Lecture 8: Linear Classifiers and More Model Validation

INFO 1998: Introduction to Machine Learning

CDS Education
We explore, learn, and educate big minds.

Agenda

1. Perceptron + SVM
2. More Cross-Validation techniques

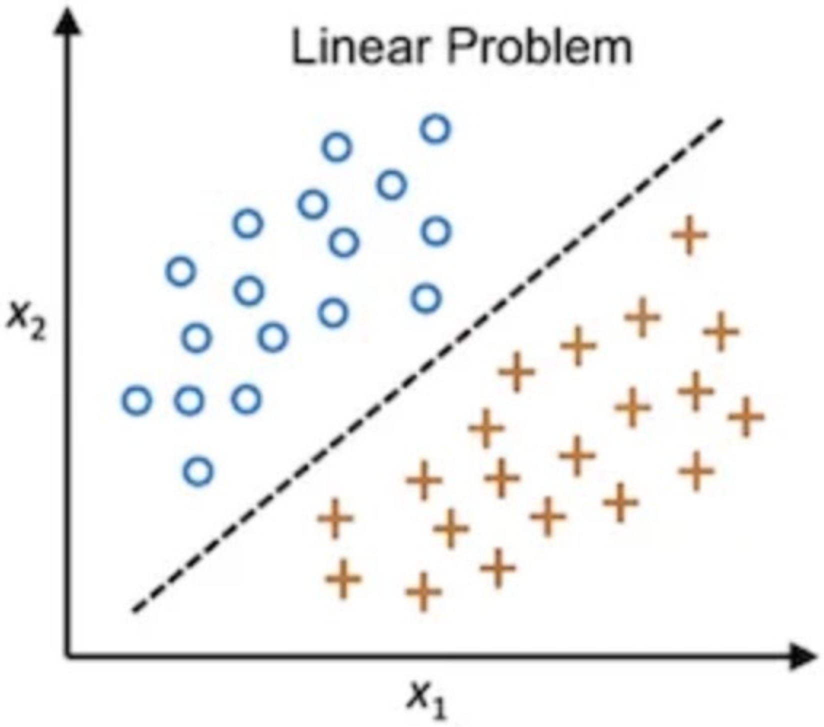
Linear Classifiers

Source

Linear Classifiers

A linear classifier is a hyper plane that is used to classify our data points

A hyperplane is our **decision boundary** and our goal is to find the hyper plane that best classifies our data



Perceptron Learning Algorithm

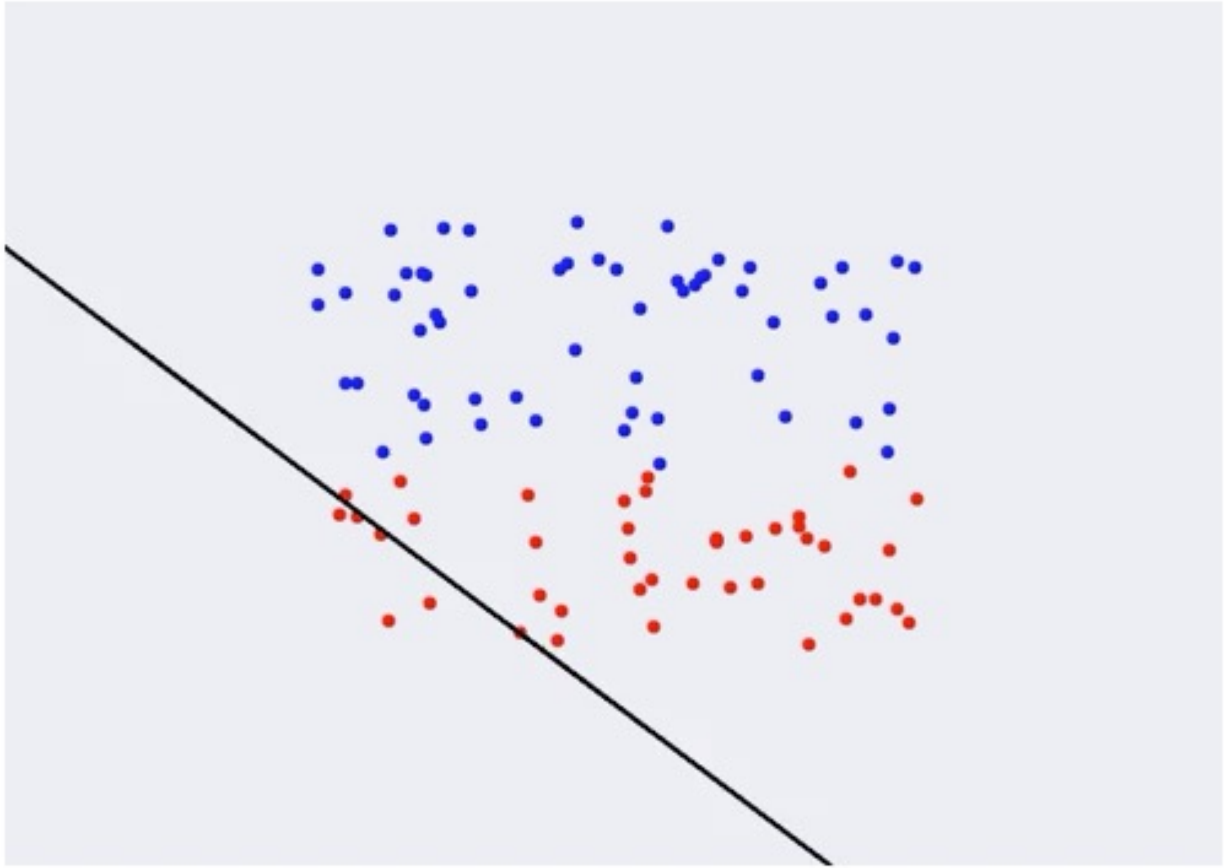
Goal: find a normal vector w that perfectly classifies all the points in our data set

Algorithm:

Initialize classifier as some random hyperplane
While there exists a misclassified point x:
    Tilt classifier slightly so that it classifies x correctly
    (or, is a little closer to classifying x correctly)
End While

“Use your mistakes as your stepping stones”

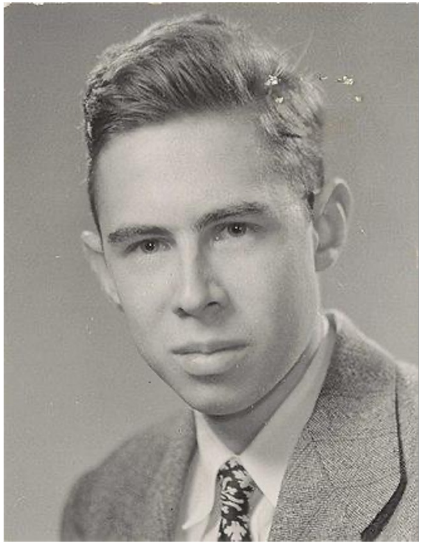
#0



Also, Frank Rosenblatt was first to implement perceptron

Gave him the title of ‘Father of Deep Learning’

He went to Cornell!!!



Limitations of Perceptron

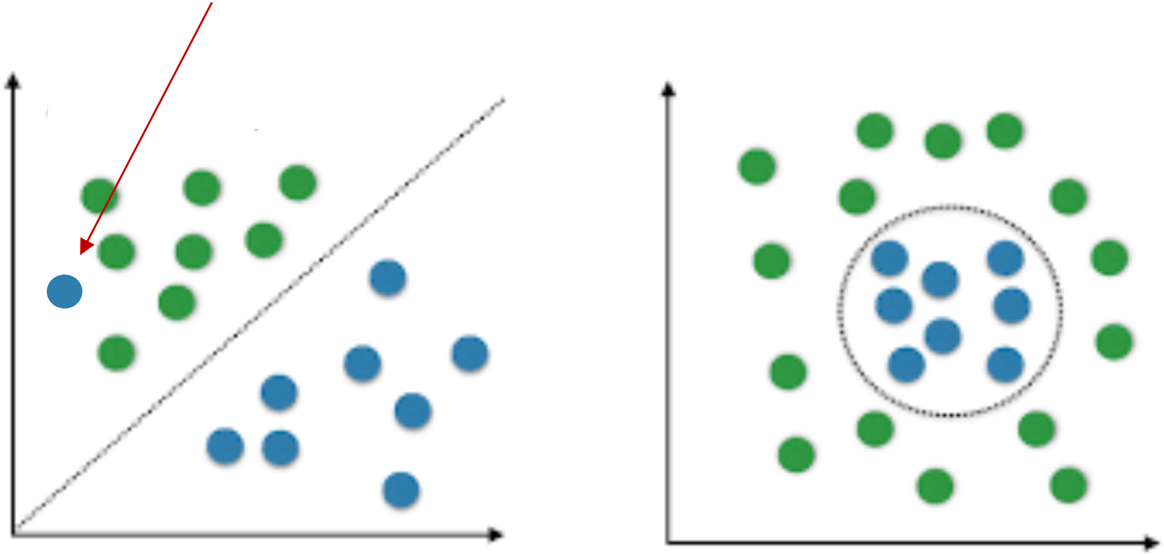
Is a great model to understand the intuition behind the training of a linear classifier: iteratively improve classifier by using misclassified points ☺

The training algorithm will never terminate if your training dataset is not linearly separable ☹️

Source

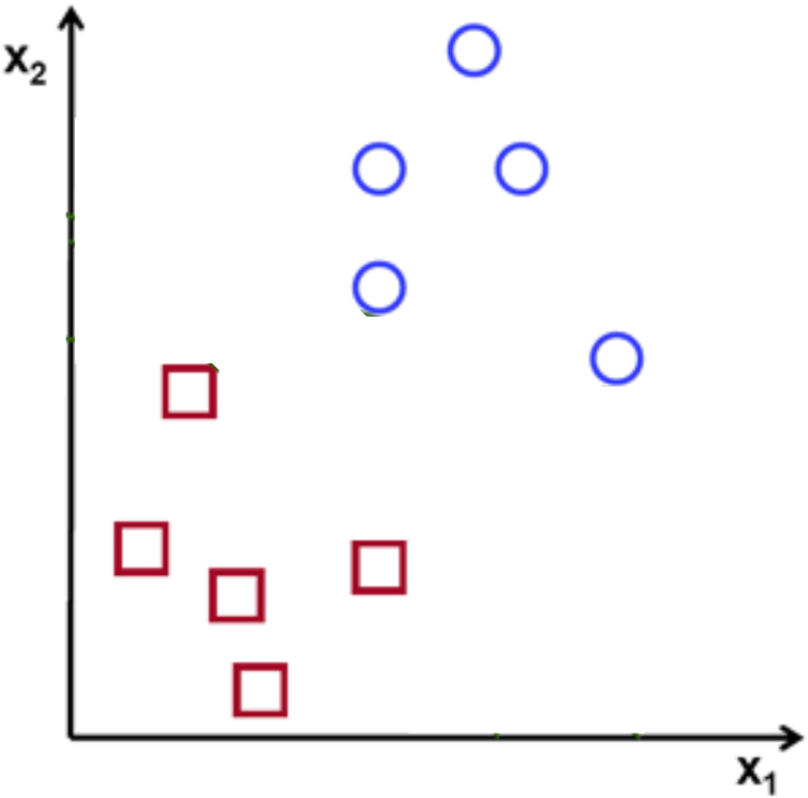
Not Linearly Separable

This data set is not linearly separable because of an outlier

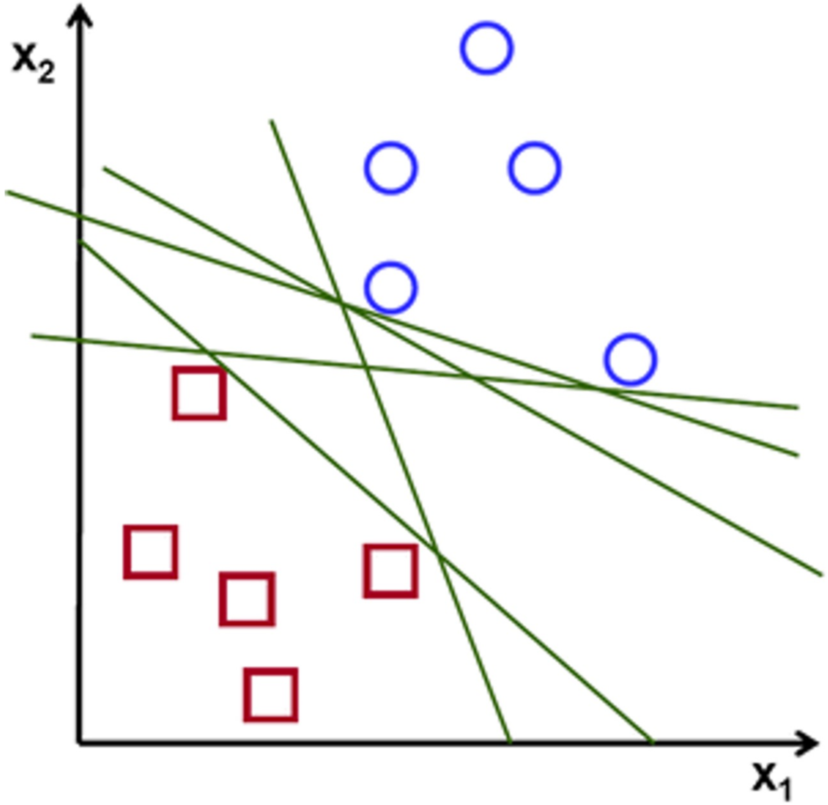


SVM

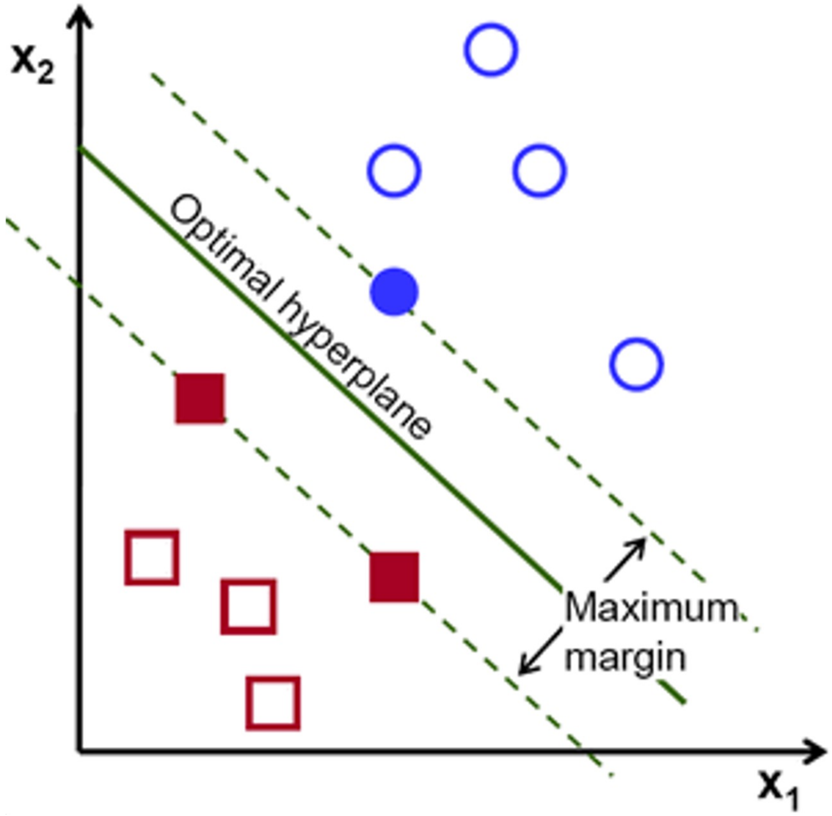
Classify (+) and (-)



Which Hyperplane?

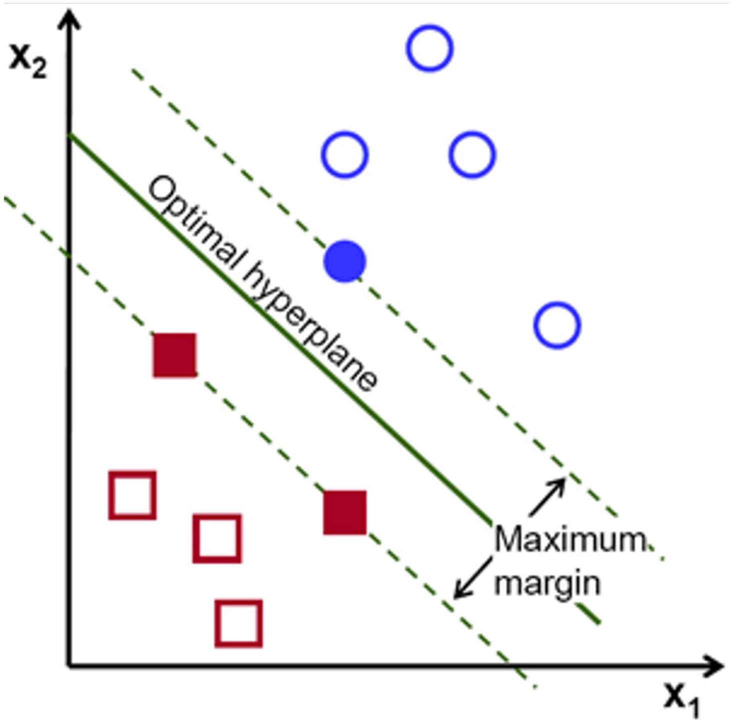


Optimal Hyperplane

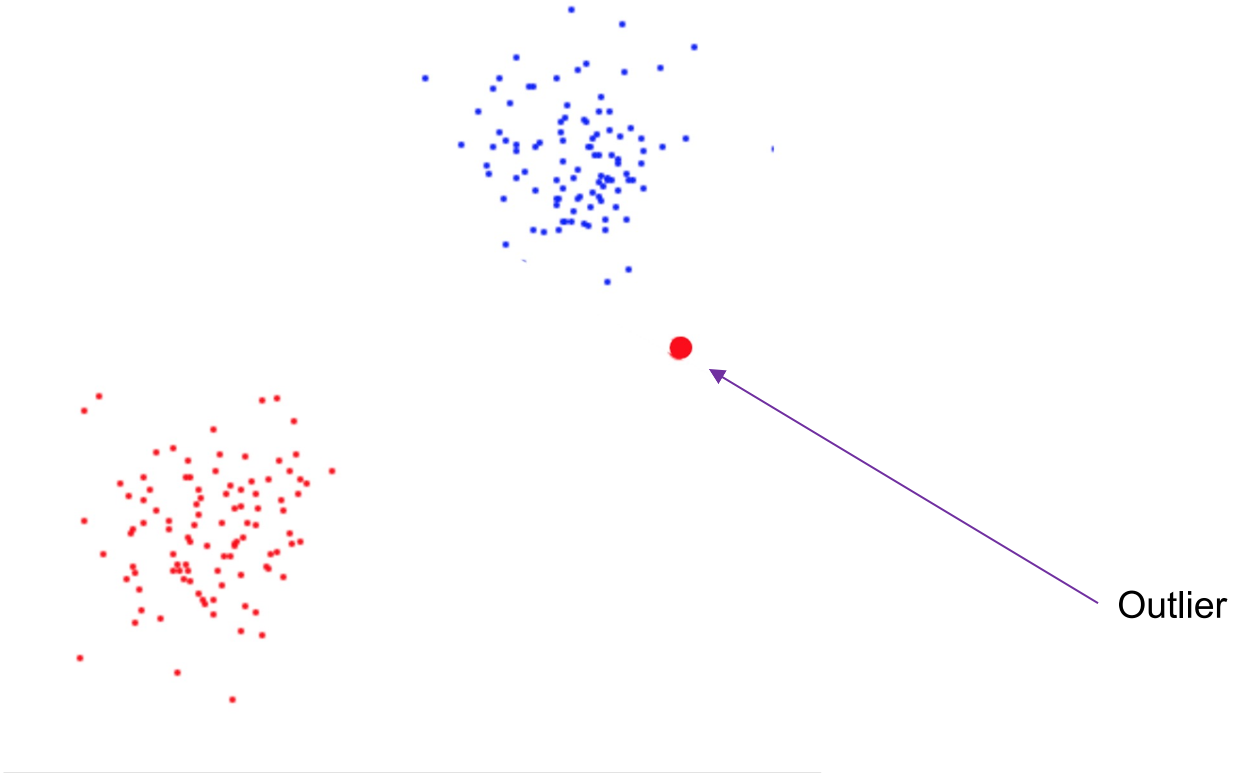


Maximal Margin Classifier

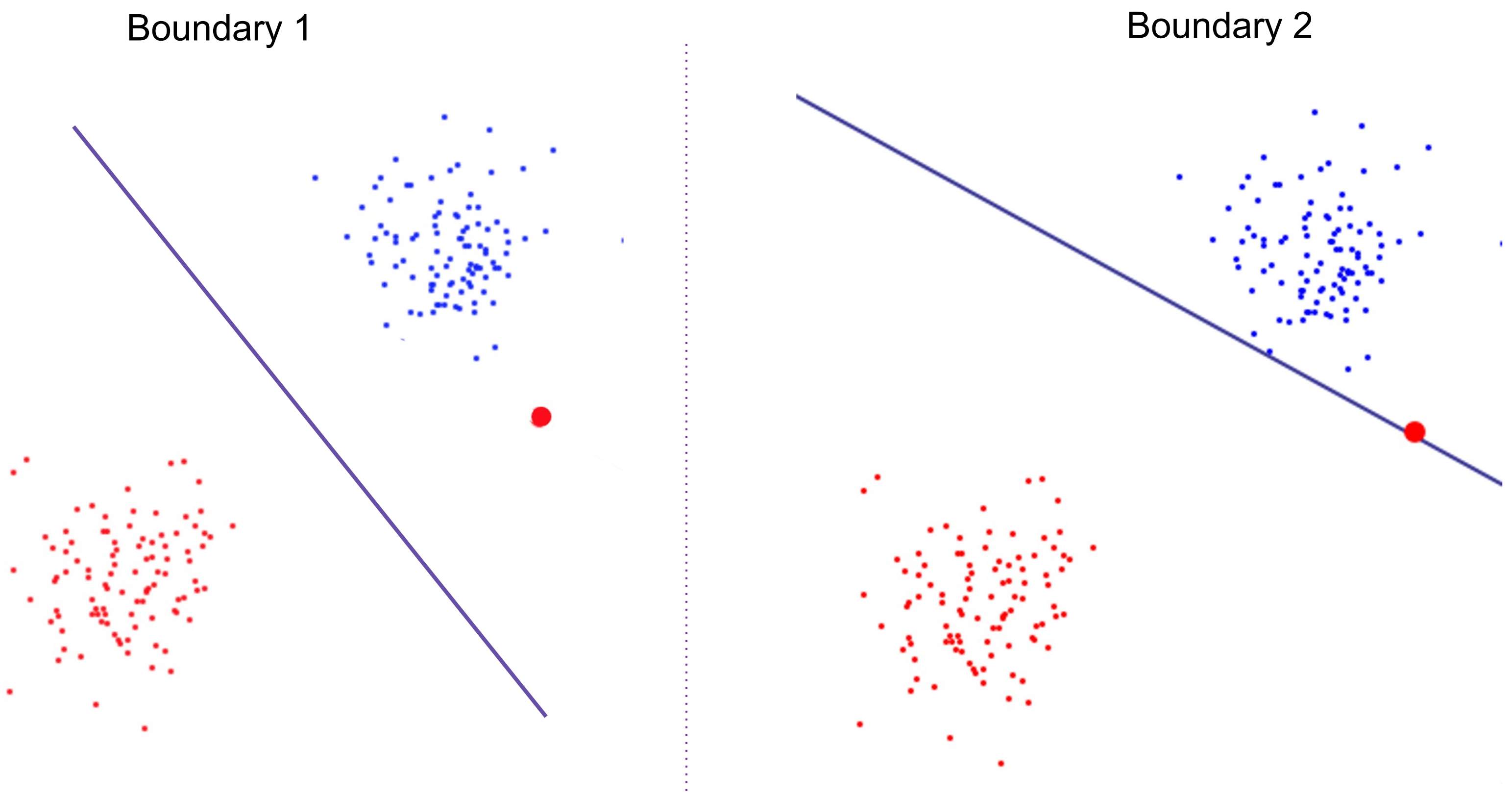
- We want to find a **separating hyperplane**
- Once we find candidates for the hyperplane, we try to maximize the **margin**, the normal distance from borderline points
  - Only **Support Vectors** matter



What if…



Which Decision Boundary is better?



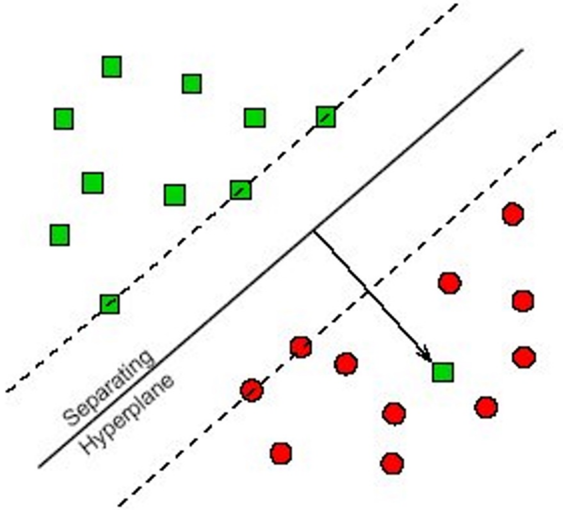
Margins

Use cost function to penalize misclassified points

Choice of cost function makes margin “hard” vs. “soft”

Non-separable training sets

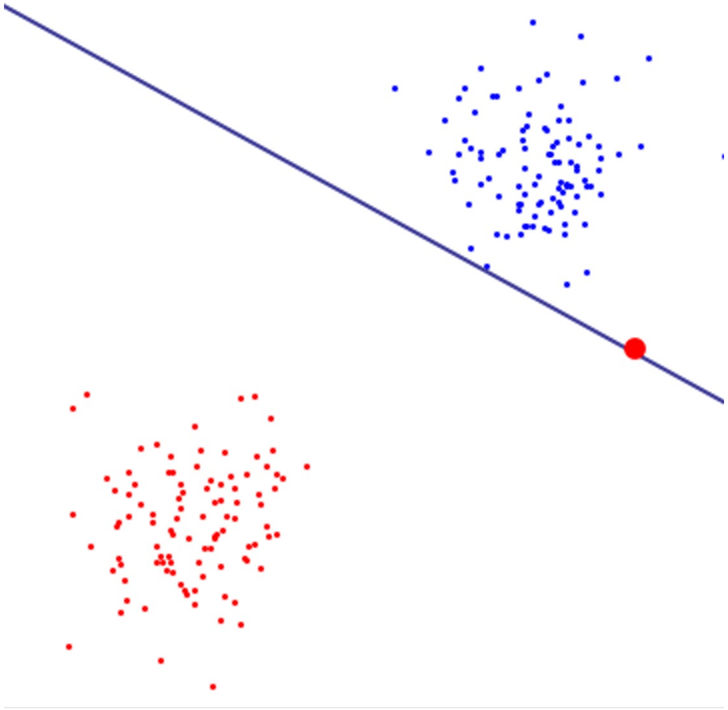
Use linear separation, but admit training errors.



Penalty of error: distance to hyperplane multiplied by *error cost* C.

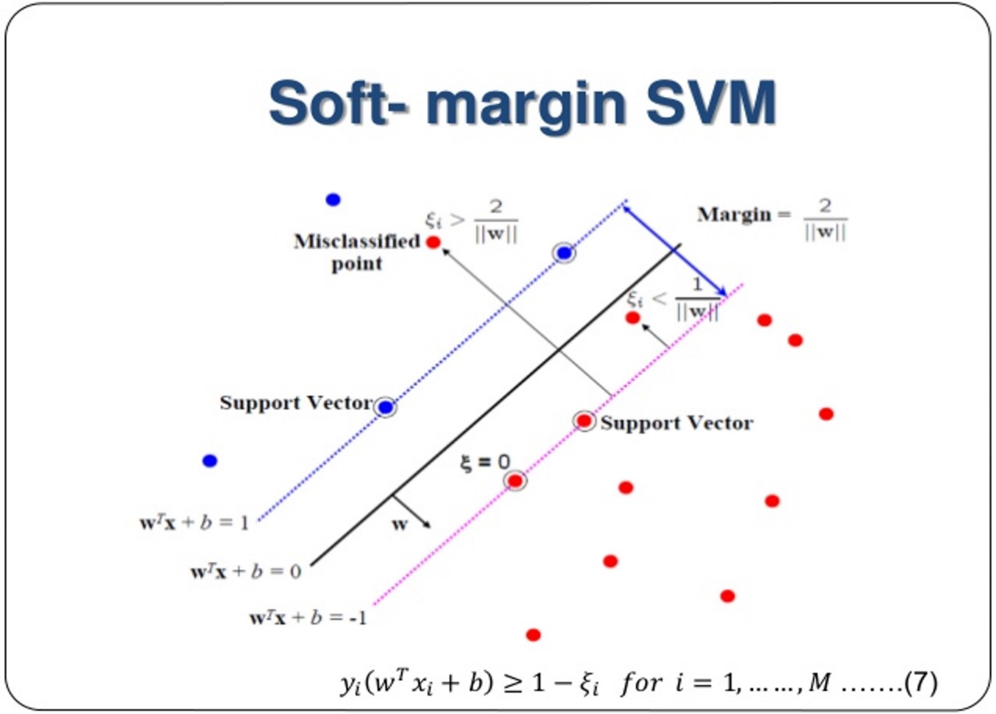
Hard Margins

- High penalty value
- The hyperplane can be dictated by a single outlier

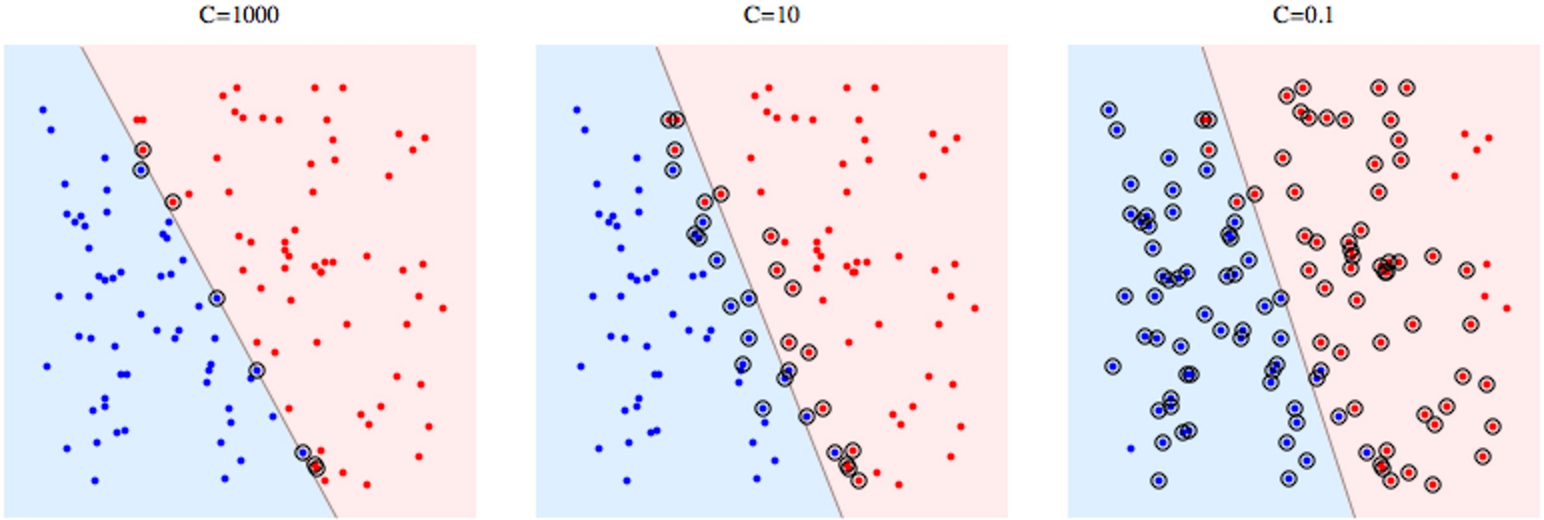


Soft Margins

- Used in non-linearly separable datasets
- Allow for misclassification
- Can account for “dirty” boundaries

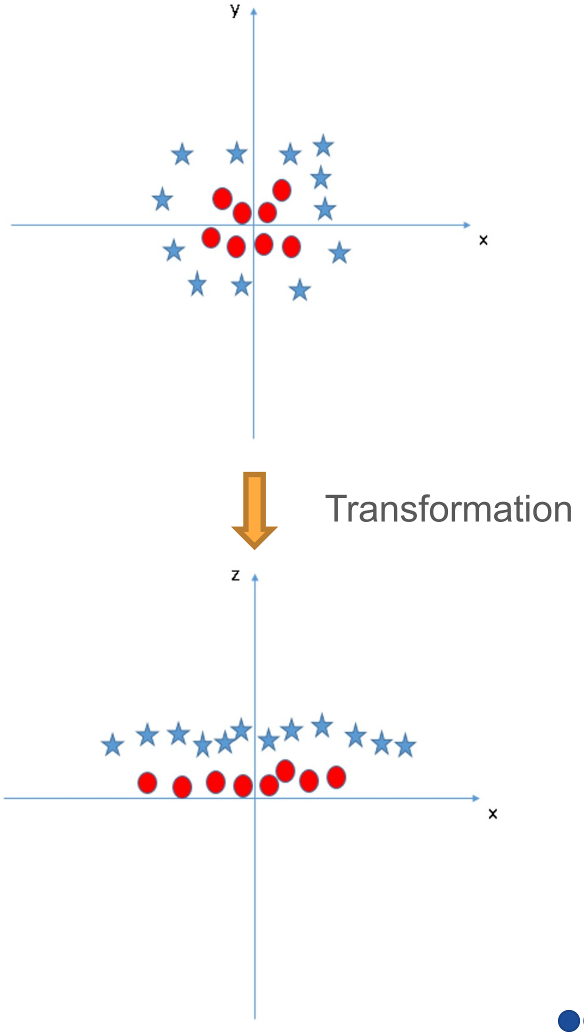


Misclassification Penalty C

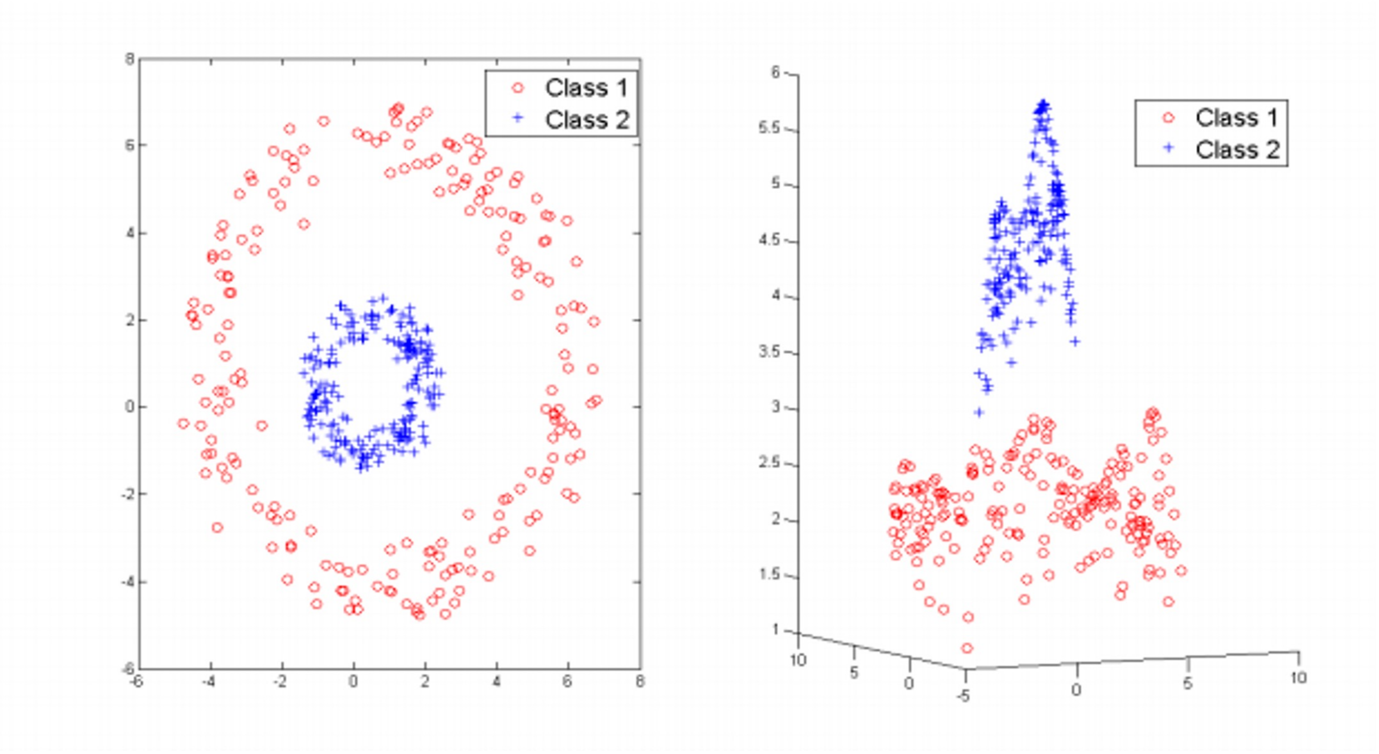


Kernels

- You cannot linearly divide the 2 classes on the *xy* plane at right
- Introduce new feature, $z = x^2 + y^2$ (**radial kernel**)
- Map 2 dimensional data onto 3 dimensional data. Now a hyperplane is easy to find. (Imagine slicing a cone!)



Kernels



SVM has MANY Hyperparameters

SVM

C	Gamma	Kernels
The “penalty cost” for misclassifications (soft margins)	How far the influence of a single training example reaches	Method of transforming our data set

Finding the Best Hyper Parameters

Grid Search

Random Search

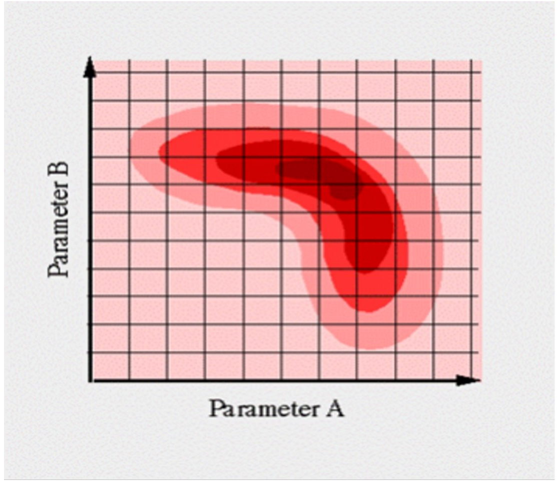
Optimize

Bayesian Optimization

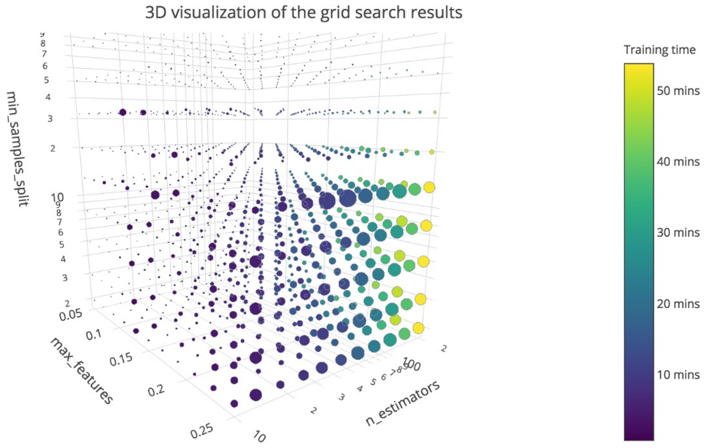
source

Curse of Dimensionality

Our search space for the optimal hyper-parameters increases **exponentially** as the number of hyper parameters we are considering increases



Add dimension



Overview

Perceptron	SVM
• A very simple model • Will perform poorly if data is not linearly separable	• More complex model because we have to choose the "penalty cost" associated with misclassifications • Can transform feature space by choosing a Kernel

Demo

More Validation Techniques

Leave-P-out Cross Validation (LpO CV)

Let **D** be our whole dataset

Choose a **P**

For every combination of **P** points in **D**:

Use a train/test split with those **P** points as test, the rest as train

Leave-P-out: different from K-fold!

Let's say **D** has a size of 4. There are four data points: *a*, *b*, *c*, and *d*.

K-fold:

- K = 2.
- Each fold has a size of 2: {*a*,*b*} and {*c*,*d*}
- So, we only have 2 possible test sets:

  {*a*,*b*} and {*c*,*d*}

Leave-P-out:

- P = 2.
- We have 6 possible test sets:

  {*a*,*b*}, {*a*,*c*}, {*a*,*d*}, {*b*,*c*}, {*b*,*d*}, and {*c*,*d*}

Leave-P-out Cross Validation (LpO CV)

Pros:

- more fine-grained estimate than k-fold
- test the model's generalization ability

Cons:

- Slow!
  - Runtime increases with larger datasets
  - Runtime explodes with larger P

Monte Carlo Cross Validation

- Need to use **new**, **random** train/test split each time
  - If you use the same train/test split each time, you're not getting any new information!
- Pros:
  - easy to implement
  - easy to make faster/slower by changing number of iterations
- Cons:
  - random -> train/test splits not guaranteed to be representative of dataset
  - harder to calculate how many iterations you need

The Bootstrap

What if we don’t have enough data?

- Use **bootstrap datasets** to approximate the test error
- **Sample with replacement** from the original training dataset (with n samples) to generate **bootstrap datasets** of size n
  - Some data points may appear more than once in the generated data
  - Some data points may not appear
- Estimate of test error = average error among bootstrap datasets

Demo

Coming Up

- **Assignment 8**: Due midnight of next class
- **Next Lecture**: Applications of *Un*supervised Learning

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