1

a) Linear: $F = -k(x - x_0)$

$v = \dot{x} \rightarrow \frac{dv}{dt} = \ddot{x} = \dot{v}$

$\rightarrow m\ddot{x} = -k(x - x_0)$

$\rightarrow \ddot{x} = \frac{-k(x - x_0)}{m}$

$$\rightarrow \begin{cases} \dot{x} = v \\ \dot{v} = \frac{-k(x - x_0)}{m} \end{cases}$$

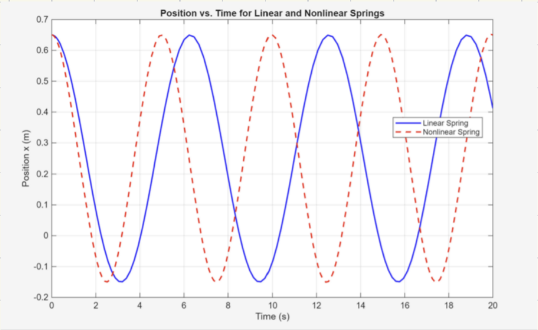
Nonlinear: $F = -k(x - x_0) - c(x - x_0)^3$

$\rightarrow m\ddot{x} = -k(x - x_0) - c(x - x_0)^3$

$\rightarrow \ddot{x} = \frac{-k(x - x_0) - c(x - x_0)^3}{m}$

$$\rightarrow \begin{cases} \dot{x} = v \\ \dot{v} = \frac{-k(x - x_0) - c(x - x_0)^3}{m} \end{cases}$$

b)



c) The difference between the graphs of the two functions demonstrates how nonlinear stiffness changes the frequency and shape of the response, even without damping.

②

a) $\ddot{x} = a - \frac{k}{m}x$

$\rightarrow \ddot{x} + \frac{k}{m}x = a$

LHS: $\ddot{x} + \frac{k}{m}x = 0$

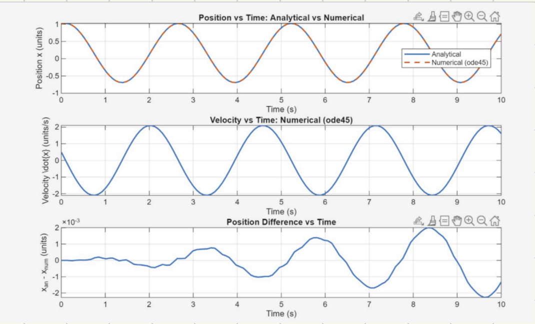
$\rightarrow x_c = c_1 \cos\left(\sqrt{\frac{k}{m}}\,t\right) + c_2 \sin\left(\sqrt{\frac{k}{m}}\,t\right)$

RHS: $0 + \frac{k}{m}x_p = a$

$\rightarrow x_p = \frac{am}{k}$

$\rightarrow x(t) = \frac{am}{k} + c_1 \cos\left(\sqrt{\frac{k}{m}}\,t\right) + c_2 \sin\left(\sqrt{\frac{k}{m}}\,t\right)$, $\dot{z} = \begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ a - \frac{k}{m}x \end{bmatrix}$

b)



c) The analytical and numerical solutions agree almost perfectly, with the velocity showing the expected sine-wave behavior and a very small, bounded position due to error

3

Frame A: $r_{p/Q} = 3\hat{a}_1 + 4\hat{a}_2$

Frame B: $\hat{b}_1 = \cos 30^\circ \hat{a}_1 + \sin 30^\circ \hat{a}_2$

$\hat{b}_2 = -\sin 30^\circ \hat{a}_1 + \cos 30^\circ \hat{a}_2$

$\rightarrow 3\cos 30^\circ + 4\sin 30^\circ = \dfrac{4 + 3\sqrt{3}}{2}$

$\rightarrow -3\sin 30^\circ + 4\cos 30^\circ = \dfrac{4\sqrt{3} - 3}{2}$

$\rightarrow r_{p/Q} = \dfrac{4 + 3\sqrt{3}}{2}\hat{b}_1 + \dfrac{4\sqrt{3} - 3}{2}\hat{b}_2$

4

$F_1 = 2\hat{e}_x - 5\hat{e}_y \, N, \; F_2 = 10(\cos\theta \hat{e}_x + \sin\theta \hat{e}_y) N, \; F_3 = (5 - 4\sqrt{6})\hat{e}_y N$

$\rightarrow F_1 = \langle 2, -5 \rangle, \; F_2 = \langle 10\cos\theta, 10\sin\theta \rangle, \; F_3 = \langle 0, 5 - 4\sqrt{6} \rangle$

$\rightarrow 2 + 10\cos\theta + 0 = 0$

$\rightarrow \cos\theta = -\frac{1}{5}$

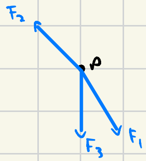
$\rightarrow \theta = 101.537^\circ$

$\rightarrow -5 + 10\sin\theta + 5 - 4\sqrt{6} = 0$

$\rightarrow \sin\theta = \dfrac{2\sqrt{6}}{5}$

$\rightarrow \theta = 78.463$

$F_2 = 10(\cos\theta \hat{e}_x + \sin\theta \hat{e}_y) = 10\left(-\frac{1}{5}\hat{e}_x + \frac{2\sqrt{6}}{5}\hat{e}_y\right) = -2\hat{e}_x + 4\sqrt{6}e_y \, N$



5

$$r_{Q/O} = a\hat{e}_1$$

$$r_{P/Q} = \rho\cos\theta\,\hat{e}_1 + \rho\sin\theta\,\hat{e}_2$$

$$\theta = \frac{s}{\rho}$$

$$\rightarrow r_{P/Q} = \rho\cos\left(\frac{s}{\rho}\right)\hat{e}_1 + \rho\sin\left(\frac{s}{\rho}\right)\hat{e}_2$$

$$r_{P/O} = r_{Q/O} + r_{P/Q} = \left(a + \rho\cos\left(\frac{s}{\rho}\right)\right)\hat{e}_1 + \left(\rho\sin\left(\frac{s}{\rho}\right)\right)\hat{e}_2$$

$$\rightarrow \boxed{x = a + \rho\cos\left(\frac{s}{\rho}\right), \quad y = \rho\sin\left(\frac{s}{\rho}\right)}$$