

Electromechanical Transduction Analysis

(Robert Brown)

Note: I was part of a team that studied a small quadcopter. The work below was my portion of a larger final report.

A main component of the quadcopter system is the brushed dc motors that actuate the propellers to move the plane around. These take input voltage from the motors and turn it into mechanical movement, hence the electromechanical transduction name. Another important aspect of this system is that Hadas Ritz has a bunch of videos where she analyzes a system very similar to this. The first part of this analysis is heavily based on those videos. I believe it's important to acknowledge this, not only to avoid plagiarism but also to emphasize that most of the system dynamics knowledge applied is the understanding of how her model applies (and where it falls short).

This system is characterized by the two equations shown below. One is for the circuit in the motor, which includes damping and inertia via a resistor and inductor. The second is for the physical mechanics of the system. These are linked due to motor torque and back emf.

$$V_a - i_a R_a - \frac{di_a}{dt} L_a - k_b \omega = 0$$
$$I_m \omega' - k_T i_a + T_{dist} + \omega c = 0$$

Equation 1.1 (Courtesy of Hadas Ritz video)

The main analysis I did here was acknowledging how this model's assumptions apply to our system. The biggest assumption here is the ωc term in the second equation. This represents a mechanical torque due to air resistance. There are a couple big assumptions here, the biggest of all is linear air drag. This really only applies to slow moving laminar flow, but we need to assume this in order to keep our differential equations linear. There are ways to analyze non-linear differential equations, which could be an interesting thing to look into, but I decided not to dive into that.

The above equations can be used to create a State Space model (see appendix). I didn't think that analyzing the state space model would be too helpful, though. Instead I used the transfer function between angular velocity and input voltage:

$$\frac{\omega(s)}{V_a(s)} = \frac{k_t}{I_m L_a s^2 + (R_a I_a + L_a c)s + R_a c + k_t k_b}$$

Equation 1.2 (also from video)

My main interest with this transfer function was plugging in estimates for all the parameters and seeing if our measurements aligned with the model. It's pretty hard to find data on tiny brushed dc hobby motors, so I found a whole spec sheet of a large brushed motor from Moog. This might not be too applicable, as there are a lot of electrical terms like "back emf" that I don't understand fully. I also used pretty coarse estimates for moment of inertia and drag constant:

$$b = 6\pi\eta r$$

Equation 1.3 Linear Drag Coefficient for Sphere in Laminar Flow

The links to the Moog DC Motor spec sheet used for constants and inertia estimate can be found in the appendix. Note that this drag coefficient is not relevant at all to this system, but I just wanted a way to determine an order of magnitude.

I then used all the parameters I found to plug in numbers to the transfer function above. I found that some terms heavily dominated the system, and where two terms were added of more than 2 orders of magnitude difference, I dropped the smaller term.

$$\frac{\omega(s)}{V_a(s)} = \frac{2.4E - 2}{(8.75E - 11)s^2 + (1.5E - 7)s + 4.56E - 4}$$

Equation 1.4 Transfer function with numerical values

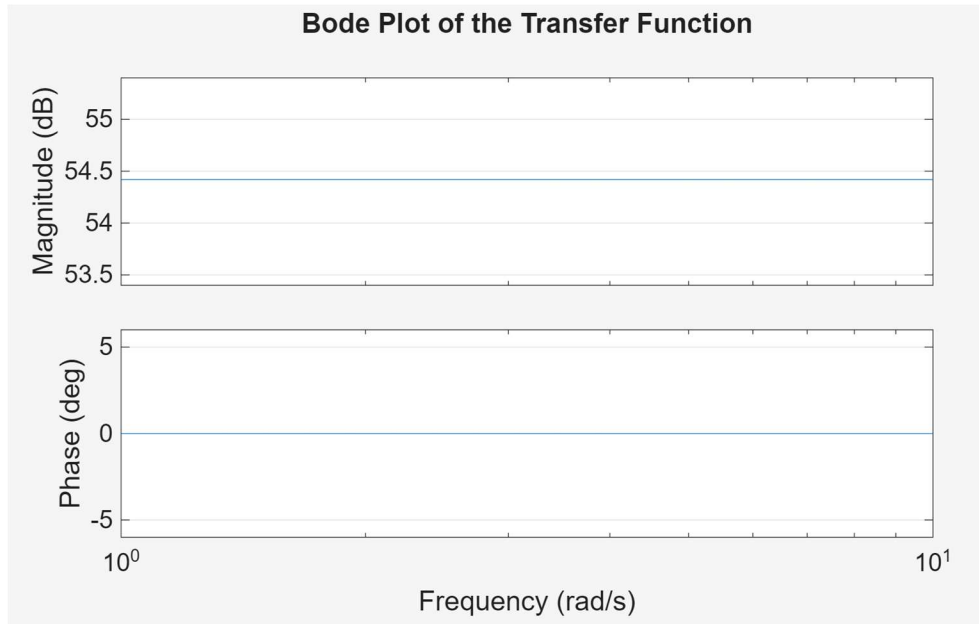
Looking at this transfer function, we can see in the denominator that the coefficients on the s terms are 3 or more orders of magnitude smaller than the constant term. This means that the system has largely just a constant gain Bode plot for 2 reasons:

- 1) The drone input voltage will almost always be constant with $s = 0$
- 2) Even if there was a frequency input, the terms with the non-zero s are dominated heavily by the constant gain.

For these two reasons, we can estimate the transfer function from input voltage (Volts) to angular velocity (rad/s) as a constant gain:

$$\frac{\omega(s)}{V_a(s)} = 526$$

Since my original goal was to make a Bode plot, this was a bit disappointing, but it does make intuitive sense. There really shouldn't be any large poles or shape to the bode plot between voltage and angular velocity.



Plot 1.1 Not very interesting but it makes physical sense.

So, does this align with data we took? We can verify it with the transfer function. Note that our data collection with the high speed camera indicated that the propellers were spinning at 628 rpm. This was a great practice in data collection, but most drone quadcopter operate around 8000-9000 rpm so I'm going to just estimate 8000 rpm.

$$\omega(s) = 7000 \text{ rpm} = 837 \text{ rad/s} = 526 * V(s) = 526 * 6.35 = 3340$$

Equation 1.5: Validation

We are less than an order of magnitude off, which I think is actually very successful. Almost all parts of this analysis were huge estimates, from looking at a much bigger motor to using linear air drag.

Appendix

Link to estimates used in electromechanical transduction:

<https://www.moog.com/literature/MCG/moc23series.pdf>

<https://engineering.stackexchange.com/questions/49367/how-to-calculate-the-moment-of-inertia-of-a-propeller>