between the upper and lower mast was identified as a possible failure point. To ensure that this connection would not fail during competition FEA was performed to ensure the maximum principle stress would not exceed our desired safety factor of 5. To run the simulation, the connection was slightly simplified for computational efficiency and the corresponding mesh is shown in Figure 9. On this simulation a convergence was performed to ensure that the results were mesh independent. In practice, the simulation actually revealed hand calculation errors which were subsequently revised.



*Model simplified for simulation efficiency

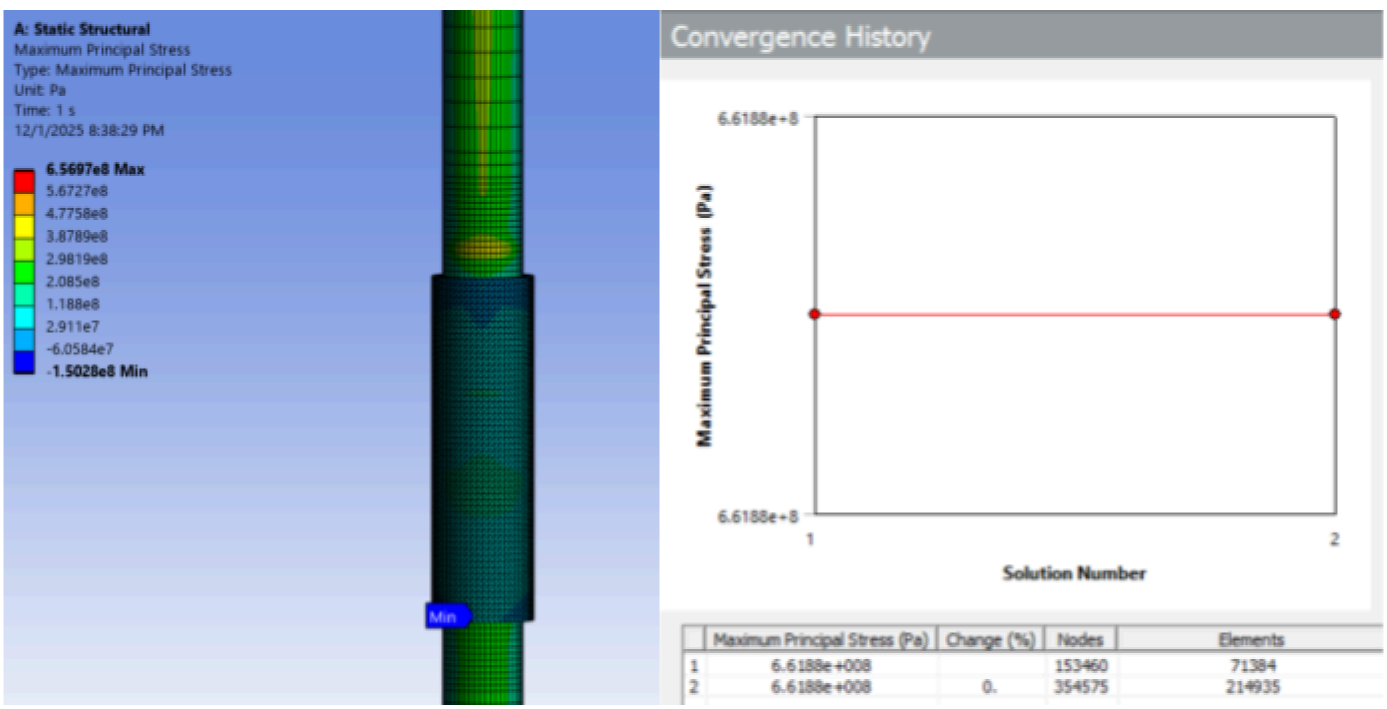


Figure 9: FEA on mast connection

CFD (Ansys Fluent)

The purpose of CFD use on this project is to visualize the flow interaction between the main and tail sail and determine if there are performance losses in the tail sail because of the mainsail wake. Additionally if there are performance losses, we will validate that our tailsail has control authority over the tailsail, meaning that the generated moment is larger. This can be achieved with the 2D model seen in figure 10, where flow is visualized at several angles of attack. The table below shows the data collected from this analysis.

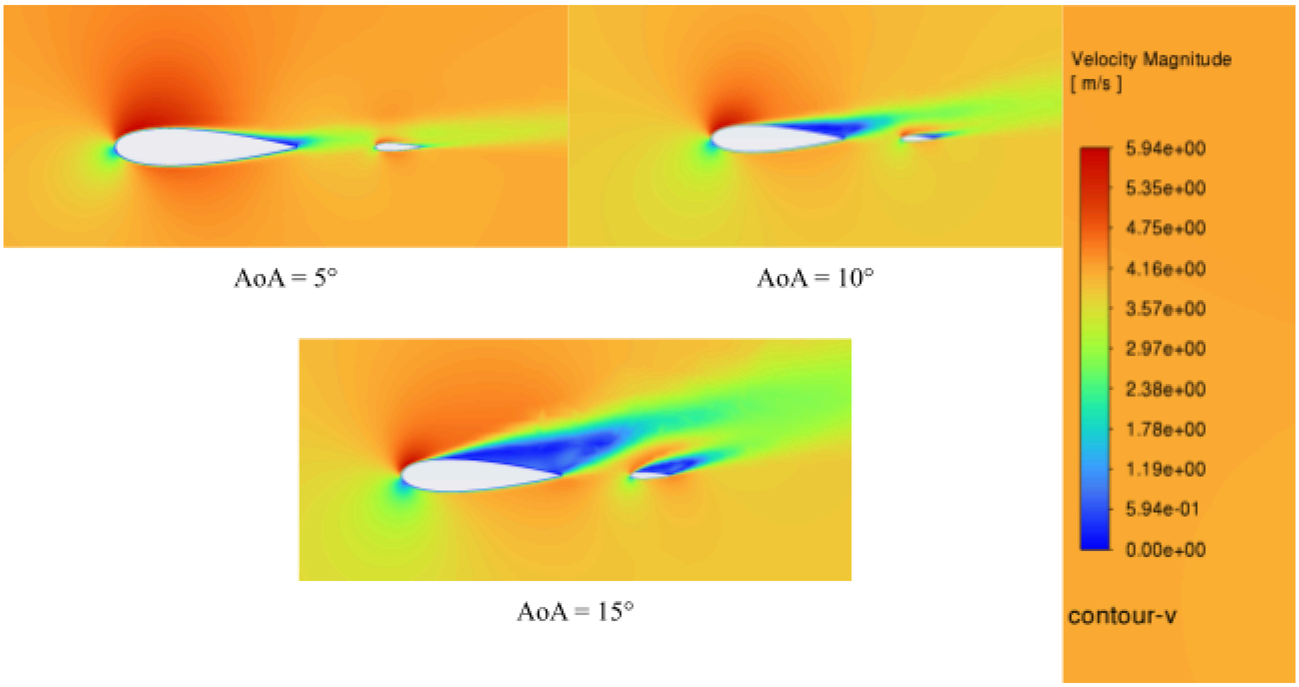


Figure 10: Pressure contours in the 2D model

Ansys Fluent Results							
Wind Speed (m/s)	AoA	Mainsail C_L	Mainsail C_D	Tailsail C_L	Tailsail C_D	Mainsail Moment (Nm)	Tailsail Moment (Nm)
4.12	5°	0.606	0.025	0.372	0.0032	0.149	0.181
4.12	10°	1.06	0.0374	0.82	0.18	0.209	0.404
4.12	15°	1.19	0.121	0.89	0.27	0.150	0.449
10.28	5°	[redacted]				0.85	1.09
10.28	15°	[redacted]				1.01	2.84

From the table above, it is clear that the mainsail has a destructive effect on the performance of the tailsail, as they are the same airfoil yet the coefficient of lift is significantly smaller compared to the mainsail. However, the tailsail is still capable of producing a greater moment at all AoA. The ratio of tailsail moment to mainsail moment is smallest at lower AoA, however in reality the tailsail can be rotated to have a greater AoA than the mainsail. However, to ensure high wind speeds would not cause unexpected loss of control a simulation was conducted at 10.28 m/s where the tailsail still created a larger moment than the mainsail.

Although lift data is not available on our prior soft sail, the table below is populated with common figures for our previous sail type which was a thin film/membrane soft sail [1]. Additionally, more data from the Ansys simulation is also provided.

AoA	Soft Sail C_L	Soft Sail C_d	Soft Sail C_T	Wing Sail C_L	Wing Sail C_D	Wing Sail C_T
2.5°	0.11	0.042	-0.0372	0.282	0.02	-0.0076
5°	0.26	0.059	-0.0361	0.606	0.025	0.0278
7.5°	0.32	0.071	-0.0286	0.865	0.03	0.083
10°	0.39	0.092	-0.0229	1.06	0.0374	0.147
12.5°	0.52	0.121	-0.00564	1.164	0.045	0.207
15°	0.69	0.176	0.0085	1.19	0.121	0.191

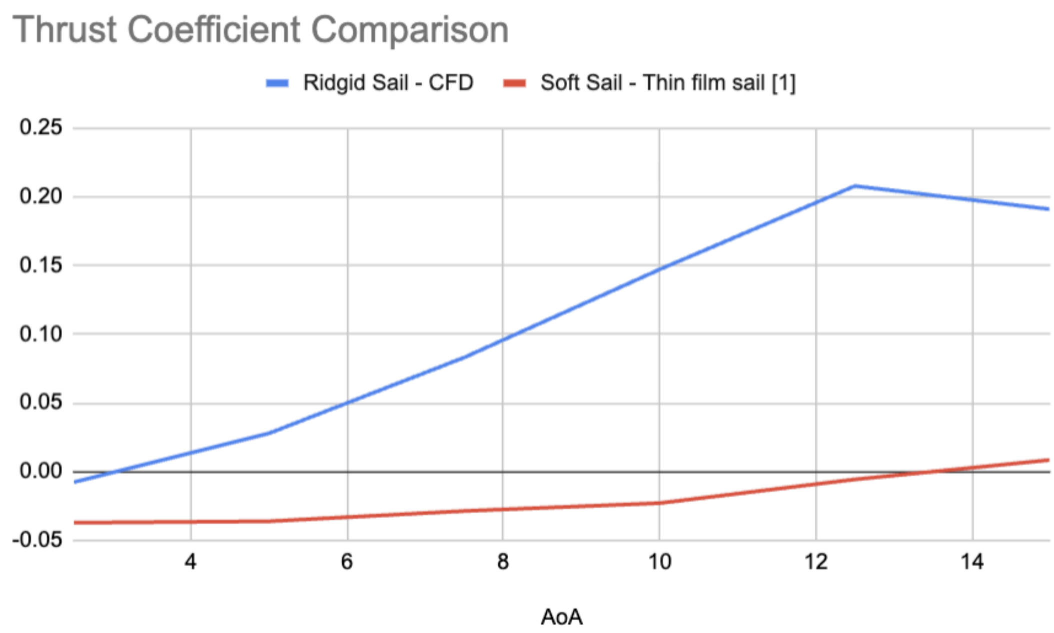


Figure 11: Graph comparing thrust coefficient calculated in CFD for the rigid sail compared to empirical results on thin film sails from Reference 1

Based on the results above, the new wing sail manages to greatly outperform soft sails such as those we previously used. Holistically, the CFD simulation validates that the sail will perform in our expected operating conditions and provide a performance benefit over the existing sail.

Conclusion and Future Work

This project successfully delivered a robust, competition-ready rigid wingsail design for the Cornell University Autonomous Sailboat (CUSail) team that meets both performance and structural requirements while addressing historical reliability issues seen in prior team designs. Through a parameterized design process, the final wingsail achieved a clear improvement in thrust coefficient relative to CUSail's existing soft sail, while remaining within strict heeling, geometric, and structural constraints. The design integrated aerodynamic optimization, structural hand calculations, FEA, and CFD to ensure the sail could withstand expected operating conditions, including gusts up to 20 knots, without structural failure or loss of control. The final parameters are listed in the table below.

	Main Sail Low Wind	Main Sail High Wind	TailSail
Span (m)	1.6	0.8	0.5
AR	4	2	5
Airfoil	NACA 0018	NACA 0018	NACA 0016

Key achievements include the selection of a NACA 0018 airfoil optimized for the relevant Reynolds number regime, the development of a modular span and aspect-ratio configuration that balances performance and stability, and a mechanically simple yet effective main-sail and tail-sail interaction strategy. CFD results confirmed that, despite wake interference from the mainsail, the tailsail retains sufficient control authority to rotate the freely-spinning mainsail under all tested conditions. Structurally, conservative safety factors were achieved for the mast and tail-sail support members, validated through both analytical calculations and finite-element analysis. Collectively, these results provide CUSail with a durable, higher-performance propulsion system and a reusable design methodology that can be adapted for different wind environments and future platforms.

Future work should focus on advancing the autonomy and efficiency of the wingsail system. A primary next step is the implementation of a closed-loop PID controller to regulate sail and tail-sail angles in real time. Such a controller would allow the system to respond smoothly to changing wind conditions, reduce oscillations, and maintain optimal angles of attack for maximum thrust while minimizing heeling. To support this control strategy, additional sensors should be integrated to measure sail position relative to the boat and the apparent wind, such as rotary encoders at the mast, IMU-based orientation sensing, and wind vane or anemometer feedback. These measurements would enable accurate state estimation and significantly improve control robustness.

Finally, further aerodynamic optimization of the wing planform represents a valuable avenue for improvement. While the current design uses a fixed linear taper, additional CFD-driven studies could investigate non-linear taper distributions or spanwise airfoil variation to reduce induced drag and improve efficiency across a wider range of operating conditions. Together, these future enhancements would transform the current wingsail from a high-performance passive design into an actively optimized, intelligent propulsion system capable of supporting CUSail's long-term goals, including extended autonomous missions and eventual ocean-crossing attempts.

Appendix A:

The Matlab function seen below was used to generate the heatmaps seen in figure 4. The function searches over wing span and aspect ratio (AR) for a rigid NACA 0018 wing and picks the design that gives the best pre-stall forward thrust performance, while rejecting any design that violates the following:

- a broadside (90° AoA) heeling moment cap Mh_cap
- a minimum Reynolds number Re_min
- a minimum chord chord_min_m

The function can optimize either maximum thrust coefficient or thrust coefficient per side force. The inputs are as follows

- Wind conditions: V_ms and apparent wind angle beta_deg
- Airfoil thickness ratio t_c (here 0.18)
- Aero assumptions: Oswald efficiency e_oswald, broadside normal force coefficient CnMax
- Heeling model: center of effort height factor k_CE, cap Mh_cap

- Geometry search ranges: span_grid, AR_grid
- Hard constraints: Re_min, chord_min_m, delta_max_frac
- A stall-angle heuristic adjustment rule stall_dAR_rule(AR)

The function then builds a grid search for span and aspect ratio. There are two nested loops, an inner for AR and an outer for span. For each candidate it computes the following:

- chord = span/AR
- S = span*chord (planform area)
- dynamic pressure q = 0.5*rho*V^2
- Reynolds number Re = rho*V*chord/mu

Before doing detailed aero, the function then rejects designs that fail the healing constraint, the minimum Reynolds number floor, and the minimum chord conditions. The aerodynamic model is a finite-wing lift slope, which estimates how the coefficient of lift changes with angle of attack. A stall angle is estimated based on the airfoil thickness, and the drag coefficient is estimated based on the Reynolds number. Finally, the AoA is swept from -stall to +stall to create lift and drag curves. The results from these curves are then used to calculate values for the thrust and side force coefficients plotted in the heat maps.

```
function optimize_ct_naca0024_realistic()
% Optimize span & AR for a NACA airfoil rigid wing, maximizing pre-stall thrust
% while enforcing a 90° broadside heeling cap and practical aero/structural
constraints.
%% ===================== USER SETTINGS (tweak here) ======================
V_ms        = 10.28;         % apparent wind [m/s]
beta_deg    = 45;         % apparent wind angle to boat forward axis [deg]
t_c         = 0.18;       % NACA 0018
e_oswald    = 0.90;       % Oswald efficiency
CnMax       = 2;        % normal-force coeff at 90° AoA
k_CE        = 0.5;        % CE height as fraction of span
Mh_cap      = 12.85;         % N·m (broadside worst-case)
washout_deg = 0.0;        % geometric washout [deg]
% Search space (keep span <= hull limit)
span_max   = 1.90;                             % [m]
span_grid = linspace(0.20, span_max, 41);     % [m]
AR_grid    = linspace(1.0, 6.0, 51);          % [-]
% Constraints
Re_min         = 1.0e5;      % floor for predictable low-Re behavior
chord_min_m    = 0.28;       % min chord for structure/packaging [m]
delta_max_frac = 0.05;       % max tip deflection as fraction of span (e.g., 5%)
% Objective: 'CT'  OR  'CT_over_CY'
objective_mode = 'CT_over_CY';
% Stall loss vs AR: example rule (can be tuned)
% Note: this example *reduces* stall penalty as AR increases a bit.
stall_dAR_rule = @(AR) 1.5 + 3.0./AR;  % [deg]
%% ======================================================================
rho = 1.225; mu = 1.81e-5;
nS = numel(span_grid);
nA = numel(AR_grid);
CTmax_map  = nan(nS,nA);
J_map      = nan(nS,nA);      % objective value
```

```
AoAstar    = nan(nS,nA);
Mh_broad   = nan(nS,nA);
feasible   = false(nS,nA);
for i = 1:nS
   span = span_grid(i);
   for j = 1:nA
       AR    = AR_grid(j);
       chord = span/AR;
       S     = span*chord;
       q     = 0.5*rho*V_ms^2;
       Re    = rho*V_ms*chord/mu;
       % ---------- Hard constraints ----------
       % Worst-case broadside heel
       Fy90  = q*S*CnMax;
       Mh90  = Fy90 * (k_CE*span);
       Mh_broad(i,j) = Mh90;
       if Mh90 > Mh_cap,       continue; end
       if Re   < Re_min,       continue; end
       if chord < chord_min_m, continue; end
       % ---------- Aerodynamics ----------
       % Finite-wing slope
       a0 = 2*pi; % [1/rad]
       a  = a0 / (1 + a0/(pi*e_oswald*AR)); % [1/rad]
       % Stall angle heuristic for 00xx (2D base), then 3D/low-Re/washout
       alpha_stall_2D = 6 + 50*t_c;             % deg (0024 ~ 18°)
       % Reynolds penalty
       if Re < 1e5, dRe = 1.5;
       elseif Re < 2e5, dRe = 0.5;
       else, dRe = 0.0; end
       dAR = stall_dAR_rule(AR);                % deg (example rule)
       alpha_stall_FW = alpha_stall_2D - dAR - dRe + washout_deg;
       alpha_stall_FW = max(alpha_stall_FW, 6); % keep sane
       % Pre-stall sweep
       alpha_deg = linspace(-alpha_stall_FW, alpha_stall_FW, 401);
       alpha_rad = deg2rad(alpha_deg);
       CL_lin = a .* alpha_rad;
       CLmax  = a * deg2rad(alpha_stall_FW);
       CL     = CLmax .* tanh(CL_lin/CLmax);
       % Drag: stronger low-Re penalty on CD0 (exponent 0.5)
       Re_ref    = 2e5;
       CD0_base  = 0.006 + 0.010*(t_c - 0.12).^2;
       CD0       = CD0_base * (Re_ref / max(Re,1e4))^0.5; % << was 0.2
       k_ind     = 1/(pi*e_oswald*AR);
       CD        = CD0 + k_ind .* CL.^2;
       % Coeffs in boat axes
       beta = deg2rad(beta_deg);
       CT   = CL.*sin(beta) - CD.*cos(beta);
       CY   = CL.*cos(beta) + CD.*sin(beta);
       % ---------- Simple structural deflection constraint ----------
```

```
       % Use worst-case *aero* uniform line load based on max side force in
sweep
       Y_max   = max(q*S.*abs(CY));          % [N]
       w       = Y_max / span;               % [N/m], uniform line load
       h_spar  = t_c*chord;                  % spar depth [m] (thickness)
       b_spar  = k_spar_w*chord;             % spar width [m]
       I_rect  = b_spar*h_spar^3 / 12;       % solid rectangle
       I_eff   = k_I_eff * I_rect;           % thin-wall / cutout effectiveness
       E       = E_GPa*1e9;                  % [Pa]
       % Cantilever tip deflection under uniform load: w L^4 / (8 E I)
       delta_tip = w * span^4 / (8 * E * I_eff);
       if delta_tip > (delta_max_frac*span)
           continue; % too flexible
       end
       % ---------- Objectives ----------
       [CTmax, idx] = max(CT);
       CTmax_map(i,j) = CTmax;
       AoAstar(i,j)   = alpha_deg(idx);
       switch lower(objective_mode)
           case 'ct'
               J_map(i,j) = CTmax;
           case 'ct_over_cy'
               epsCY = 1e-3;
               J_map(i,j) = max(CT ./ max(abs(CY),epsCY)); % best CT/CY in
sweep
           otherwise
               error('Unknown objective_mode.');
       end
       feasible(i,j) = true;
   end
end
% Pick best feasible
J_map(~feasible) = -inf;
[J_best, linIdx] = max(J_map(:));
if isinf(J_best)
   error('No feasible design. Loosen constraints (Re_min/chord_min/deflection)
or reduce V/beta.');
end
[i_best, j_best] = ind2sub(size(J_map), linIdx);
% Extract best design and re-evaluate for report
span_best    = span_grid(i_best);
AR_best      = AR_grid(j_best);
chord_best   = span_best/AR_best;
S_best       = span_best*chord_best;
Re_best      = rho*V_ms*chord_best/mu;
q            = 0.5*rho*V_ms^2;
% Rebuild curves for summary
a0 = 2*pi;  a  = a0 / (1 + a0/(pi*e_oswald*AR_best));
alpha_stall_2D = 6 + 50*t_c;
```

```
if Re_best < 1e5, dRe = 1.5; elseif Re_best < 2e5, dRe = 0.5; else, dRe = 0.0;
end
dAR = stall_dAR_rule(AR_best);
alpha_stall_FW = max(alpha_stall_2D - dAR - dRe + washout_deg, 6);
alpha_deg = linspace(-alpha_stall_FW, alpha_stall_FW, 401);
alpha_rad = deg2rad(alpha_deg);
CL_lin = a .* alpha_rad;
CLmax  = a * deg2rad(alpha_stall_FW);
CL     = CLmax .* tanh(CL_lin/CLmax);
CD0_base = 0.006 + 0.010*(t_c - 0.12).^2;
CD0      = CD0_base * (2e5 / max(Re_best,1e4))^0.5;
k_ind    = 1/(pi*e_oswald*AR_best);
CD       = CD0 + k_ind .* CL.^2;
beta = deg2rad(beta_deg);
CT  = CL.*sin(beta) - CD.*cos(beta);
CY  = CL.*cos(beta) + CD.*sin(beta);
[CTmax, idx] = max(CT);
alpha_best = alpha_deg(idx);
% Worst-case broadside
Fy90  = q*S_best*CnMax;
Mh90  = Fy90*(k_CE*span_best);
% Structural numbers at best point
Y_max   = max(q*S_best.*abs(CY));
w       = Y_max / span_best;
h_spar  = t_c*chord_best;
b_spar  = k_spar_w*chord_best;
I_rect  = b_spar*h_spar^3 / 12;
I_eff   = k_I_eff * I_rect;
E       = E_GPa*1e9;
delta_tip = w*span_best^4 / (8*E*I_eff);
%% ----------------------- Report ------------------------
fprintf('\n=== Optimized NACA 0024 (realistic constraints) ===\n');
fprintf('Objective: %s\n', upper(objective_mode));
fprintf('V=%.1f m/s, beta=%.1f deg, CnMax=%.2f, k_CE=%.2f, e=%.2f\n', V_ms,
beta_deg, CnMax, k_CE, e_oswald);
fprintf('Span = %.3f m,  AR = %.2f,  Chord = %.3f m,  Area S = %.3f m^2\n', ...
       span_best, AR_best, chord_best, S_best);
fprintf('Re (chord) ≈ %.0f   |   alpha_stall_FW ≈ %.1f deg\n', Re_best,
alpha_stall_FW);
fprintf('CT_max (pre-stall) = %.3f at AoA ≈ %.1f deg\n', CTmax, alpha_best);
fprintf('Worst-case broadside Mh = %.2f N·m (limit %.1f)\n', Mh90, Mh_cap);
fprintf('Tip deflection ≈ %.1f mm (limit %.1f mm)\n', 1e3*delta_tip,
1e3*delta_max_frac*span_best);
%% ----------------------- Plots -------------------------
% ---- Heatmap (smooth) ----
%CTplot = J_map;                  % or CTmax_map
%CTplot(~feasible) = NaN;         % hide infeasible
CTplot = CTmax_map;               % already masked to -inf for infeasible
%CTplot(CTplot==-inf) = NaN;       % hide infeasible, show only feasible
```

```
figure('Name','Max pre-stall C_T over feasible region','Color','w');
contourf(AR_grid, span_grid, CTplot, linspace(0,1,60), 'LineStyle','none');
set(gca,'YDir','normal'); colormap(turbo(512)); clim([0 1]);
cb = colorbar; cb.Label.String = 'Max C_T (pre-stall)';
grid on; box on;
xlabel('Aspect Ratio (AR)'); ylabel('Span (m)');
title('Objective (C_T) over feasible region');
hold on;
contour(AR_grid, span_grid, Mh_broad, [Mh_cap Mh_cap], 'k--', 'LineWidth',1.4);
plot(AR_best, span_best, 'p', 'MarkerSize',12, 'MarkerFaceColor','w',
'MarkerEdgeColor','k');
legend('','M_h = cap','Best','Location','southoutside');
figure('Name','CT and CY vs AoA (best design)','Color','w');
plot(alpha_deg, CT, 'LineWidth',1.8); hold on;
plot(alpha_deg, CY, 'LineWidth',1.8); grid on;
xlabel('\alpha (deg)'); ylabel('Coefficient');
legend('C_T','C_Y','Location','best');
title(sprintf('Best: span=%.2f m, AR=%.2f, chord=%.3f m', span_best, AR_best,
chord_best));
end
```

References:

[1]: Fang S, Tian C, Zhang Y, Xu C, Ding T, Wang H, Xia T. Aerodynamic Analysis of Rigid Wing Sail Based on CFD Simulation for the Design of High-Performance Unmanned Sailboats. Mathematics. 2024; 12(16):2481. https://doi.org/10.3390/math12162481

[2]: Sailbot. Sailbot International Robotic Sailing Competition, sailbot.org, https://www.sailbot.org/. Accessed 1 Dec. 2025.